

# Measurements, memories and time



## Quantum measurement: a dialog of big and small

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Dip. Fisica, INFN Sez. Pavia,  
Universita' di Pavia

FQxI Foundation,  
“The physics of what happens”

**QUIT**  
quantum information  
theory group  
[www.qubit.it](http://www.qubit.it)



# Quantum measurement

- What is it?
- Measurement problem
- Subjectivity of the Born rule
- What is the role of time?
- Quantum metrology



# What is a quantum measurement?



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# Quantum measurement

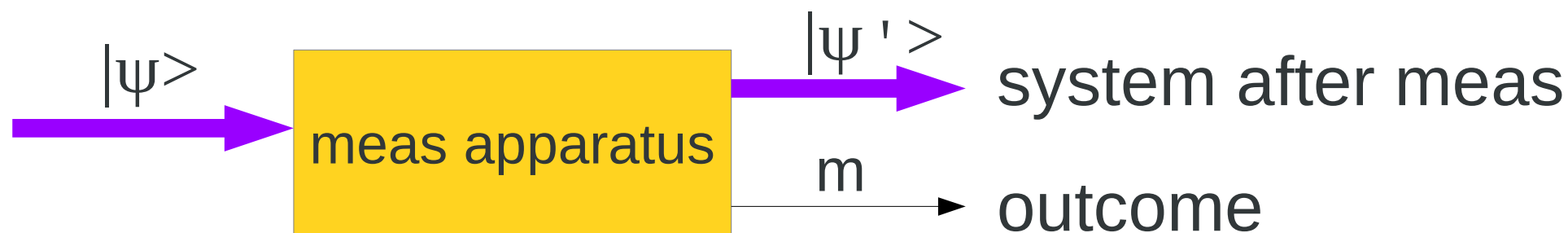
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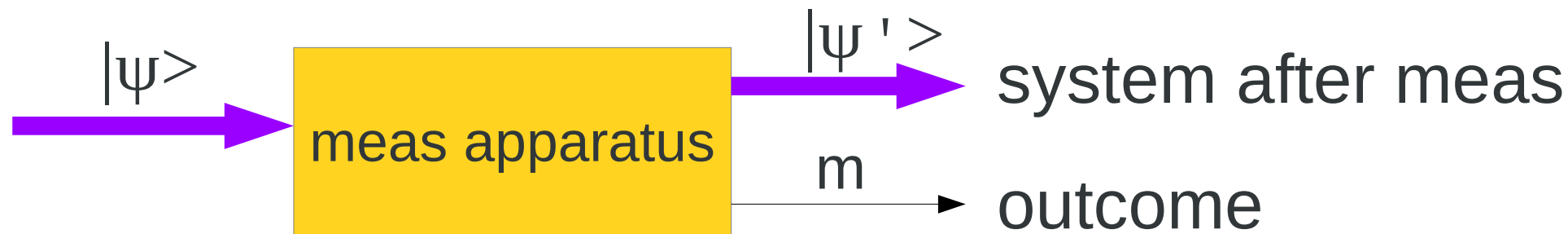
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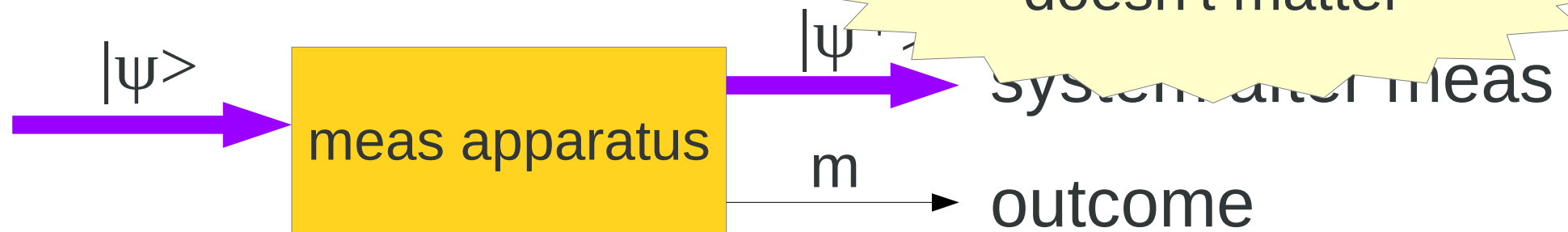


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We cannot directly compare things at different times, but only different records at the same time. We cannot know the past except through its records in the present, so it is only present records that we can really test.

