

Opinion Dynamics & Evolving Networks

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Opinion model

Opinion of individual i

Social network

Interaction function

$$\frac{dx_i}{dt} = \frac{1}{k_i(t)} \sum_{j=1}^N w_{ij} \phi(|x_j - x_i|) (x_j - x_i)$$

$$k_i(t) = \sum_{j=1}^N w_{ij}(t)$$

Degree of individual i

Opinion model

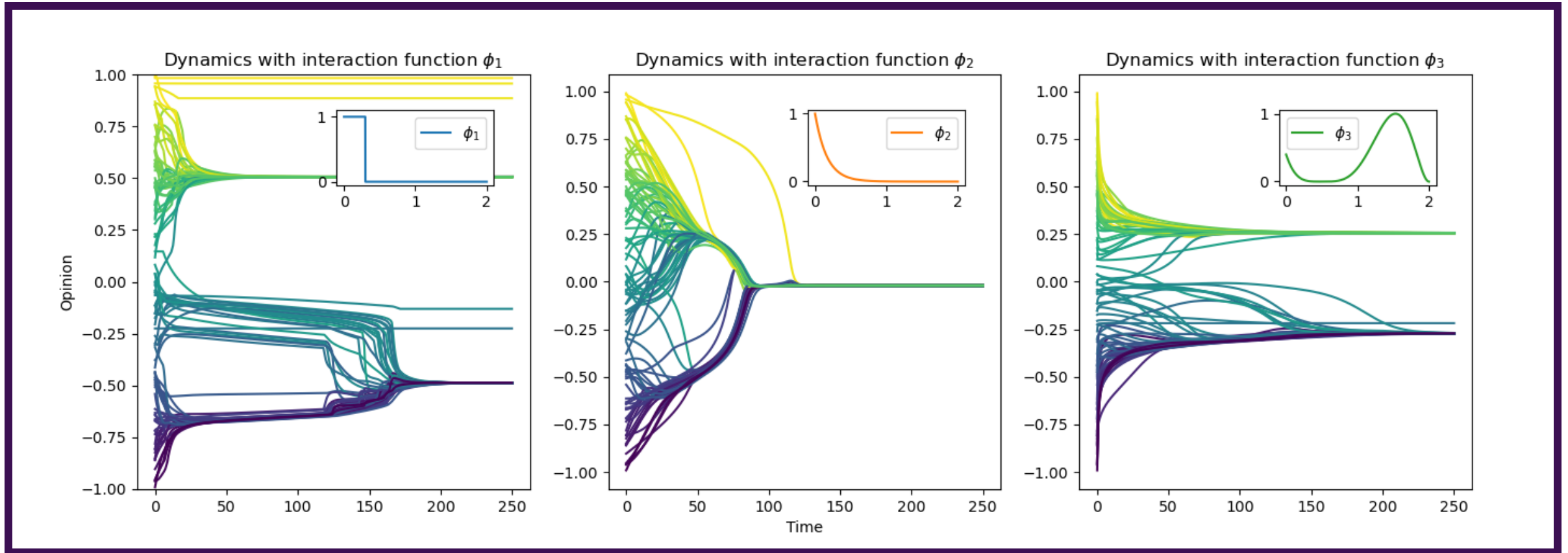


Figure: Example dynamics on an Erdos-Renyi random graph (N=100, p=0.1) with various interaction functions.

Opinion model

- The opinion diameter $D(t) = \max |x_i(t) - x_j(t)|$ is a non-increasing function, and therefore opinions always remain in the interval $[-1, 1]$.
- For all $\epsilon > 0$ there exists a time t^* such that for all i, j we have

$$w_{ij} \phi(|x_j(t^*) - x_i(t^*)|) |x_j(t^*) - x_i(t^*)|^2 < \epsilon.$$

Hence one of the **edge weight**, **interaction function**, or **distance between opinions** must become arbitrarily small*.

This means that if the network is connected, and the interaction function is bounded away from zero then consensus is guaranteed.

