

# Evidence synthesis for a single randomized controlled trial and observational data in small populations

Steffen Unkel, Christian Röver, Tim Friede

Department of Medical Statistics  
University Medical Center Goettingen, Germany



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## Motivation

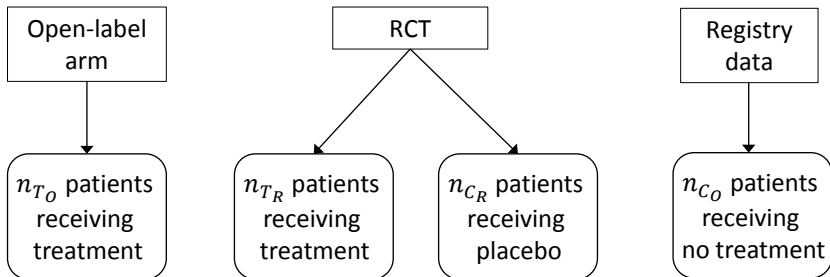
- Usually two independent randomized controlled trials (RCTs) are required to demonstrate efficacy and safety for marketing authorization.
- In **small populations** the conduct of a single RCT with a sufficient sample size might be difficult or not feasible.
- This is particularly the case
  - ① in paediatric studies,
  - ② if the intervention is to treat a rare disease, or
  - ③ if randomization is challenging.
- In situations where randomization is difficult to achieve, methods that **incorporate data from other sources** in the estimation of the treatment effects may be beneficial.

## Examples

Examples where the required number of patients were not randomized include

- 1 several RCTs in patients with **ankylosing spondylitis**.  
Patients were randomly assigned (in a 4:1 ratio) to either treatment or placebo. To support the small placebo control, data from eight previous trials in patients with ankylosing spondylitis were included (Baeten et al. 2013).
- 2 an RCT in patients with **Creutzfeldt-Jakob disease**  
Meta-analysis combining evidence on the effects of a certain treatment in patients with Creutzfeldt-Jakob disease from both a randomized study and a non-randomized study (Varges et al. 2017).
- 3 the EARLY PRO-TECT trial in **paediatric Alport patients** (Gross et al. 2012a).

# Trial design that mimics the Alport trial



# Endpoints

- The primary efficacy endpoint in the EARLY PRO-TECT Alport trial is “time-to-progression to the next disease level”.
- This **time-to-event endpoint** will be assessed in 6-monthly intervals over the treatment period of 3 years.
- The second efficacy endpoint “albuminuria after 3 years corrected for baseline albuminuria for patients randomized to receive ramipril compared to placebo” is **continuous**.
- One might also think of **binary endpoints** such as “progression to the next disease level within 3 years (yes/no)”.

# Data

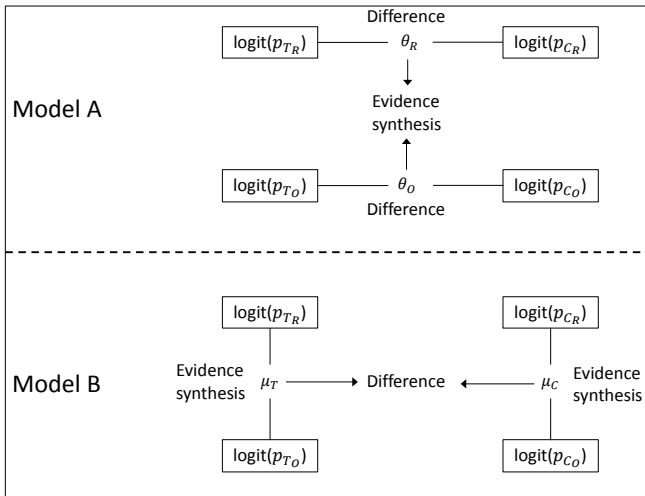
- We consider a **binary** endpoint.
- **Randomized arms**: let  $X_{iR}$  be the number of events and  $p_{iR}$  denote the probability of an event in group  $i$  ( $i = T, C$ ).
- **Non-randomized arms**: let  $X_{iO}$  be the number of events and  $p_{iO}$  denote the probability of an event in group  $i$  ( $i = T, C$ ).

