

Modelling peloton dynamics in competitive cycling: a quantitative approach

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Abstract. We propose an agent-based model for peloton dynamics in competitive cycling. It aims to generate the very complex behaviour observed in real-life competitive cycling from a collection of agents with simple rules of behaviour. Cyclists in a peloton try to minimize their energy expenditure by riding behind other cyclists, in areas of reduced air resistance. Drafting cyclists spend considerably less energy than front-runners, making the strategies in the sport to be based around trailing as much as possible. We quantify energy expenditure and recovery in relation to cyclists' positions in the peloton. Finally, we analyse the results and try to compare them to real-life behaviour of competitive pelotons.

Keywords: Agent-based Modelling, Computer Simulation, Peloton Dynamics, Flocking, Emerging Complexity

1 Introduction

Despite the appearance of a purely individual sport where the strongest tends to win, there is a very deep tactical layer to competitive cycling. A very simple fact generates complex behaviours and strategies: the fact that air resistance dominates the energy expenditure of athletes. The drafting effect, thus, plays a big role on peloton dynamics. The existence of a peloton itself can be explained by this effect: energy expenditure when drafting in a single line is reduced by approximately 18% at 32km/h, 27% at 40km/h, when drafting a single rider, and by as much as 39% at 40km/h in a group of eight riders[4].

From this fact, the behaviour observed in a real-life cycling race emerges: riders tend to group together and rotate in front of the peloton, trying to minimize the average energy expenditure of the group. When someone tries to break away from the pack of riders, the peloton decides either to chase them or let them open a time gap. If the group is deemed too dangerous, the typical behaviour is to chase it, and by having a bigger number of athletes to share the time spent in non-drafting positions, they normally succeed in this task. Only when a break-away group is considered harmless enough a gap is established, and the tired riders from this group are normally caught close to the finish line.

Based on contributions[5][1], we take a mathematical approach trying to quantify energy expenditures based on different factors as speed, gradient of the terrain, cross-section area and weight of the cyclist, and specific physiological characteristics of each individual. We regard the rider basically as a static engine on the bike, and consider the external factors that try to impede motion, such as drag resistance, rolling resistance and changes in the potential energy (such as when climbing a hill). Based on these opposing forces and in physiological data from professional cyclists, we establish an energy balance consistent with real-life values.

This is not the first agent-based approach for modelling competitive cycling; another work [1] does this, albeit with a different approach and with different goals. Their work has a focus on final results of a race and a game-theoretical, best-response model for elaborating strategies; it does not concern itself with peloton dynamics, nor with the relationship between those and the energy expenditures. In this work, we focus not on the actual results of a race, but rather on simulating the behaviour of a peloton during a race, under different circumstances.

Here, we take the peloton as a complex dynamical system. We establish general principles and rules for the behaviour of each agent, roughly based on flocking models already existing[9], calculating cohesive and separating forces for each agent, so that they stay together as a group, but having spatial gaps between them. We alter this model so that it makes sense in the context of this work and observe a self-organizing pattern emerge, resembling other natural dynamical systems[7].

We develop a model proposal for the dynamic behaviour of a competitive cycling peloton, in an agent-based fashion, based on a small set of basic rules for each agent. These rules are derived from the mathematical, physiological and dynamical concepts presented, and the emerging patterns are roughly similar to those observed in real life and in other systems.

Finally, we discuss the results based on data generated during simulations for many different parameters of the system. Starting from this data, we try to establish similarities with real-life cycling behaviour and validate the constructed model. We see this work as a first step to possible further analysis of this sport via computational simulation in the future, integrating even more parameters for natural influence and specific behaviour of the agents, based on further empirical data.

2 The Model

When constructing our model, we search for different kinds of parameters in order to replicate and extend the real-life behaviour of cyclists in a peloton. This work proposes an agent-based model for that. Agent-based models are very well suited for situations where dynamics emerge from simple interactions between different individuals, or agents[10].

2.1 Dynamical parameters

In this section, we describe the utilized dynamical parameters for our model; that is, the set of parameters responsible for interaction between agents and the peloton behaviour. This is a central part in our model. A non-desirable choice of values here leads to unordered behaviour from the agents, rendering our energy balance equations completely useless and turning the model away from the dynamics it expects to simulate.