* Proof based off Motsch, Tadmor (2013)

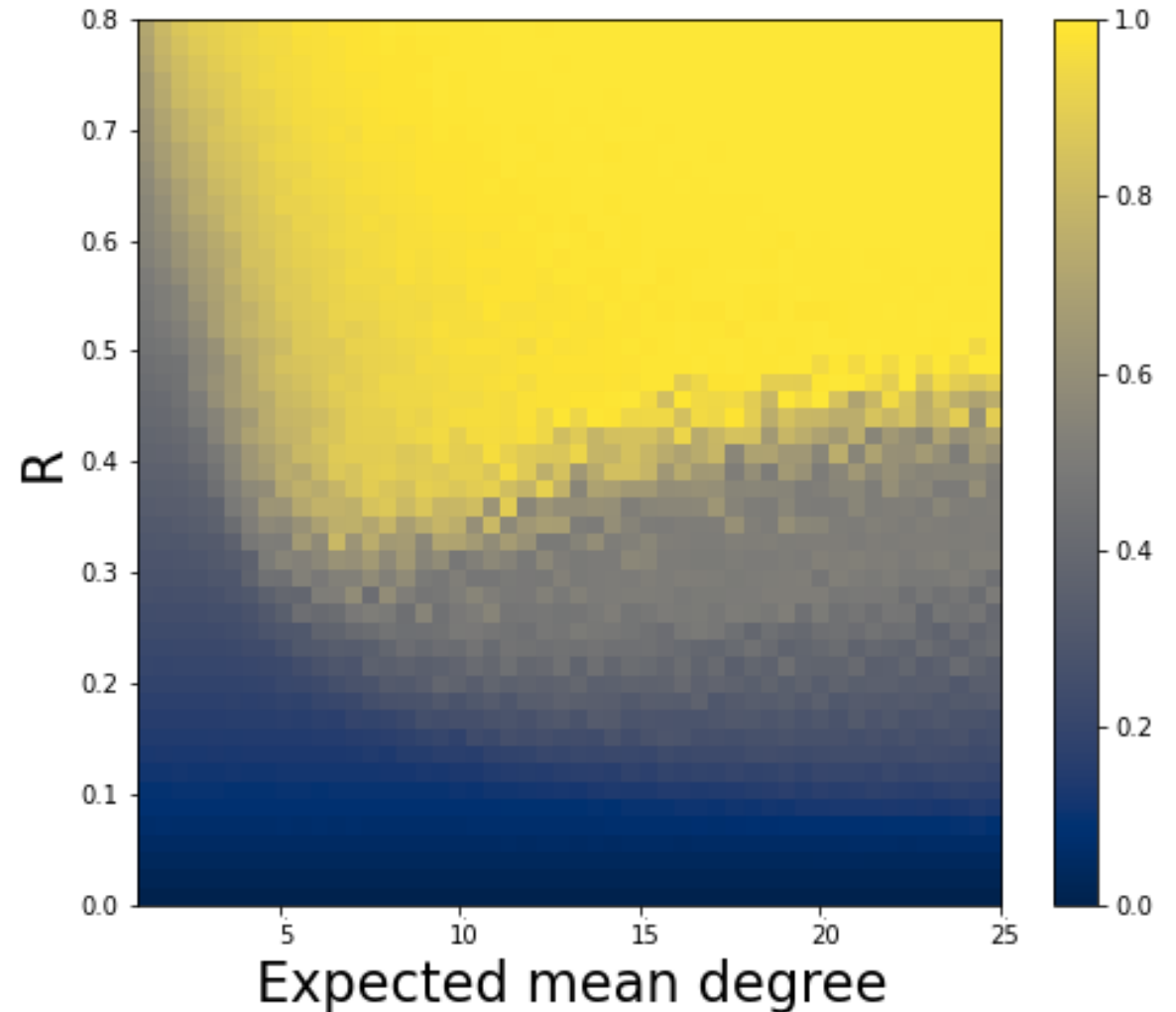
Opinion model

The opinion distribution can be quantified using an order parameter*

$$Q = \frac{1}{N^2} \sum_{i,j} \phi(|x_j - x_i|).$$

which takes values $Q \in [0,1]$, with Q^{-1} corresponding roughly to the number of clusters.

Figure: Erdos-Renyi graph with 500 agents, uniform initial distribution of opinions.



