Steering opinion dynamics through control of social networks

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Model Formulation

An ODE system with coupled equations for opinions x_i and network edge weights w_{ij}, inspired by the structure in [1,2]. Edge dynamics are driven by control variables u_{ij} that determine when edges are strengthened or weakened.

$$\frac{dx_i}{dt} = \frac{1}{k_i(t)} \sum_{j=1}^{N} w_{ij}\, \phi(|x_j - x_i|)\,(x_j - x_i) \qquad i \in \Lambda\,,$$

$$k_i(t) = \sum_{j=1}^{N} w_{ij}(t) \qquad i \in \Lambda\,,$$

$$\frac{dw_{ij}}{dt} = s(u_{ij})\,\big(\ell(u_{ij}) - w_{ij}\big) \qquad i,j \in \Lambda\,, i \neq j\,.$$

The functions s and ℓ determine the speed and direction of controls respectively, while the interaction function ϕ describes how individuals respond to each others' opinions.

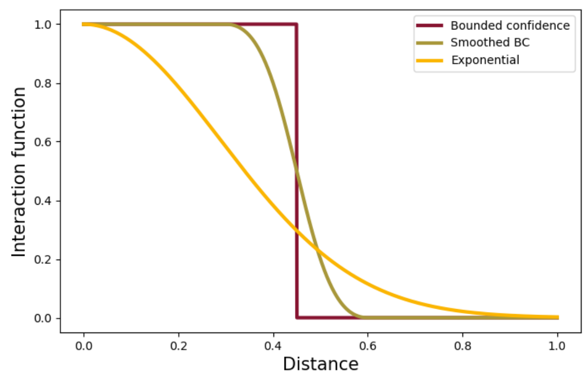
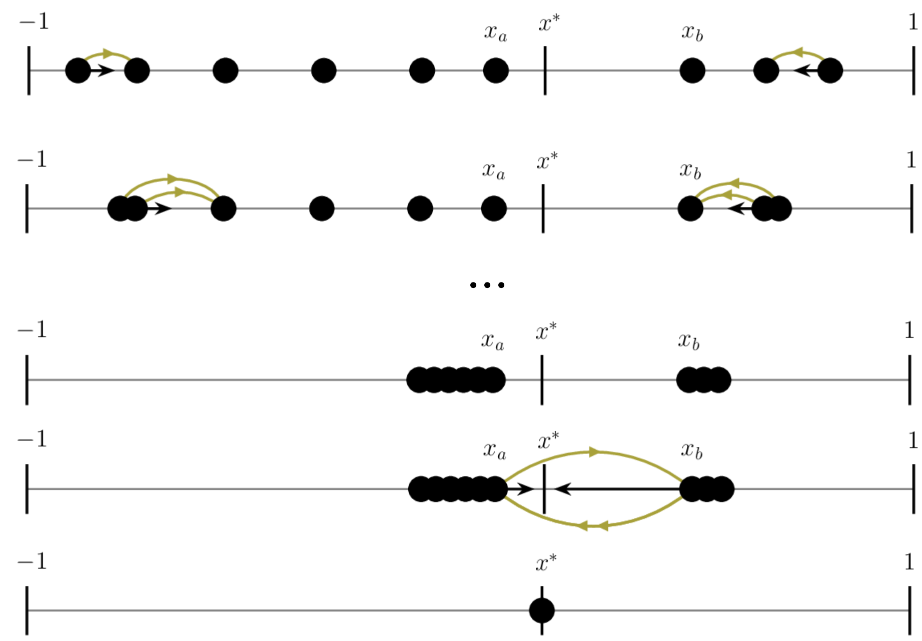


Fig 1. Examples of common interaction functions (ϕ).

Controllability

Proposition: Assume the initial network is empty except for self-edges, that the target x* is inside the initial opinion interval, and that the initial opinions form an r*-chain. Then there exists a control that achieves consensus at x*.



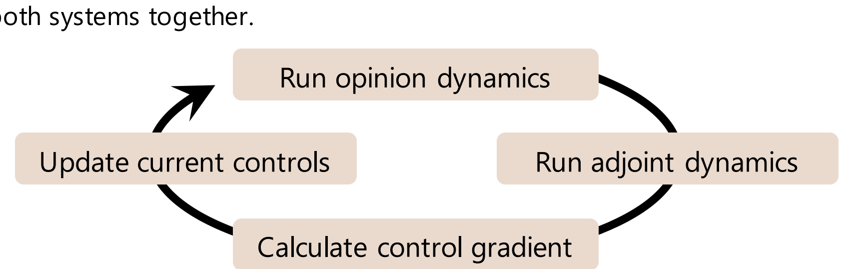
We can prove a similar result for more general initial networks if the control can remove edges sufficiently quickly [3], essentially returning us to the empty network case.