As a starting point for our dynamics, we take a simple flocking model [9]. In this model, agents are subject to three different kinds of forces: separating, aligning and cohesive forces. The separating force is intended to make agents keep a minimum separation between them, so that they do not collapse to a single point. This is well-suited to our model, since cyclists will try to steer away from other fellow cyclists, to avoid crashes.

In the original flocking model we have, then, an aligning force. This force is applied to each agent to make it follow the direction of their nearest neighbours. In our case, this seems a bit out of place. Naturally, in a bicycle race, all competitors are following the same direction, and steering inside the peloton is a relatively small change in direction. This way, we have decided to model intra-peloton movement purely by lateral movement, without change of heading direction. There is no need, then, for an aligning force.

Finally, we have a cohesion force. In the flocking model, each agent turns in to become closer to its mates, making the group coherent. As we want to simulate a peloton behaviour, it makes sense to apply this kind of force in our model.

In the original model, the resulting behaviour of the agents is to move around freely when far from other agents. However, when approaching other agents, this group tends to become a coherent flock, with agents exhibiting similar headings and staying together, but with some spatial separation between them.

As an improvement on this flocking model, we take the Swarm Chemistry model [6]. This time, different sets of agents have different parameters for cohesive, separating and alignment forces, making the proportion between those different, as it can be seen in Algorithm 1. This makes different sets behave in different ways, and they tend to stay together in space, even though they are not coupled together by anything other than the similarity of their parameters.

We take the way of calculating separating and cohesive forces, with minor adjustments, from this model. We calculate an acceleration for each agent, based on its relative position to the others. However, this code calculates it taking into account any other agent inside R_i , a vision radius. This way, vision is isotropic. As we are trying to model human behaviour, it is useful to tweak this so that we only take into account agents that could possibly be seen by a human, that is, contained inside a visual cone. We also want to take different scales of distance when calculating cohesive and separating forces. Cohesive force should be a long-range calculation, taking into account pretty much any other agent in the visual field, while separating forces should be more local, with only close agents being taken into account.

Algorithm 1 Sayama's algorithm for cohesive and separating forces

```

for  $i \in \text{agents}$  do
   $N \leftarrow \{j \neq i, |x_i - x_j| < R_i\}$ 
  if  $N \neq \emptyset$  then
     $\langle x \rangle \leftarrow \sum_j \frac{x_j}{|N|}$ 
     $a \leftarrow c_1^i (\langle x \rangle - x_i) + c_3^i \sum_j \left( \frac{x_i - x_j}{|x_i - x_j|^2} \right)$ 
  end if
end for

```

Fig. 1. MOPEDs algorithm for cohesive and separating forces

Based on these assumptions, we present the pseudo-code for these forces, defined here as Algorithm 2.

Algorithm 2 MOPED's algorithm for cohesive and separating forces

```

for  $i \in \text{agents}$  do
   $N_c \leftarrow \{j \neq i, x_j \in C_i^c\}$ 
   $N_s \leftarrow \{l \neq i, x_l \in C_i^s\}$ 
  if  $N_c \neq \emptyset$  then
     $\langle x \rangle \leftarrow \sum_j \frac{x_j}{|N_c|}$ 
     $a_c \leftarrow c_1^i (\langle x \rangle - x_i)$ 
     $a_s \leftarrow c_3^i \sum_l \left( \frac{x_i - x_l}{|x_i - x_l|^2} \right)$ 
     $a \leftarrow a_c + a_s$ 
  end if
end for

```

Fig. 2. MOPEDs algorithm for cohesive and separating forces

In Algorithm 2, N_c and N_s are, respectively, the neighbourhoods used for the calculation of cohesive and separating forces. These neighbourhoods are cones with an aperture angle of 140° , defined to mimic a human visual cone, and different radii. The neighbourhood for cohesive force is being used with a radius of 20m, and the separating one with a radius of 2m, so that the rider only tries to avoid contact with other agents in the immediate neighbourhood.