* Order parameter first introduced by Wang, Li, Chazelle et al. (2017)

Dynamic Network

$$\frac{dx_i}{dt} = \frac{1}{k_i(t)} \sum_{j \neq i} w_{ij} \phi(|x_i - x_j|) (x_j - x_i)$$

$$k_i = \sum_{j=1}^N w_{ij}$$

$$\frac{dw_{ij}}{dt} = \underbrace{\phi(|x_i - x_j|) f^+(w)_{ij}}_{\text{Edge creation}} - \underbrace{\left(1 - \phi(|x_i - x_j|)\right) f^-(w)_{ij}}_{\text{Edge removal}}$$

Edge creation

Edge removal

Dynamic Network

Logistic weight dynamics

$$\begin{aligned}\frac{dw_{ij}}{dt} &= \phi(|x_i - x_j|) w_{ij} (1 - w_{ij}) - (1 - \phi(|x_i - x_j|)) w_{ij} (1 - w_{ij}) \\ &= w_{ij} (1 - w_{ij}) (2\phi(|x_i - x_j|) - 1)\end{aligned}$$

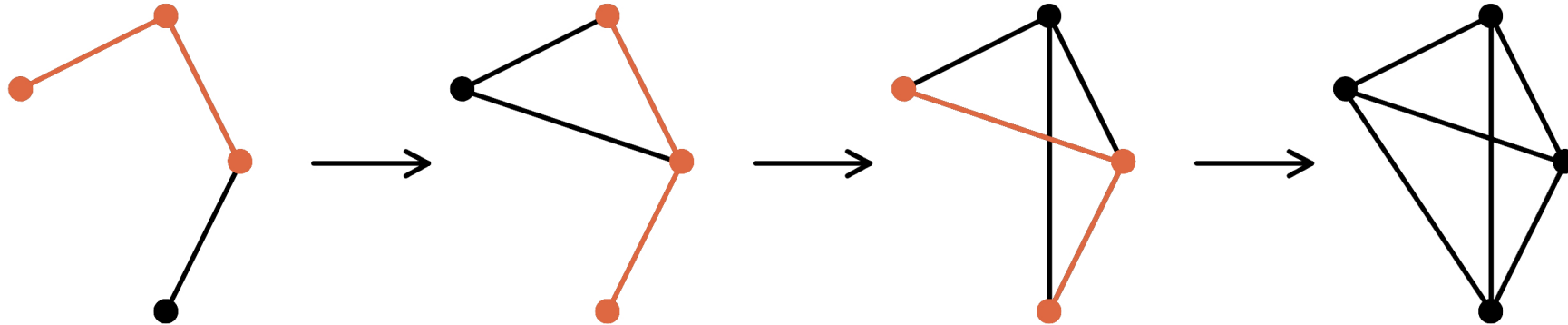
Dynamic Network

Friend-Of-A-Friend weight dynamics

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|) \left(w_{ij} + N^{-1} (w^2)_{ij} \right) (1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|) \right) w_{ij}$$