- **Binomial model**:

$$X_{ij} \sim \mathcal{B}(n_{ij}, p_{ij}) \quad , \quad i = T, C; j = R, O \quad .$$

- Let  $\theta_R = \log \left( \frac{p_{TR}(1-p_{CR})}{p_{CR}(1-p_{TR})} \right)$  and  $\theta_O = \log \left( \frac{p_{TO}(1-p_{CO})}{p_{CO}(1-p_{TO})} \right)$  denote the **log odds ratio** for the randomized and observational data, respectively.

# Model frameworks



# Arm-based versus contrast-based synthesis of data

## Commentary

## Research Synthesis Methods

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# Absolute or relative effects? Arm-based synthesis of trial data

S. Dias\* and A. E. Ades

We congratulate Hwanhee Hong and colleagues on another fascinating paper (Hong *et al.*, 2015a) arguing the case for arm-based models for meta-analysis.

The standard approach to meta-analysis is the *contrast-based* model where the information that is pooled over trials is the information of the trial-specific *relative* treatment effect, expressed for example as a log relative risk, log odds ratio, or as a mean treatment difference. In an *arm-based* model, it is the *absolute* log risk, log odds, or mean outcome on each arm that are pooled.



## Methods for evidence synthesis

- 1 The **power prior** approach assigns a weight to the external data somewhere in between the cases of irrelevance and full equality.
- 2 **Bias allowance models** assume that the external data are potentially biased and the potential bias is modelled using an extra variance component that represents the bias.
- 3 **Meta-analytic approaches** or **hierarchical models** for evidence from different study designs are an extension of standard random-effects meta-analysis that explicitly model between-study-type variability.

## Hierarchical models

- The **hierarchical structure of model A** may be stated as

$$y_j | \theta_j, s_j \sim \mathcal{N}(\theta_j, s_j^2) ,$$
$$\theta_j | \mu, \tau \sim \mathcal{N}(\mu, \tau^2) , \quad j = R, O ,$$

where  $y_j$  is an estimate of  $\theta_j$  and  $s_j$  is its standard error.

The  $\theta_j$  differ from study to study and are distributed around a **common mean**  $\mu$  with **between-study-type variability** or **heterogeneity**  $\tau$ .

- The framework for **model B** consists of **two hierarchical structures** with parameters  $(\mu_T, \tau_T)$  and  $(\mu_C, \tau_C)$ .

The overall treatment effect is computed as a contrast:  $\mu_T - \mu_C$ .

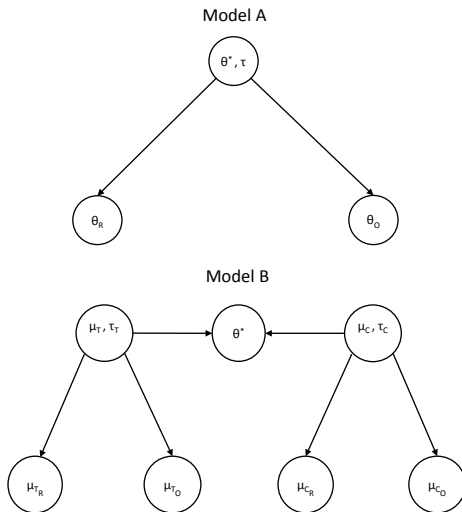


Figure: Hierarchical structures for model A (top) and model B (bottom).

# Fitting Bayesian hierarchical models

- We use a **Bayesian approach** for fitting the hierarchical models.
- Inference for  $\mu$  and  $\tau$  is captured by the joint **posterior distribution**, from which the marginal distribution of  $\mu$  is used to derive point estimates and probability intervals for  $\mu$ .
- Our approach requires **prior distributions** for  $\mu$  and  $\tau$ :
  - For  $\mu$  one may use a noninformative (improper) uniform prior or a normal prior with mean zero and large variance.
  - For  $\tau$  we use half-normal (HN) prior distributions.
- The R package `bayesmeta` provides a collection of functions to facilitate Bayesian inference in the random-effects meta-analysis model.