# measurement problem



measurement problem

# Quantum mechanics



# Quantum mechanics

1. States postulate
2. tensor product postulate
3. Schroedinger eq: an isolated q system evolves **unitarily** (hence reversibly and deterministically).
4. Measurement postulate: during a measurement the probability of an outcome is the **Born rule**



Parenthesis

Measurement problem

Quantum mechanics



Born rule vs. "collapse" of the wave fn

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**Born rule:** the outcome  $a$  of an observable

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B.R. +  $p(x, y) = p(x|y)p(y) \Rightarrow$  collapse

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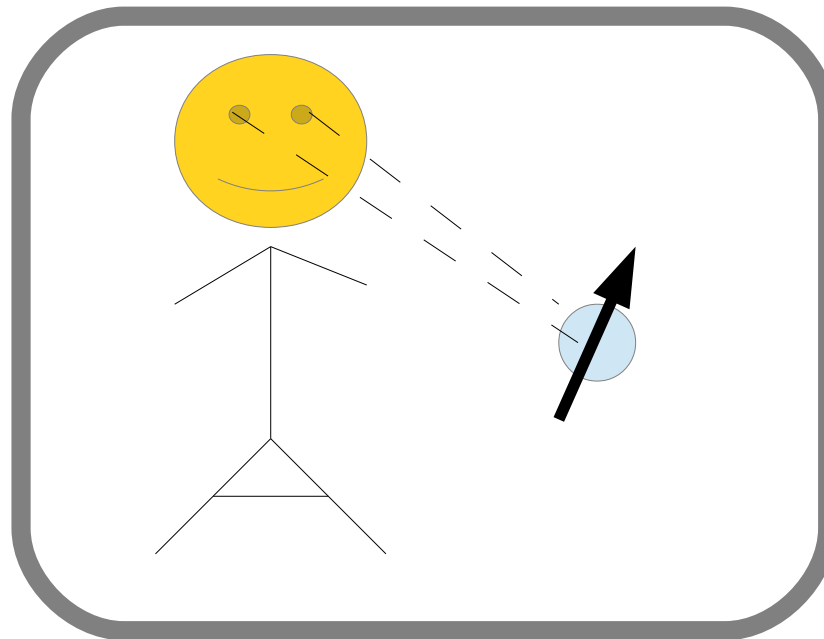


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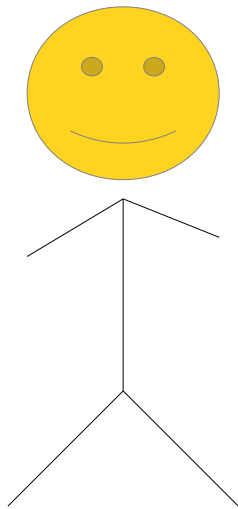


