Corralled Active Brownian Particles: Whirligig Beetles show a density dependent speed with MIPS-like, co-existing high and low density phases

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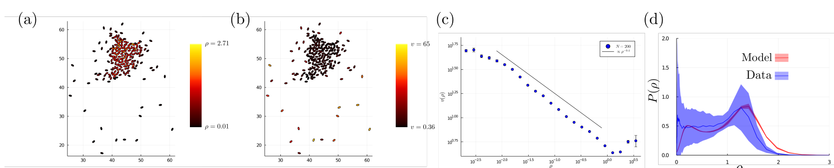

Fig. 1. (a) A snapshot of the positions and orientations of $N = 200$ beetles with individuals shown on a colour scale to indicate local density ρ, in units of per beetle length squared. Each individual is plotted as an ellipse with aspect ratio 2:1. Lengths on the axes are scaled by the average beetle length 12mm. (b) The same snapshot as in (a) but with colour scale showing the beetles current self propulsion speed in units of beetle lengths per second. Speeds are computed using a central difference of neighbouring video frames. The video frame-rate is $\frac{1}{30}$ s. (c) The self-propulsion speed varies with local density and seems to follow a power-law dependence over two orders of magnitude in density. The solid line indicates a dependence of $\rho^{-0.4}$, as a guide to the eye. (d) CABP model fitted to the density distribution we observe in the $N = 200$ beetle group, the model is shown in red and the data in blue, both with error ribbons of one standard deviation.

1 Introduction

Aggregation is a generic phenomenon seen in systems of active matter across many length scales. In bird flocks, e.g. Starlings [1], up to hundreds of thousands of individuals come together, exploring complex three-dimensional patterns. Bacterial systems [2], granular systems [3], colloids [4] and robotic groups [5] also show characteristic swarm behaviour. Recently there has been significant interest in the emergence of co-existing high and low density phases. Such coexistence can arise in the presence of attractive interactions between particles, such as chemical signalling (bacteria) or physical forces (colloids). This is fairly trivial insofar as there is inter-particle attraction. Most strikingly, however, such phase coexistence can also arise in the presence of purely repulsive interactions (e.g steric contact forces) [6, 7]. This is known as motility induced phase separation (MIPS) and is a relatively generic feature of systems of purely repulsive self-propelled particles (SPPs). Inertia in active particle systems has been show to suppress MIPS while promoting a different phase coexistence between low and high density phases with a signature difference in kinetic energies [8, 9], but are only beginning to be understood, with experimental realisations limited to vibrated systems of self-propelled robots [5].

Here we report experiments on group ($N = 50, 100, 200$) dynamics of *Dineutus discolor (Coleoptera: Gyrinidae)* "Whirligig beetles" swimming on the surface of water in a circular container without external stimulus. We see evidence of self-propulsion speed which is dependent on local density by a power law $v(\rho) \sim \rho^{-\nu}$ where $\nu \approx 0.4$, a coexistence of high and low density regions (accompanied by low and high speeds respectively), and a minor inertial delay between body axis orientation and velocity of approximately 13 ms. The speed density relationship and coexistence of high and low density regions are consistent with MIPS, moreover the explicit measurement of self propulsion speed dependent on local density is to our knowledge not widely reported in other living systems. Liu *et al* [10] is a notable exception. We measure the local density using a new method based upon the Delaunay Triangulation. To understand these experimental observations we propose a new model of Corralled Active Brownian Particles (CABPs) that modifies a standard Active Brownian Particle model, known to exhibit MIPS [6], so that it functions in unbounded 2D space. We achieve this by incorporating a torque on each individual particle's orientation dynamics that depends on the local density. This torque tends to rotate particles towards the geometric centre of the group and depends on an inverse power of density, having no explicit dependence on the particle's distance from the geometric centre.