# Generating data

| RCT      | Treatment     | Control       |
|----------|---------------|---------------|
| No event | 31            | 9             |
| Event    | 9             | 11            |
| $\sum$   | $n_{TR} = 40$ | $n_{CR} = 20$ |

| Observational data | Treatment     | Control       |
|--------------------|---------------|---------------|
| No event           | 29            | 29            |
| Event              | 11            | 31            |
| $\sum$             | $n_{TO} = 40$ | $n_{CO} = 60$ |

Log odds ratio  $y_R = 1.4374$   
 Standard error  $s_R = 0.5877$

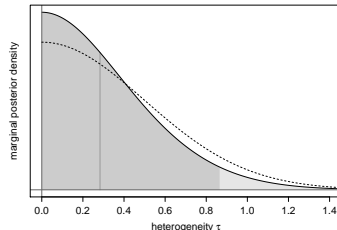
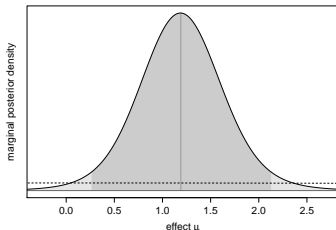
Log odds ratio  $y_O = 1.0361$   
 Standard error:  $s_O = 0.4383$

## Fitting model A

```
bma <- bayesmeta(y, s, mu.prior.mean=0, mu.prior.sd=10,  
tau.prior=function(t){dhalfnormal(t,scale=0.5)})
```

- Marginal posterior summary:

|           | tau    | mu     |
|-----------|--------|--------|
| mode      | 0.0000 | 1.1870 |
| median    | 0.2833 | 1.1960 |
| mean      | 0.3428 | 1.1931 |
| sd        | 0.2680 | 0.4699 |
| 95% lower | 0.0000 | 0.2637 |
| 95% upper | 0.8651 | 2.1278 |

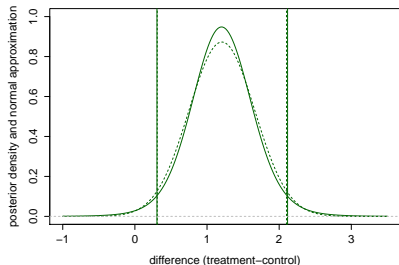
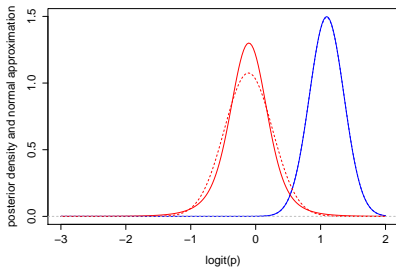


## Fitting model B

- Compute estimates for the logits( $p_{ij}$ ) ( $i = T, C; j = R, O$ ) and associated standard errors.
- ```
bma.t <- bayesmeta(y=yt, s=st, labels=names(yt),  
                  mu.prior.mean=0, mu.prior.sd=10,  
                  tau.prior=function(t){dhalfnormal(t, scale=0.1)})  
  
bma.c <- bayesmeta(y=yc, s=sc, labels=names(yc),  
                  mu.prior.mean=0, mu.prior.sd=10,  
                  tau.prior=function(t){dhalfnormal(t, scale=0.5)})
```
- Compute the **convolution**, that is, the distribution of the difference (treatment - control).

# Fitting model B (2)

|   |            |                |         |        |
|---|------------|----------------|---------|--------|
| • | Difference |                | Model A |        |
|   | mean       | standard error | mean    | sd     |
|   | 1.2056     | 0.4571         | 1.1931  | 0.4699 |



|   |               |        |        |
|---|---------------|--------|--------|
| • |               | 2.5%   | 97.5%  |
|   | Normal approx | 0.3097 | 2.1015 |
|   | Convolution   | 0.3059 | 2.1165 |
|   | Model A       | 0.2637 | 2.1278 |