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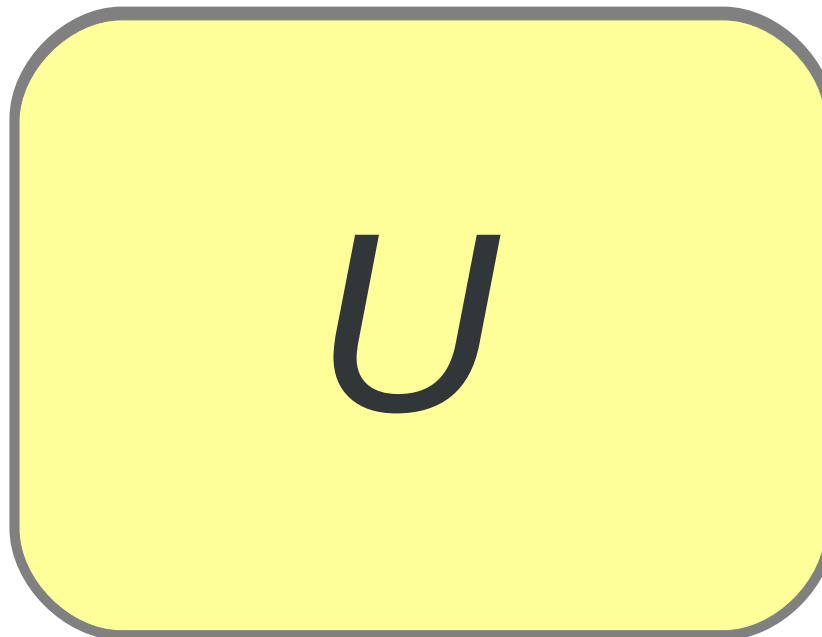
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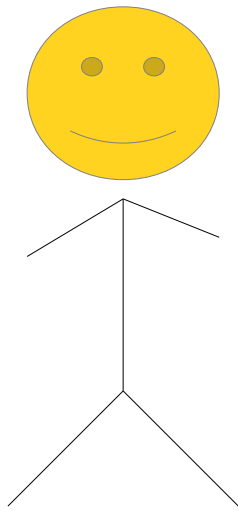


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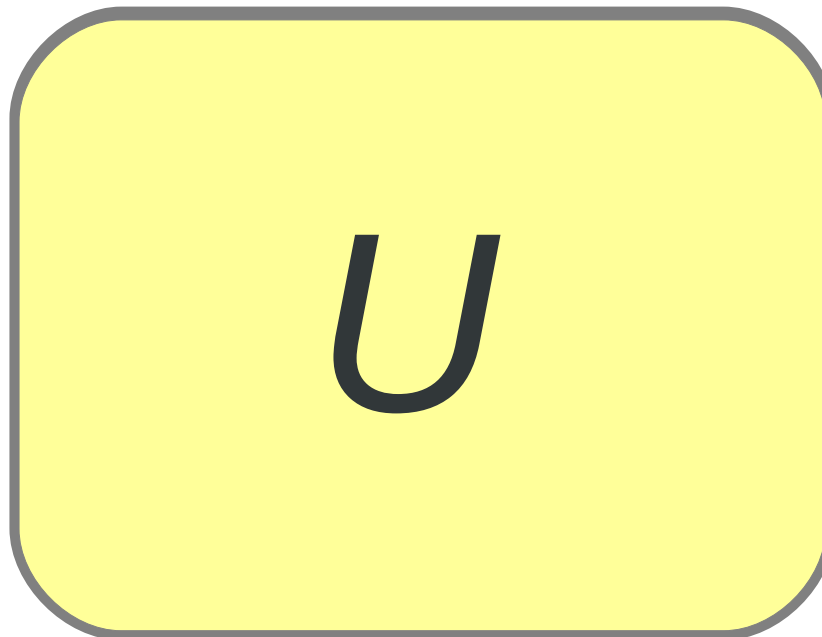


whether the scientist sees a definite outcome depends on who you ask. Alice: yes, Bob: no