A last point to be addressed in this section is some kind of bias to the center; real cyclists tend to be as close to the center of the road as possible in the case where no wind is blowing, so as to avoid danger on the fringes of the road. For that, a very small bias is put in place. Also, as in any flocking model, a small, random acceleration is put into place.

As a final addendum, we introduce the concept of *active* riders: a rider is considered active if he is willing to cooperate and be the frontrunner of the peloton, being out of drafting positions in order to protect the other cyclists.

This is modelled by adding a small bias to the front for active cyclists, as far as they are not already the frontrunners.

We observe that this algorithm, with the right set of parameters, yields a very life-like behaviour, with a narrow peloton front giving place to a wider formation behind, as it can be seen at Figure 3.



Fig. 3. View of a simulation of the peloton.

2.2 Energetic parameters

We will now group together mechanical and physiological parameters, since both sets of parameters affect the energetic balance of the cyclist, making it easier to analyse them together.

Physiological data about cyclists is abundant; the literature on how the human body behaves under these circumstances is extensive and it is not our idea to redefine anything on this domain, but rather to use existing results and adjust our model to reflect them.

Many mathematical models for competitive cycling exist already. For this model, we will use elements from Olds[5], who has a validated, well-behaved set of equations for energy expenditures. We also use specific results from Hoenigman et al.[1], who extended Olds' results and applied it on an agent-based model, and Martin et al.[3] for the potential-energy equation. By taking an agent-based approach and simulating both peloton dynamics and energy expenditures, we believe it is possible to have a more complete model and simulations that yield results that are closer to real life.

Our first point is to calculate the drafting coefficient, that is, a correction factor relating the air resistance when drafting and when not drafting. Kyle[2] measured this coefficient experimentally and affirms that the reduction in air resistance diminishes when wheel spacing increases. This is a fairly intuitive result. He mentions that this reduction obeyed a second-order polynomial, but he does not present the equation. Olds[5] reconstructs the equation from the graphical data, arriving at

$$CF_{draft} = 0.62 - 0.0104d_w + 0.0452d_w^2 \quad (1)$$

where d_w is the wheel spacing (in meters) between the bicycle and the preceding rider, and CF_{draft} is the correction coefficient. However, eq. 1 can be applied only when drafting happens in a paceline, that is, when riders are exactly behind

one another. In a peloton, drafting occur also in other ways; riders in a diagonal have a "comet's tail" effect, with drafting bonuses decreasing when the rider behind moves backwards or sideways. There is no extensive study of drafting coefficients when a cyclist is not directly behind another one, as there is scarce data about drafting multiple riders; we only know for sure that drafting behind multiple riders is more beneficial than behind only one[4]. In this situation, we have decided to assign weights depending on the angle of view to the preceding rider. It is important to notice that drafting benefits are negligible in a distance over 3m, and therefore we limit our calculations to this radius.

Having calculated CF_{draft} , we can now go on to the power equations. As we are using a scale of one iteration per second of simulation, there is no need to account for the difference between power and energy: they are numerically the same. From Hoenigman et al.[1], we have the following equations:

$$P_{air} = kCF_{draft}v^3 \quad (2)$$

$$P_{roll} = C_r g(M + Mb)v \quad (3)$$

On eq. 2, k is a lumped constant for aerodynamic drag, dependant, between other things, on the cross-section area of the cyclist. This constant is generally reported with the value 0.185 kg/m, and we are following this value. Of course, v is wind speed (and, as we are considering only a situation without wind, is equal to ground speed). On eq. 3, C_r is a lumped constant for all frictional losses on a bike, and is generally reported with a value of 0.0053. Of course, g is the usual gravitational constant (9.81 m/s^2). The variables M and M_b represent, respectively, the mass of the cyclist and of the bicycle. On this model, we are using values of 63kg and 7kg, respectively, for these variables, without any variation between cyclists.