## Simulation setup: meta-analysis scenario

- We investigate the performance of models A and B by means of a so-called **general meta-analysis scenario**.
- We assume that we observe four logit estimates  $(y_{T_R}, y_{C_R}, y_{T_O}, y_{C_O})$  and associated standard errors  $(s_{T_R}, s_{C_R}, s_{T_O}, s_{C_O})$ .
- The underlying true effects are  $\mu_{ij} = \text{logit}(p_{ij})$  ( $i = T, C; j = R, O$ ).
- The effects  $(\mu_{T_R}, \mu_{C_R}, \mu_{T_O}, \mu_{C_O})^\top$  are assumed to follow a multivariate normal distribution with mean  $(\mu_T, \mu_C, \mu_T, \mu_C)^\top$  and covariance matrix  $\Sigma$ .

## Simulation setup: meta-analysis scenario (2)

- We consider a scenario in which there are dependencies between the two randomized arms and between the two observational arms only.
- That is, the covariance matrix of  $(\mu_{T_R}, \mu_{C_R}, \mu_{T_O}, \mu_{C_O})^T$  is assumed to be

$$\Sigma = \begin{pmatrix} \sigma^2 & \sigma^2 - \frac{\tau_R^2}{2} & 0 & 0 \\ \sigma^2 - \frac{\tau_R^2}{2} & \sigma^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & \sigma^2 - \frac{\tau_O^2}{2} \\ 0 & 0 & \sigma^2 - \frac{\tau_O^2}{2} & \sigma^2 \end{pmatrix},$$

where  $\tau_R^2 = \text{Var}(\mu_{T_R} - \mu_{C_R})$  and  $\tau_O^2 = \text{Var}(\mu_{T_O} - \mu_{C_O})$ .

## Simulation setup: data and evaluation criteria

- Four groups motivated by the EARLY PRO-TECT study protocol:
  - $n_{TR} = 40$ : 8 failures / 32 successes
  - $n_{CR} = 20$ : 10 failures / 10 successes
  - $n_{TO} = 40$ : 8 failures / 32 successes
  - $n_{CO} = 60$ : 30 failures / 30 successes
- This leads to standard errors:  $s_{RT} = 0.4$ ,  $s_{RC} = 0.45$ ,  $s_{OT} = 0.4$  and  $s_{OC} = 0.26$
- Using 2000 simulation runs per parameter combination, we computed
  - 1 observed coverages for 95% confidence intervals for the pooled effect.
  - 2 lengths of meta-analytic confidence intervals relative to the interval length of the RCT.

## Simulation setup: between-study heterogeneity

- For the **between-study-type variability** the choice of the prior distribution can be critical.
- For log-odds ratios, values for  $\tau$  equal to 0.25, 0.5, 1 and 2 represent moderate, substantial, large, and very large heterogeneity.
- For example,  $\exp(1.09\tau)$  is the median ratio of the maximum to the minimum of any random pair of odds ratios.
- **Sensitivity analysis**: we choose two half-normal priors for log-odds ratios with the following characteristics:

| prior                  | median | 95% interval  |
|------------------------|--------|---------------|
| Half-normal(scale=0.5) | 0.337  | (0.016, 1.12) |
| Half-normal(scale=1.0) | 0.674  | (0.031, 2.24) |

# Coverages of confidence intervals (nominal level = 95%)

• **Model A:**

| $\tau_R \backslash \tau_O$ | 0           | 0.1         | 0.2         | 0.5         | 1    | 2    |
|----------------------------|-------------|-------------|-------------|-------------|------|------|
| 0                          | <b>98.9</b> | <b>99.2</b> | <b>98.7</b> | <b>97.0</b> | 89.8 | 73.2 |
| 0.1                        | <b>99.1</b> | <b>99.2</b> | <b>98.8</b> | <b>96.9</b> | 89.2 | 73.0 |
| 0.2                        | <b>98.9</b> | <b>99.0</b> | <b>98.7</b> | <b>96.7</b> | 89.8 | 73.0 |
| 0.5                        | <b>97.9</b> | <b>97.7</b> | <b>97.5</b> | <b>95.4</b> | 87.6 | 70.9 |
| 1                          | 93.9        | 94.2        | 94.0        | 89.8        | 82.4 | 69.0 |
| 2                          | 81.8        | 81.0        | 80.3        | 78.2        | 71.7 | 62.9 |