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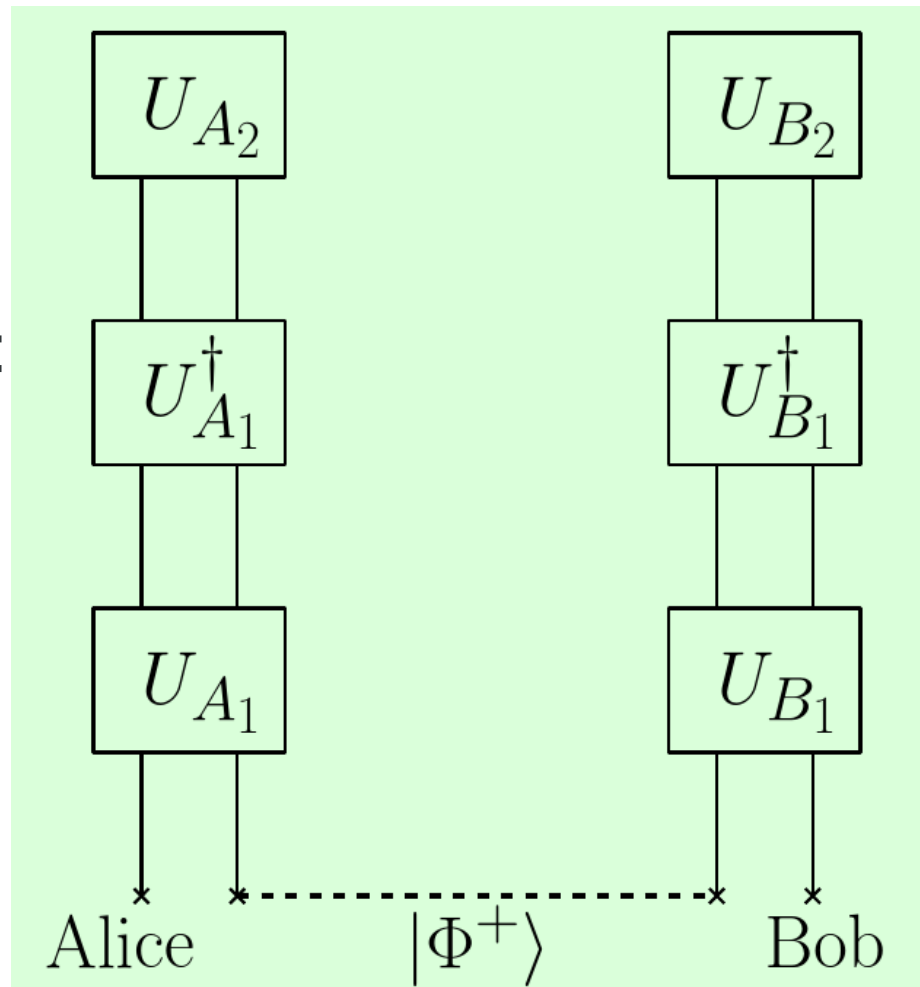