These equations are enough for modelling the energy expenditure of a cyclist in a flat, non-windy situation. However, we want to model also the behaviour of the peloton in uphill and downhill, and therefore we need an extra equation for that. Martin et al.[3] present this equation, for grades up to 10% (where we consider $\sin(\arctan(G_r)) = G_r$) :

$$P_{PE} = G_r g(M + Mb)v \quad (4)$$

and, therefore, we can introduce this on eq. 3, obtaining the following:

$$P_{roll+PE} = (C_r + G_r)g(M + Mb)v \quad (5)$$

and a total energy expense of:

$$P_t = E_t = (C_r + G_r)g(M + Mb)v + kCF_{draft}v^3 \quad (6)$$

But this is only taking into account the energy expenditure. We need to model how the cyclists react to this and how much energy they can spend without exhausting themselves. For that, we will introduce the concept of lactate or

anaerobic threshold, very well-known in any endurance sport. Roughly speaking, the lactate threshold is the power output an athlete is capable of without accumulating lactic acid in his muscles, that is, without getting tired[8]. In this model, we will assess how tired a rider is through a simple "energy-left" variable, so that, at lactate threshold, the value of this variable is roughly unchanged.

As presented in Hoenigman et al.[1], a speed of $0.7S_m$ is slightly under the lactate threshold, where S_m is the speed at which a rider can travel at his Max_{10} power output. The Max_{10} represents the 10-minute maximum power a rider can generate, and is generally regarded as an indicator of a rider's skill level. We are using, as in that work, a mean value for Max_{10} $\mu = 7.1\text{W/kg}$. That represents, for a rider with 63kg, a Max_{10} of approximately 450W. This is equivalent, on flat ground, to $S_m = 12.96\text{m/s}$. Therefore, $0.7S_m$ is approximately 9m/s, and this should be slightly under lactate threshold. Finally, we take 10m/s as a representative value for this threshold, and set a "recovery" variable, normally distributed with $\mu = 225\text{W}$, that will be deduced from the actual spent energy. Only an energy expenditure over this limit will make the rider grow tired.

We are still faced with the challenge of determining how long does it take for a rider over his anaerobic threshold to be exhausted. For that, we will use the concept of time to exhaustion (T_{lim}), as defined in Olds[5]. The defining equation for T_{lim} is

$$\ln(T_{lim}) = -6.351 \ln(fVO_{2_{max}}) + 2.478 \quad (7)$$

In this equation, $fVO_{2_{max}}$ is the fraction of the $VO_{2_{max}}$ (maximum oxygen consumption) being used. We can substitute that for Max_{10} generating then:

$$\ln(T_{lim}) = -6.351 \ln\left(\frac{P_{tot}}{Max_{10}}\right) + 2.478 \quad (8)$$

For establishing which would be the initial value of "energy-left" to each rider, we decided to take an average situation: a sole rider at 45km/h (or 12m/s). From that, with our typical Max_{10} of 450W, we calculate which would be the time to exhaustion. From there, considering our average recovery of 225W, we calculated how much reserves a cyclist should have at the beginning of the simulation to achieve this typical time to exhaustion.

2.3 An overview of the model

We have presented the two parts of the model: the dynamical parameters and equations and the energetic parameters and equations. Now, we present how these two parts are interconnected.

It is clear that the position of an athlete inside the peloton greatly affects his energy balance: if a rider spends the whole time in front of the peloton, he should use more energy than another one sitting safely behind another cyclist. This way, the dynamical parameters and equations are influential on the energy balance. On the other hand, in our model, the energy balance is, usually, not relevant for the positioning of the cyclist: position calculation depends only on neighbouring

agents. However, when the cyclist becomes tired, this is relevant to his position. We have postulated that a rider with less than 100kJ of energy left is declared "exhausted". An exhausted rider has a backward bias and is effectively slower than the peloton average. This way, he tends to hang at the back of the peloton, eventually letting the group go altogether. A rider with 0 energy left quits the peloton altogether. In our model, he is positioned in the leftmost coordinate and becomes even slower. However, if he can somehow recover energy enough to get out of this condition altogether, he will rejoin the peloton. We present figure 4, a small, schematic flowchart representing the relation between the two "sides" of our model.

