Problem sheet (1)

1. Show that the quantity $a_\mu a^\mu$ is invariant under a Lorentz boost along the x-axis.

[2]

2. Show that the Lorentz invariant phase-space for spinless electron-muon scattering,

$$e^-(p_A)\mu^-(p_B) \to e^-(p_C)\mu^-(p_D) \ ,$$

given by

$$\mathrm{d}Q = (2\pi)^4\delta^4(p_C + p_D - p_A - p_B)\frac{\mathrm{d}^3\vec{p}_C}{(2\pi)^3 2E_C}\frac{\mathrm{d}^3\vec{p}_D}{(2\pi)^3 2E_D}$$

can be reduced in polar coordinates to

$$\mathrm{d}Q = \frac{1}{4\pi^2}\frac{|\vec{p}_C|}{4\sqrt{s}}\mathrm{d}\Omega \ ,$$

where $\mathrm{d}\Omega$ is the element of solid angle and $s = (p_A + p_B)^2$. Hence, show that the differential cross-section for the process is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_C|}{|\vec{p}_A|}|\mathcal{M}|^2 \ .$$

[4]

3. In the centre of mass system, in the very high-energy limit, show that this cross-section can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4s}\left(\frac{3 + \cos\theta}{1 - \cos\theta}\right)^2 \ ,$$

where $\alpha = e^2/4\pi$ and θ is the scattering angle.

[4]