• **Model B:**

| $\tau_R \backslash \tau_O$ | 0           | 0.1         | 0.2         | 0.5         | 1    | 2    |
|----------------------------|-------------|-------------|-------------|-------------|------|------|
| 0                          | <b>99.5</b> | <b>99.6</b> | <b>99.3</b> | <b>98.2</b> | 92.0 | 71.5 |
| 0.1                        | <b>99.5</b> | <b>99.6</b> | <b>99.3</b> | <b>97.9</b> | 91.6 | 70.8 |
| 0.2                        | <b>99.4</b> | <b>99.5</b> | <b>99.2</b> | <b>98.1</b> | 91.8 | 70.8 |
| 0.5                        | <b>98.9</b> | <b>98.4</b> | <b>98.3</b> | <b>97.0</b> | 90.1 | 69.5 |
| 1                          | 94.8        | 94.9        | <b>95.3</b> | 91.6        | 85.4 | 67.1 |
| 2                          | 79.3        | 78.4        | 78.0        | 76.5        | 70.9 | 58.5 |

# Lengths of confidence intervals (relative to RCT, in %)

- **Model A** (67% relative length means an effective sample size gain of  $\approx 123\%$ ):

| $\tau_R \backslash \tau_O$ | 0    | 0.1  | 0.2  | 0.5  | 1    | 2    |
|----------------------------|------|------|------|------|------|------|
| 0                          | 66.0 | 66.0 | 66.1 | 67.0 | 70.1 | 80.3 |
| 0.1                        | 66.0 | 66.0 | 66.2 | 67.1 | 70.2 | 80.2 |
| 0.2                        | 66.1 | 66.1 | 66.3 | 67.3 | 70.4 | 79.9 |
| 0.5                        | 67.0 | 67.1 | 67.3 | 68.2 | 71.3 | 80.2 |
| 1                          | 70.2 | 70.2 | 70.4 | 71.2 | 74.1 | 82.2 |
| 2                          | 79.6 | 79.9 | 79.9 | 80.6 | 82.0 | 88.5 |

- **Model B** (80% relative length means an effective sample size gain of  $\approx 56\%$ ):

| $\tau_R \backslash \tau_O$ | 0    | 0.1  | 0.2  | 0.5  | 1    | 2    |
|----------------------------|------|------|------|------|------|------|
| 0                          | 79.1 | 79.3 | 79.4 | 79.1 | 79.9 | 80.3 |
| 0.1                        | 79.4 | 79.5 | 79.5 | 79.5 | 79.8 | 80.1 |
| 0.2                        | 79.5 | 79.4 | 79.4 | 79.5 | 79.7 | 80.1 |
| 0.5                        | 79.6 | 79.5 | 79.1 | 79.7 | 79.9 | 79.7 |
| 1                          | 79.4 | 79.9 | 79.9 | 79.7 | 80.3 | 79.9 |
| 2                          | 80.0 | 80.1 | 80.0 | 80.2 | 80.0 | 79.3 |

# Coverages (prior: $\tau \sim \text{HN}(\text{scale} = 1.0)$ )

• **Model A:**

| $\tau_R \backslash \tau_O$ | 0     | 0.1   | 0.2   | 0.5  | 1    | 2    |
|----------------------------|-------|-------|-------|------|------|------|
| 0                          | 100.0 | 100.0 | 100.0 | 99.9 | 98.5 | 96.2 |
| 0.1                        | 100.0 | 100.0 | 99.9  | 99.7 | 98.8 | 95.5 |
| 0.2                        | 100.0 | 99.9  | 100.0 | 99.7 | 98.6 | 94.8 |
| 0.5                        | 100.0 | 99.8  | 99.6  | 99.6 | 97.6 | 93.0 |
| 1                          | 99.7  | 99.7  | 99.5  | 98.2 | 95.1 | 88.3 |
| 2                          | 98.1  | 98.4  | 98.0  | 95.3 | 89.5 | 79.9 |