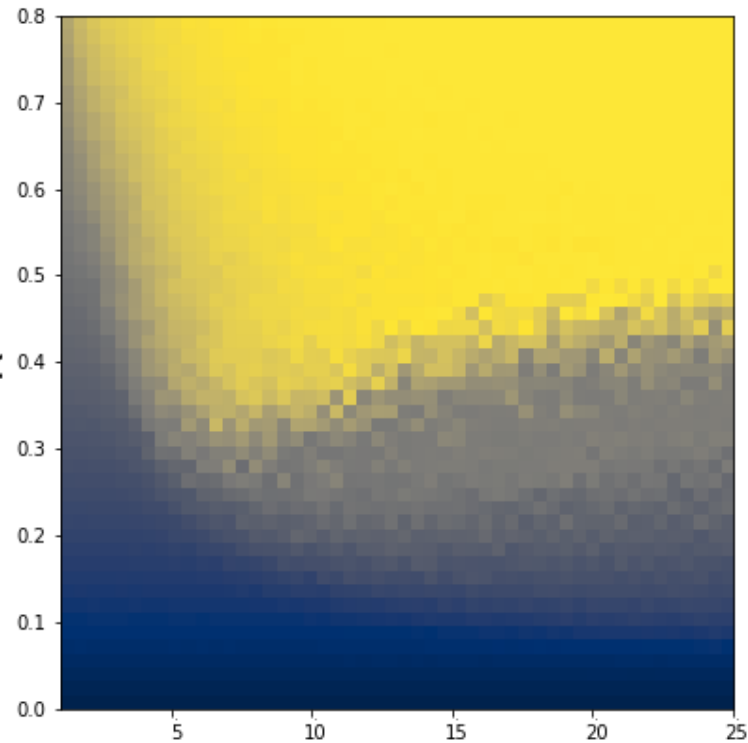
Immediate connections

Two-step connections

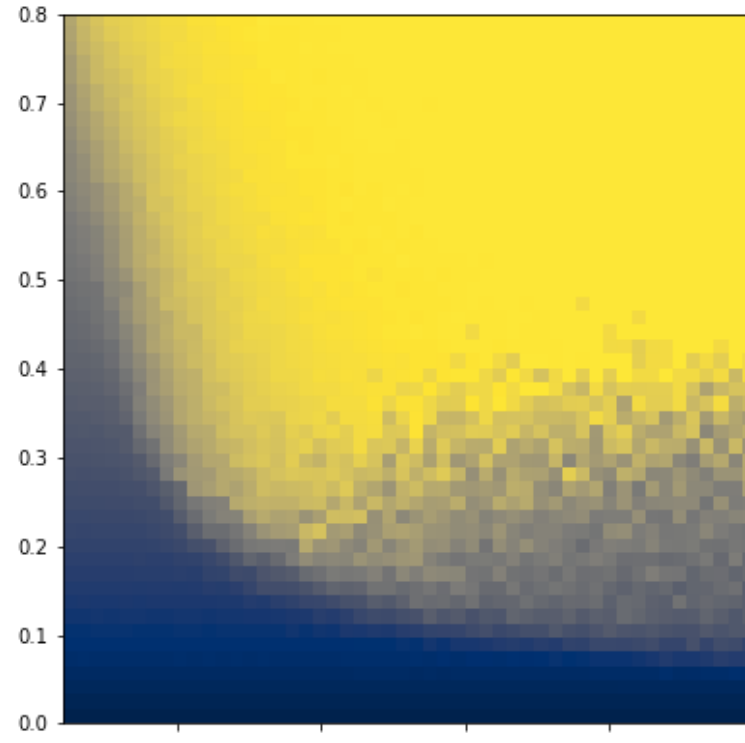


Dynamic Network

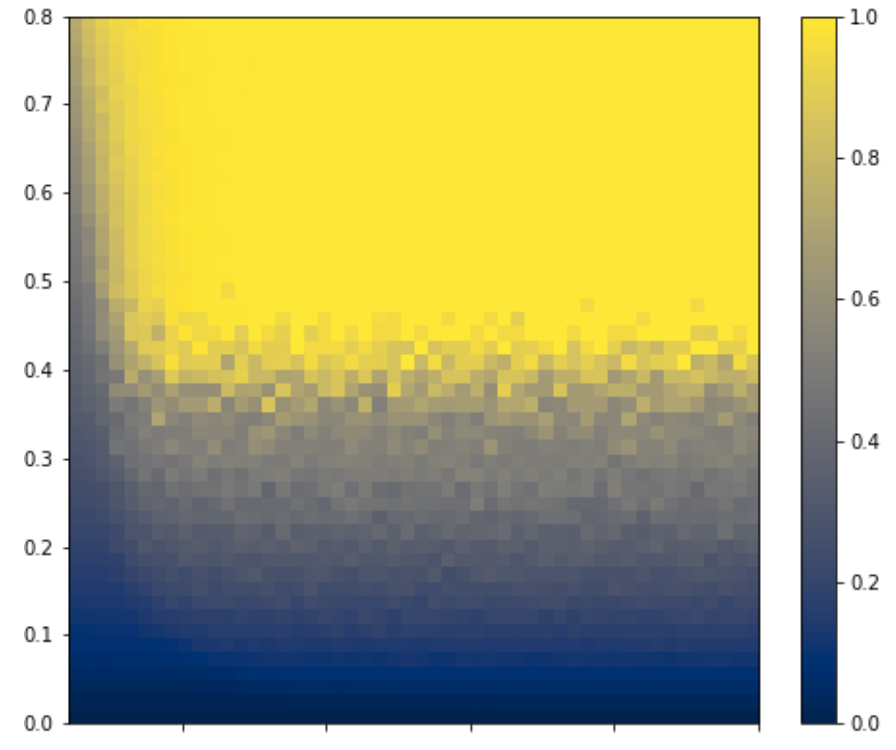
No weight dynamics



Logistic weight dynamics



FOAF weight dynamics



Expected mean degree (of initial network)