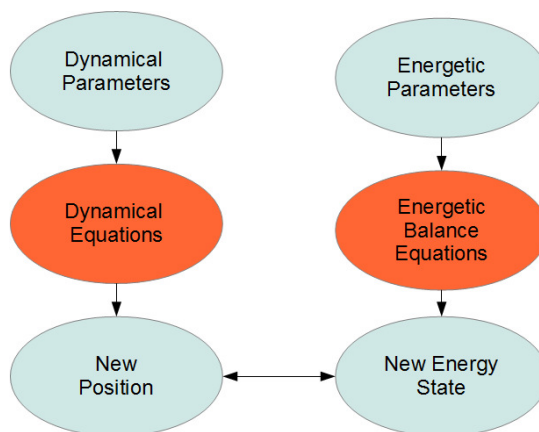


Fig. 4. Schematic diagram showing the relation between parts of the model.

3 Results

In the following section, we summarize the results obtained from the simulations using MOPED. These are divided between general behaviour, dynamical and energetic results, for clarity purposes.

3.1 General Behaviour

In general, the emerging complex behaviour from the agents is very promising; riders rotate back when they feel they are under the average energy of the peloton. The peloton itself conforms to the general form of real-life pelotons, with a very narrow front (usually a 3-5 long single line of riders) and a widening profile as we look backwards.

We rapidly see a convection dynamic settling in. Riders at the back of the peloton wishing to move forwards take the sides to do that, since the middle is

cluttered with cyclists and going through is virtually impossible. When they get to the front, they normally settle down in the center of the road, and as they stay there other riders coming from the back start to overtake them. This way, we have forward-moving cyclists taking the sides, backward-moving cyclists staying in the middle. Of course, to move forwards through the sides, the athletes end up spending more energy than those in the center of the peloton.

Trenchard[7] compares this dynamics to a convection roll, with "warming" riders in the peripheries and "cooling" riders in the middle. He then presents other natural systems with similar dynamics, such as Rayleigh-Bernard cells and penguin huddle rotations, and hints that this is the way for achieving optimal energy dissipation in the whole system. Even though our model does not take any energy considerations into account for the dynamics, we still establish similar patterns, which is quite interesting.

Independently of the number of active cyclists chosen, they rapidly take over the head of the peloton. Even though not all active agents gather at the front of the peloton at a single time, most of the time the frontrunner will be an active cyclist.

When a sustained effort is maintained, cyclists start to get exhausted and a sizeable group soon forms at the back of the peloton. Not much after, the first spent cyclists start to appear, giving up on the peloton. If this effort is maintained for long enough, only the strongest riders stay in the peloton, with all the rest giving up. This is consistent with long climbs in professional races, where the front group is normally smaller than ten cyclists. Also as in real life cycling, a long descent after a climb has the effect of bringing lots of cyclists back to the peloton, as they recover energy.

3.2 Dynamical parameters

Most of the results about the dynamics are difficult to quantify. As commented in the section above, the general behaviour of the peloton seems coherent with real life and, furthermore, coherent with an optimal energy dissipation.

As an addendum, we can show graphically the convection dynamics. On figure 5, we have a graph of average draft coefficient for agents moving forwards (blue) and backwards (red). It is clear that, on average, forwards-moving cyclists on the periphery will have less opportunities to draft behind another cyclists, and the opposite happens with the central, backwards-moving agents.

3.3 Energetic parameters

At first, for illustrating which kind of behaviour our model generates, we start with an example. We submit 100 agents to a 3-hour race at a constant speed of 45km/h. Of course this speed is too high for uphill parts, making them spend much more energy than humanly possible, but we are only interested in the qualitative behaviour that exhaustion generates. We generate a profile for the race, plotting the elevation at each time. As we are at constant speed, it is the