2 Corralled Active Brownian Particles

Our CABP model in 2D is given by the following equations of motion for the location $\boldsymbol{r}_i$ and orientation $\hat{\boldsymbol{n}}_i(t) = [\cos\theta_i(t), \sin\theta_i(t)]^T$ (body axis) of particle i at time t, with the hat $(\hat{})$ indicating a unit vector.

$$\partial_t \boldsymbol{r}_i = v_0 \hat{\boldsymbol{n}}_i + \mu \sum_{j \neq i} \boldsymbol{F}_{ij} \tag{1}$$

$$\partial_t \theta_i = \eta_i(t) + \tau \rho_i(t)^{-\alpha} (\hat{\boldsymbol{R}}_i \times \hat{\boldsymbol{V}}_i) \cdot \hat{\mathbf{z}} \tag{2}$$

Particles are treated as circular disks with radius a, v_0 a constant self-propulsion speed, $\boldsymbol{F}_{ij}$ is a repulsive contact force acting between all pairs of particles given by $\boldsymbol{F}_{ij} = k(2a - r_{ij})\hat{\boldsymbol{r}}_{ij}$ when the inter particle distance $r_{ij} = ||\boldsymbol{r}_i - \boldsymbol{r}_j||$ satisfies $r_{ij} < 2a$, i.e. there is contact, and $\boldsymbol{F}_{ij} = 0$ otherwise. The total force on the i^{th} particle is then $\boldsymbol{F}_i = \sum_{j \neq i} \boldsymbol{F}_{ij}$, effectively a sum over contact forces. The mobility coefficient μ can be combined with the harmonic force spring constant k as μk and only appears in this combination. For the rotational dynamics $\eta_i(t)$ is Gaussian white noise with zero mean and correlations $\langle \eta_i(t)\eta_j(t') \rangle = 2D_r\delta_{ij}\delta(t - t')$ for rotational diffusion constant D_r. The torque term is modelled by a prefactor τ, controlling overall strength, and inverse exponent $\alpha > 0$. Here $\boldsymbol{R}_i = \boldsymbol{r}_i - \langle \boldsymbol{r}_j \rangle_j$ is the vector pointing from the swarms geometric centre to particle i and $\boldsymbol{V}_i$ is particle i's instantaneous velocity. Finally the dot product with $\hat{\boldsymbol{z}}$, a vector pointing out of the plane of motion, converts the cross product to a signed scalar, positive for anticlockwise turns and negative for clockwise. To evaluate our model we first extract the rotational diffusion coefficient D_r and the mean collision free speed v_0 from our trajectory data. Then we fit the parameters α, τ, and μk by minimising the mean square error of the simulation ($N = 200$) density distribution with the $N = 200$ experimental data. The optimal fit parameters are recorded in Table 1. Using the fitted parameters from Table 1 we can compare the model's predictions with the data, see Fig 1. This figure shows a typical snapshot from the data, with the local density (a) and velocity (b) shown on a colour scale. Note the presence of a “corona” of low density, fast moving particles co-existing with a “core” of high density, slowly moving particles. This is highly reminiscent of MIPS. We argue that this is natural, given that the beetles in the corona systematically turn back towards the core so that the flock doesn't dissipate, even if on an arbitrarily large body of water. This effect is provided in our model by the density dependent turning torque. Panel (c) shows that an analysis of the experimental data reveals a robust power law dependence of the velocity with the local density that extends over two orders of magnitude in density. This result may be particularly useful: Continuum theories used to study MIPS often need to invoke a phenomenological velocity-density dependence [7]. Our experiments provide evidence for the use of a power law. Panel (d) shows a comparison between simulations using the CABP model and the experimental data. Note in both cases the presence of a high density phase (peak) coexisting with a lower density phase

Parameter	α	τ (s^{-1})	μk (s^{-1})	v_0 (s^{-1})	D_r ($\text{rad}^2 s^{-1}$)
Best fit value	1.1	19.6	316	13.19	2.34

Table 1. Best-fit values of the control parameters. These are inferred by fitting the results of simulations performed on $N = 200$ particles, using equations of motion 1 and 2, to the $N = 200$ beetle dataset. The fit metric is a least-squares measure of the density PDF. We include v_0 and D_r in this table but note that they are fitted to data *before* using Bayesian optimisation to find τ, α, and μk to reduce dimensionality. All lengths are measured in units of the average body length.

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