Network Control

$$\frac{dx_i}{dt} = \frac{1}{k_i(t)} \sum_{j=1}^N w_{ij} \phi(|x_j - x_i|) (x_j - x_i)$$

$$k_i(t) = \sum_{j=1}^N w_{ij}(t)$$

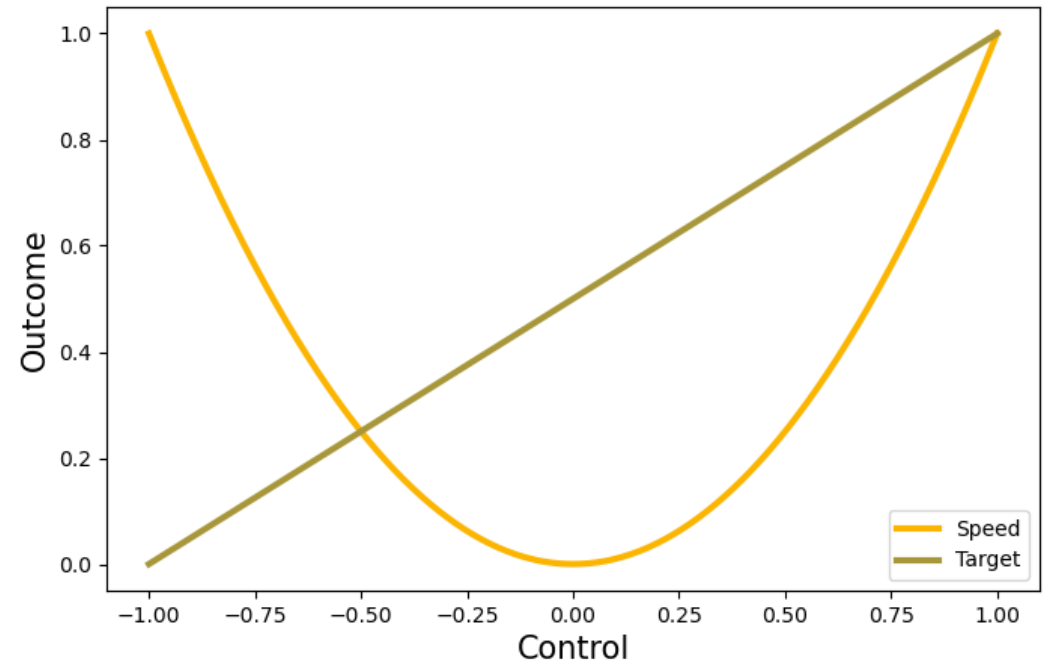
$$\frac{dw_{ij}}{dt} = \underbrace{s(u_{ij})}_{\text{yellow}} \left(\underbrace{\ell(u_{ij})}_{\text{green}} - w_{ij} \right)$$

Network Control

$$\frac{dx_i}{dt} = \frac{1}{k_i(t)} \sum_{j=1}^N w_{ij}$$

$$k_i(t) = \sum_{j=1}^N w_{ij}(t)$$

$$\frac{dw_{ij}}{dt} = \underbrace{s(u_{ij})}_{\text{orange}} \left(\underbrace{\ell(u_{ij})}_{\text{green}} - w_{ij} \right)$$