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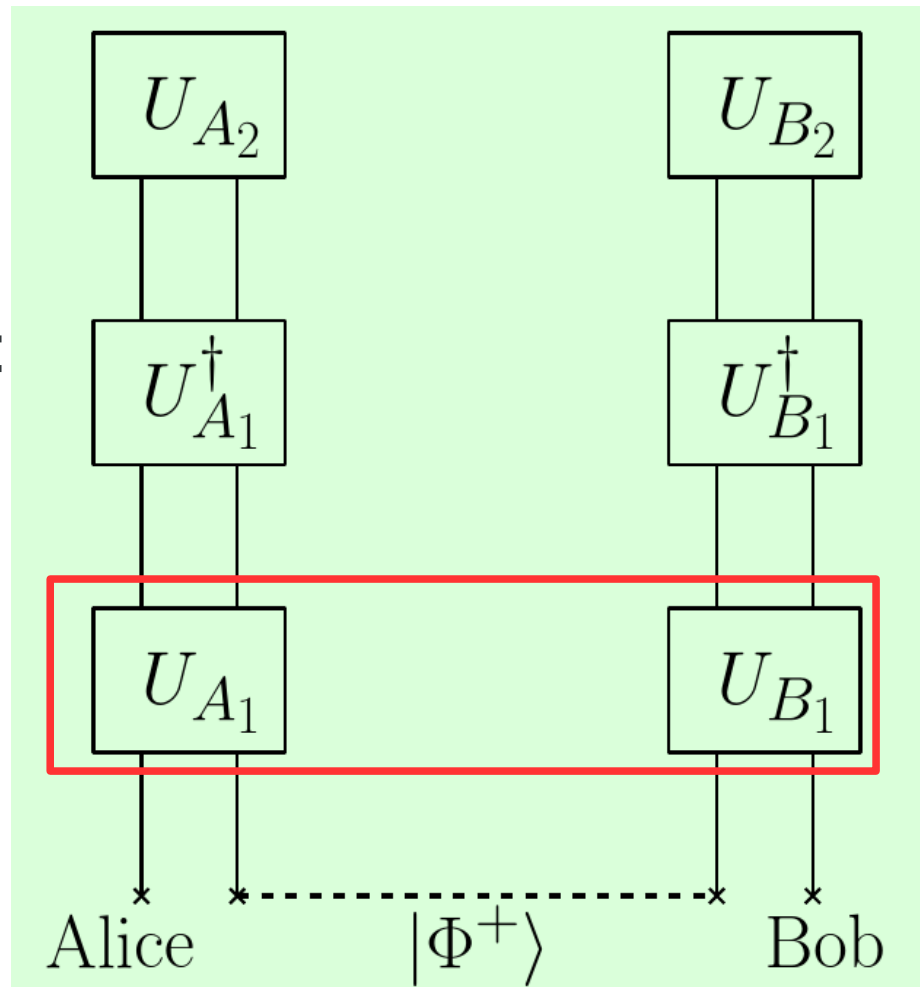


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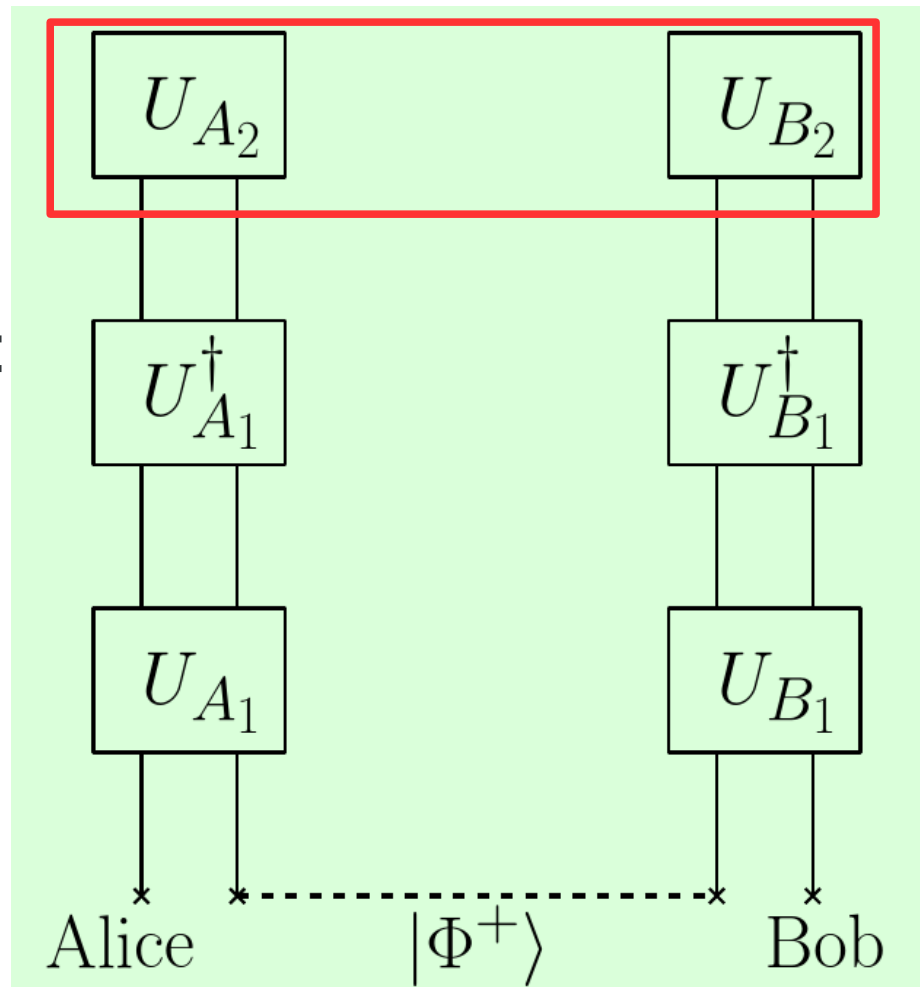