Optimal Control

We look for a control that minimises the cost functional

$$\mathcal{C}(u) = \int_0^T \underbrace{\alpha \sum_{i=1}^{N}\sum_{j=1}^{N} u_{ij}(s)^2}_{\text{Cost of control}} + \underbrace{\beta \sum_{i=1}^{N} \big(x_i(s) - x^*\big)^2}_{\text{Cost of distance from x*}}\, ds$$

This is a **constrained optimization problem**, with the system dynamics and set of admissible controls providing constraints. This problem can be solved using Lagrange multipliers and the **Pontryagin Maximum Principle** [4], which in this case means introducing an adjoint system of ODEs and solving both systems together.



This currently yields an effective but dense bang-bang control, see Fig 2 for an example.

In future it would be more realistic to look for **sparse controls** with **imperfect information** about individuals' opinions. We could also introduce a cost of deviating from the initial network or try to use the patterns observed in optimal control to **inspire analytic results**.

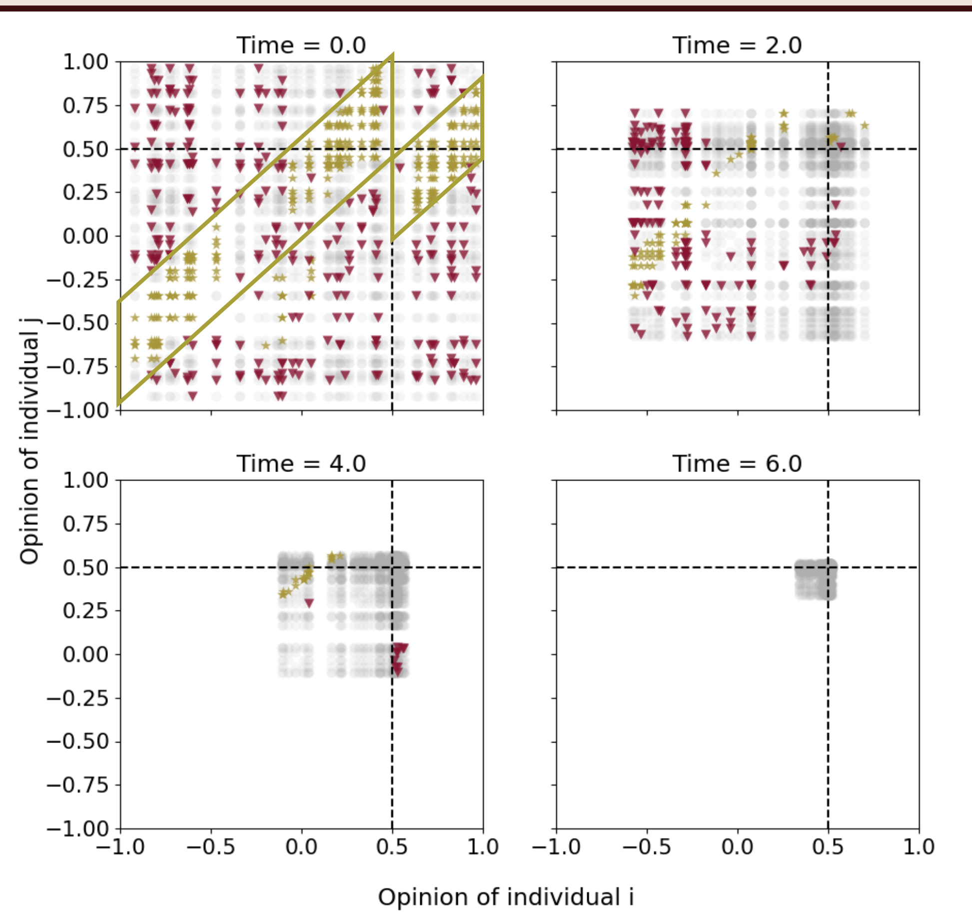


Fig 2. Example optimal control solution at four timepoints. A point describing the control applied to the edge w_{ij} is positioned at (x_i, x_j). Green, grey and red points indicate edge growth, inaction and decay, respectively.

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2. Piccoli, Benedetto, and Nastassia Pouradier Duteil. "Control of collective dynamics with time-varying weights." *Recent Advances in Kinetic Equations and Applications*. Springer International Publishing, 2021.
3. Nugent, Andrew, Susana N. Gomes, and Marie-Therese Wolfram. "Steering opinion dynamics through control of social networks." *arXiv preprint arXiv:2404.09849* (2024).
4. Evans, Lawrence C. "An introduction to mathematical optimal control theory version 0.2." *Lecture notes available at http://math. berkeley. edu/evans/control. course. pdf* (1983).