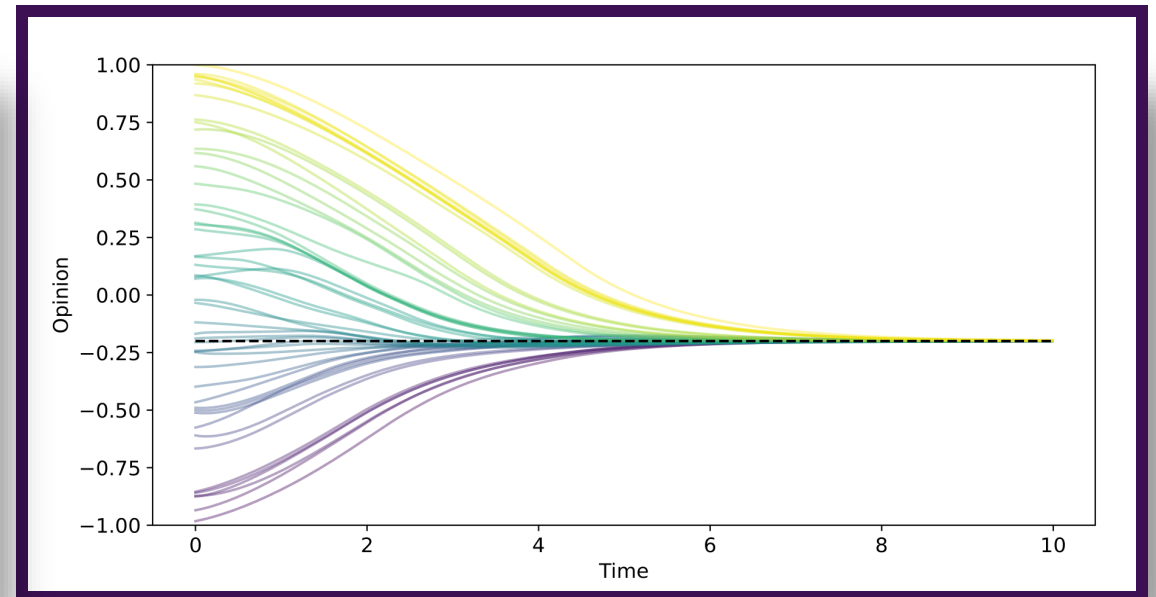
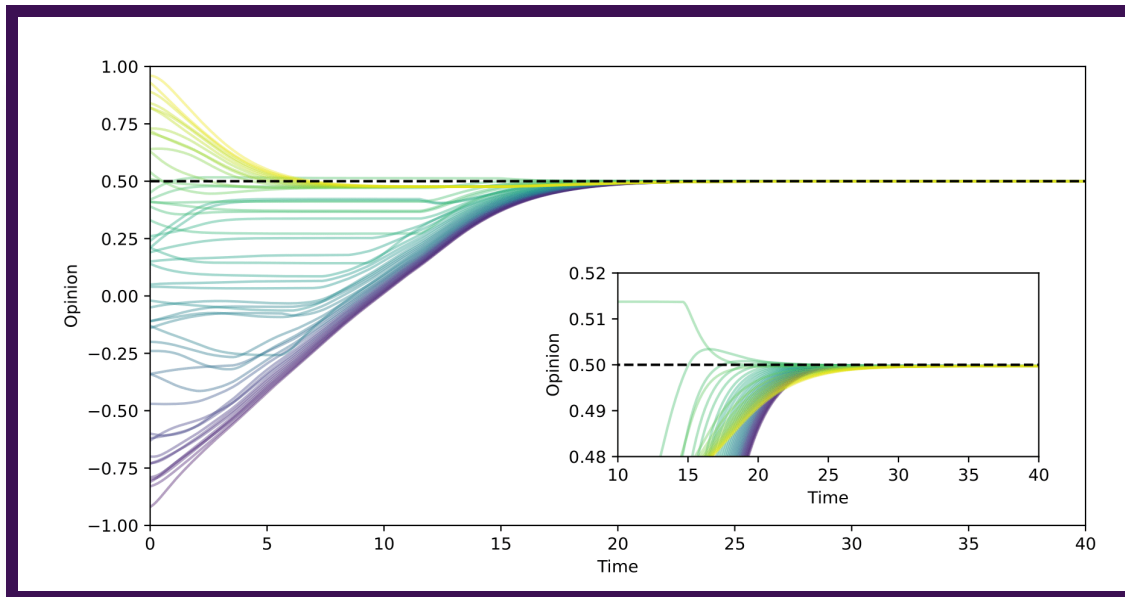


Network Control

Goal: For given initial conditions, find a control

$$u \in L^\infty(\mathbb{R}^+; \mathbb{R}^{N \times N})$$

that brings the whole population to consensus at a preselected target opinion x^* .