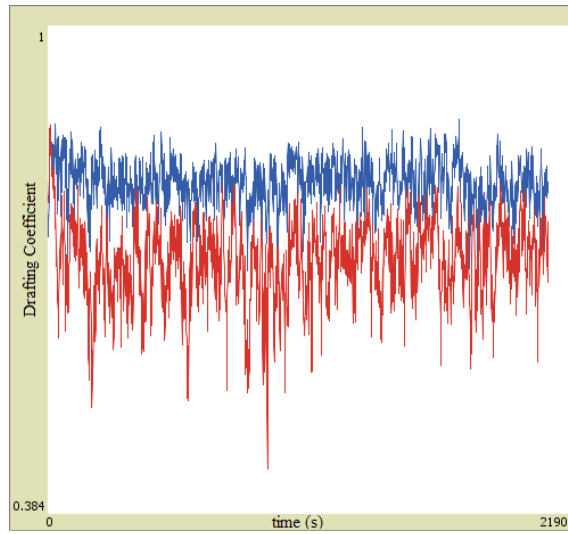


Fig. 5. Temporal distribution of draft coefficient for backward and forward-moving agents.

same as plotting elevation per distance. The generated profile is represented on figure 6.

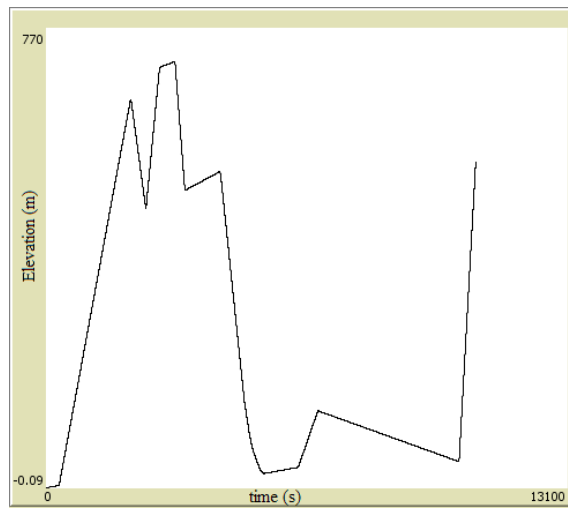


Fig. 6. Temporal profile of our test race.

Subjecting our cyclists to this profile, we generate, then, a graph illustrating the amount of exhausted riders at any given time. The plot is shown as figure 7.

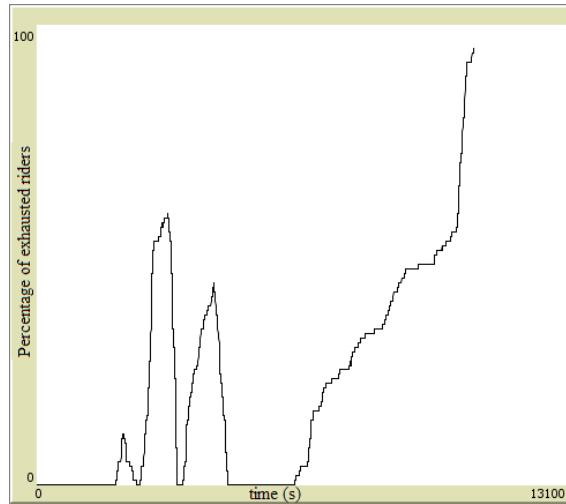


Fig. 7. Exhausted riders (out of 100) at any given time.

We can now establish correlations between both plots. In the very long initial uphill, relatively few riders were exhausted. That is, of course, because at the beginning of the race, every athlete is still fresh. A slight downhill follows, enough to recover all cyclists. However, a second, steepest uphill follows, and this time the exhaustion is much bigger in the peloton. Even during the shallow part close to the summit, the number of exhausted riders is still increasing considerably. Another downhill follows and, again, it is enough to recover all riders. Now, a short, not too steep climb is presented, and the result is a big increase again in exhaustion, this time much more due to the accumulation of climbs than because of the difficulty of this one. A very long downhill is next, and of course all riders arrive down there in conditions. A false-flat (very shallow gradient) do not break them, but as soon as the road gets steeper exhaustion increases. Even a long, slight downhill after the climb is not enough to recover, and exhaustion keeps increasing even during this part. Finally, a relatively short, but very steep climb to finish the race. This time, only the very best stay in the front group at the top.

The whole behaviour of the peloton described on the paragraph above will certainly sound familiar for anyone who already watched a mountain stage in a professional cycling race. The patterns are quite similar, even if not identical, which indicates that the energy balance of the model is sound.