• **Model B:**

| $\tau_R \backslash \tau_O$ | 0     | 0.1   | 0.2   | 0.5   | 1    | 2    |
|----------------------------|-------|-------|-------|-------|------|------|
| 0                          | 100.0 | 100.0 | 100.0 | 100.0 | 99.2 | 92.7 |
| 0.1                        | 100.0 | 100.0 | 100.0 | 99.8  | 99.1 | 92.7 |
| 0.2                        | 100.0 | 100.0 | 100.0 | 99.9  | 98.6 | 93.0 |
| 0.5                        | 99.9  | 99.9  | 100.0 | 99.9  | 98.1 | 90.9 |
| 1                          | 99.8  | 99.7  | 99.7  | 99.1  | 95.8 | 87.2 |
| 2                          | 94.8  | 96.2  | 94.9  | 94.2  | 89.0 | 75.0 |

# Lengths of intervals (prior: $\tau \sim \text{HN}(\text{scale} = 1.0)$ )

• Model A:

| $\tau_R \backslash \tau_O$ |     | 0     | 0.1   | 0.2   | 0.5   | 1     | 2     |
|----------------------------|-----|-------|-------|-------|-------|-------|-------|
| 0                          | 0   | 94.9  | 94.7  | 94.9  | 97.2  | 103.5 | 118.8 |
|                            | 0.1 | 95.0  | 94.8  | 94.9  | 97.3  | 102.6 | 118.6 |
|                            | 0.2 | 95.0  | 94.8  | 95.5  | 97.4  | 103.5 | 118.4 |
|                            | 0.5 | 96.7  | 97.0  | 97.1  | 99.4  | 104.3 | 119.8 |
|                            | 1   | 102.5 | 102.7 | 103.4 | 104.1 | 109.5 | 121.6 |
|                            | 2   | 118.9 | 119.1 | 118.6 | 119.1 | 123.2 | 132.4 |

• Model B:

| $\tau_R \backslash \tau_O$ |     | 0     | 0.1   | 0.2   | 0.5   | 1     | 2     |
|----------------------------|-----|-------|-------|-------|-------|-------|-------|
| 0                          | 0   | 118.4 | 117.4 | 117.0 | 118.2 | 117.7 | 119.1 |
|                            | 0.1 | 118.5 | 117.5 | 117.7 | 117.9 | 118.9 | 118.2 |
|                            | 0.2 | 117.9 | 116.8 | 117.1 | 118.2 | 118.3 | 118.3 |
|                            | 0.5 | 117.5 | 117.8 | 118.1 | 118.7 | 117.8 | 119.2 |
|                            | 1   | 117.8 | 118.4 | 118.6 | 117.2 | 118.1 | 118.3 |
|                            | 2   | 118.4 | 118.5 | 118.8 | 118.5 | 119.5 | 118.2 |



## Summary and conclusions

- We have synthesized evidence from a single RCT and observational data in small populations.
- We presented two model frameworks within which evidence synthesis can be performed. Our simulation results indicate that framework A should be preferred over framework B.
- Recent computational advances in evidence synthesis facilitate the application of Bayesian hierarchical models.
- A meta-analysis of only two studies is a challenging problem, in particular the choice of a prior distribution for  $\tau$ .
- Risk of bias due to lack of comparability of treatment groups or confounding.
- Adjustments for covariates can be done before the models are fitted.

## Further work (some of it under way)

- 1 Simulations: consider a scenario, in which there is no correlation between the two observational arms, but a correlation between the randomized arms and open-label arm instead.
- 2 We will also consider continuous and time-to-event endpoints.
- 3 We estimated a pooled effect,  $\theta^*$ . Other quantities of interest:
  - effect,  $\theta_R$ , of an RCT in the light of observational data (shrinkage estimator),
  - effect,  $\theta_{k+1}$ , of a future study (prediction / extrapolation).
- 4 Our frameworks bear some similarities to a comprehensive cohort-study design (Olschewski et al. (1992)). We may also want to consider a trial design with an additional observational open-label arm but no registry.

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