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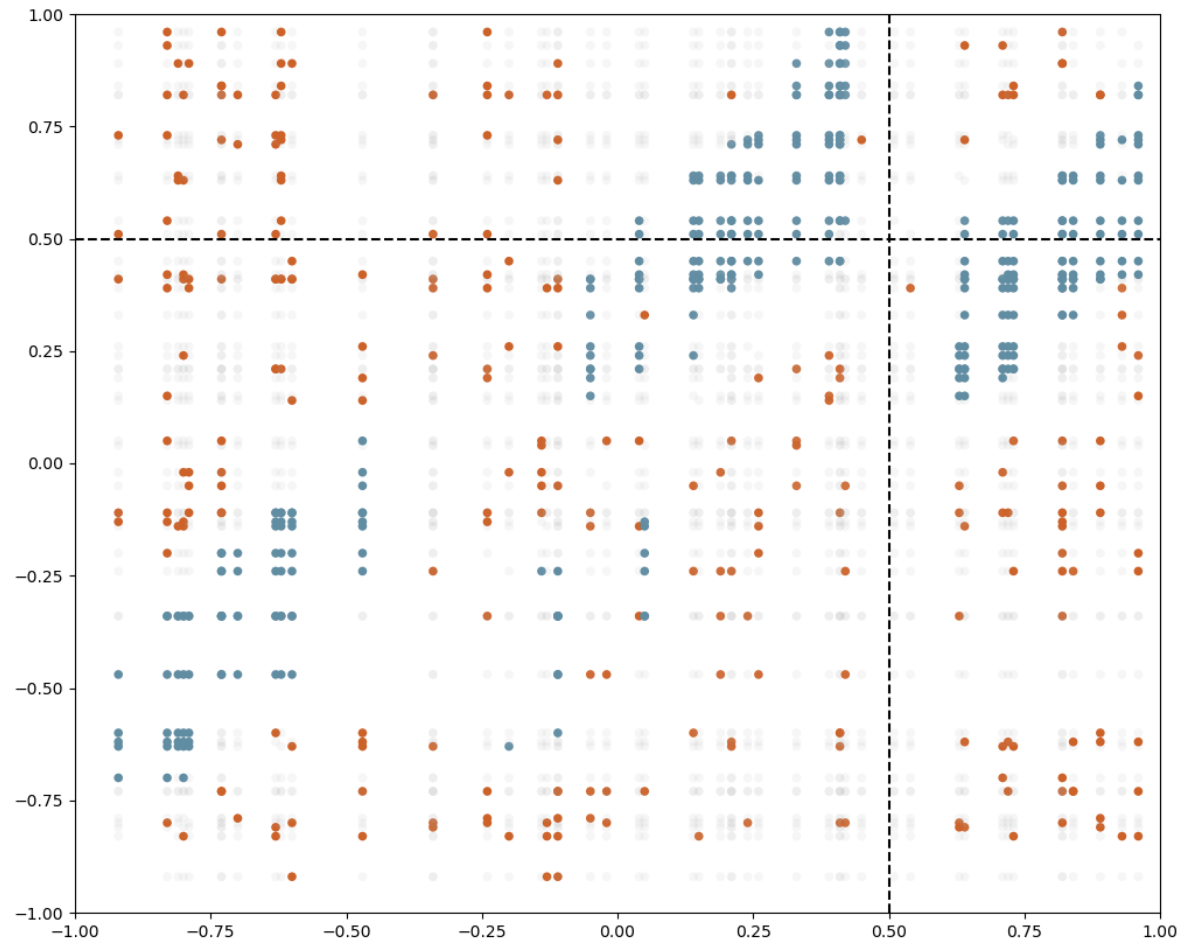
Minimise the cost functional

$$\mathcal{C}(u) = \int_0^T \alpha \sum_{i=1}^N \sum_{j=1}^N u_{ij}(s)^2 + \beta \sum_{i=1}^N (x_i(s) - x^*)^2 ds$$

Subject to the opinion dynamics, controlled weight dynamics, and box constraints on controls.

Optimal Control

- Optimal controls are bang-bang controls.
- Blue shows edge strengthening, red shows edge weakening and grey shows no control.
- Optimal control has clear stripes of edge creation, but edge removal is less clearly structured.



Thank you!

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Dynamic Network



Network Control