As a visual representation of what was described here, we present figures 8 and 9. Figure 8 shows what the peloton looks like at the summit of a long climb.

You can see a relatively small group of 14 riders in front, followed by a bigger group barely hanging at the back, and many agents already at the leftmost coordinate, representing they have no more energy.

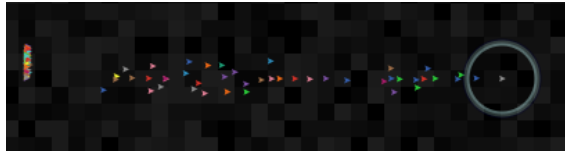


Fig. 8. Peloton at the top of a climb.

As a comparison, figure 9 shows the peloton during the descent, when the lesser riders have already recovered. You can see the peloton shape is still longest than normal due to the fact that the back riders are still coming back, but there are no more riders with no energy left and the group is unified again.



Fig. 9. Peloton during the descent.

Finally, we present an interesting, counter-intuitive result derived from the model. We start by drawing, at figure 10, the average draft coefficient for active riders (considering 20 out of 100 to be active riders) compared to the average draft coefficient of non-active ones.

The horizontal reference level is 100%, which would mean that, on average, active and non-active riders have similar draft coefficients. However, that is not what the graph shows: even if it is not that easy to see on this plot, active riders have lower draft coefficients on average, which means they spend less energy. To confirm that, we plot a quotient between average energy for active and non-active cyclists, and, even though it starts at 1 as expected, it undoubtedly increases with time, indicating active riders are, indeed, spending less energy on average. This plot is presented as figure 11.

This result is surprising, but there are reasons for it: active riders spend more time on the well-organized, single line part of the peloton. This way, they have constantly medium-to-low draft coefficients in these positions. Even if they do have to go to the front more frequently than non-active riders, this is offset by the fact that they are consistently drafting some other cyclists. Non-active riders spend more time in the convection-like part of the peloton, constantly going back and forth and spending considerable amount of time in non-drafting positions

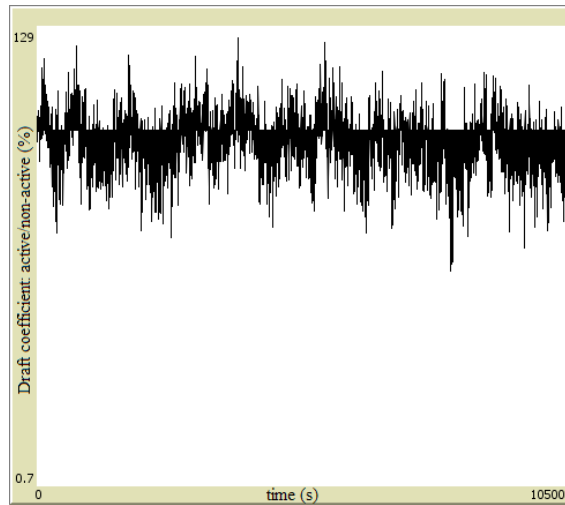


Fig. 10. Draft coefficient of active riders, as a percentage of DC of non-active riders.

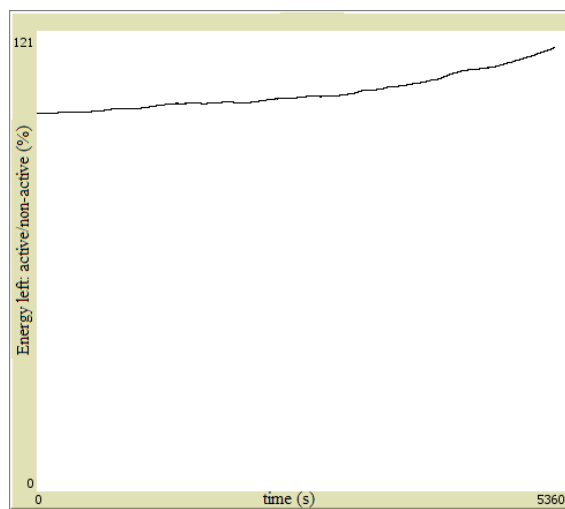


Fig. 11. Quotient of energy left between active and non-active cyclists.

at the periphery of the peloton. Furthermore, if we can imagine the speed of a cyclist as an oscillatory function around the average speed of the peloton (with the oscillations being in the periods where the cyclist move forwards or backwards), cyclists in the peloton have a bigger oscillatory amplitude than active ones. As the air resistance scales with the third power of speed, this by itself would cause a bigger air resistance.

Besides that, with much more movement inside the peloton, they are not guaranteed to have another rider right in front of them at all times, as someone in the single-line part of the peloton does. This way, their draft coefficient is much more subject to variations.

This can also be shown by figure 12, where we plot a quotient between average neighbourhood size for active and non-active agents. At the beginning, when the active agents are still organizing themselves, the plot goes under and over 1, but it quickly settles under 1 as soon as the active riders organize themselves. This shows that they are getting smaller draft coefficients in spite of drafting less cyclists, which is coherent with the idea that they spend more time exactly behind another cyclist.

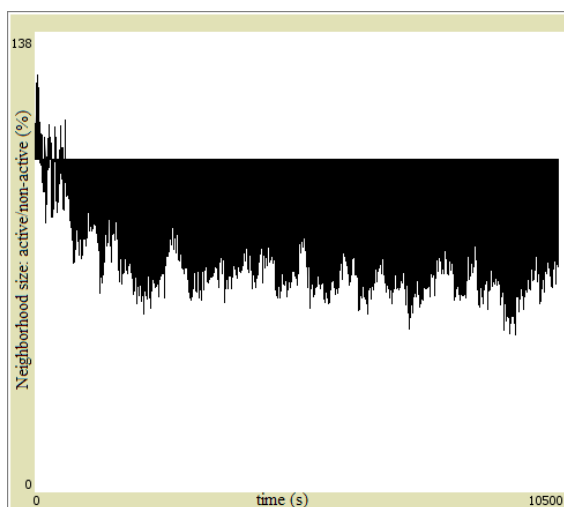


Fig. 12. Average size of active-agent neighborhood compared to nonactive agents.

4 Discussion

This work presents a model for peloton dynamics in competitive cycling, using an agent-based approach. Based on a few simple rules for dynamics and energy balance, we derived a rather complex pattern of convection in the peloton, and coherent results in terms of energy. Some interesting, counter-intuitive results arise from the proposed model.

The result where active cyclists spent less energy than non-active ones does not conform to general knowledge in cycling, There is, however, room for improvement and calibration in the model. For instance, when moving around, agents do not look for favourable positions in terms of drafting possibilities, and

the lack of data for drafting in diagonal positions and behind multiple cyclists hinder the accuracy of energy distributions.

Besides that, we do not account for many real-life factors that affect competitive cyclists. An example is wind, a factor that has major impact in many professional races. Frontal wind can be modelled as a difference in speed for the aerodynamical factor in our energy expenditure equation, but sidewinds require a whole new approach that was not within the scope of this work. Of course, there is a different dimension of cycling that was also not modelled here: the strategic part. "Intelligent" agents, who know what their best response to the circumstances of a race is, could create breakaways, become active or non-active mid-way during a race, save as much energy as possible for a final sprint. This is certainly feasible as a future model.

This is probably the first time an agent-based approach is used to try and simulate large-scale cycling peloton dynamics. Another work[1] take a similar approach, using an agent-based model to simulate results of cycling races. Many ideas between this work and their work are similar; in special, the energy balance is probably quite similar, even though they do not model uphill and downhill and this work takes a different approach for modelling the time to exhaustion of a cyclist. However, their focus is on obtaining final results of the races, while we want to simulate the dynamics of a peloton during the race. A different work[7] makes some proto-simulations of pelotons with drafting, looking for hysteresis on phase transitions. In spite of presenting interesting results, this model is not interested in simulating the complex dynamics of a peloton, but only in illustrating a concept.

As the first work to explore simulation of such a complex system as a cycling peloton, we do not expect this to be a complete work in any way, but rather to be a first step on exploring this fascinating phenomenon of collective behaviour. The results presented are certainly promising and show that a more complete model of this system is feasible and can even show similarities with other natural systems.

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