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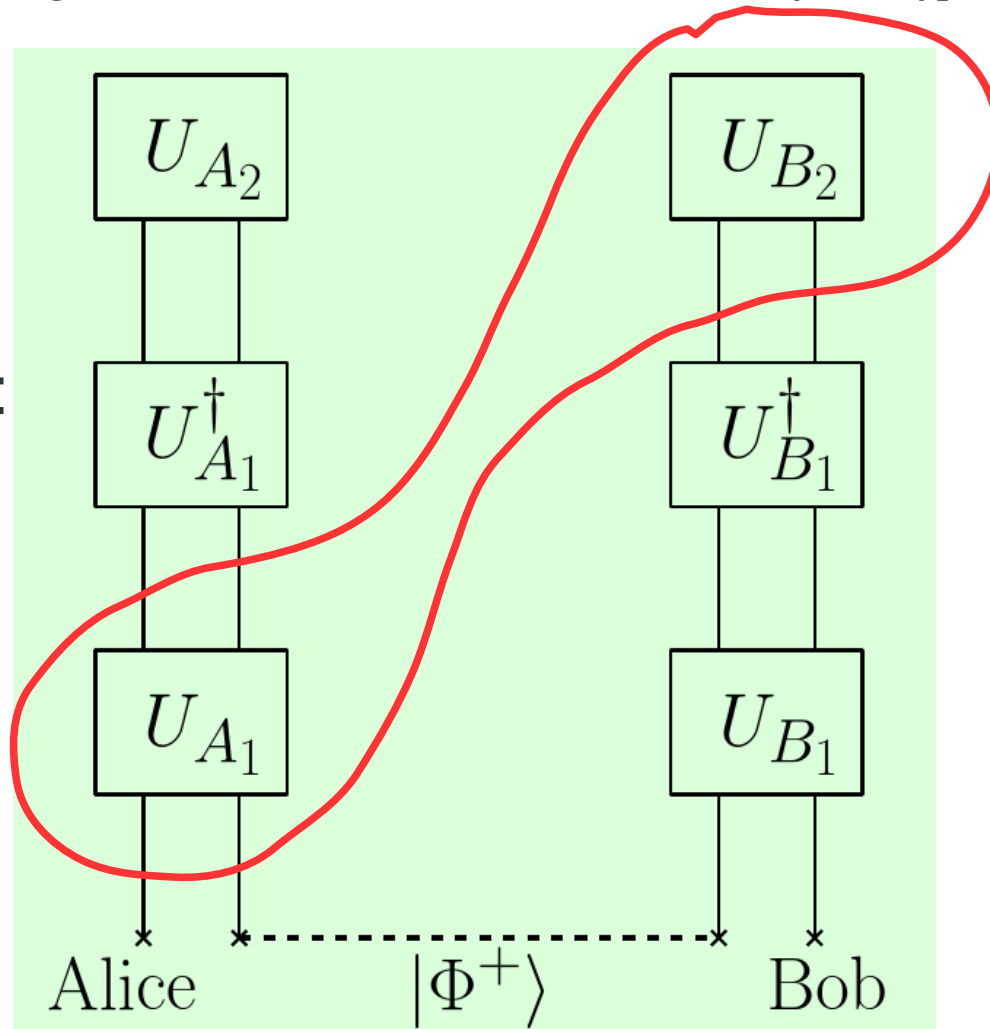


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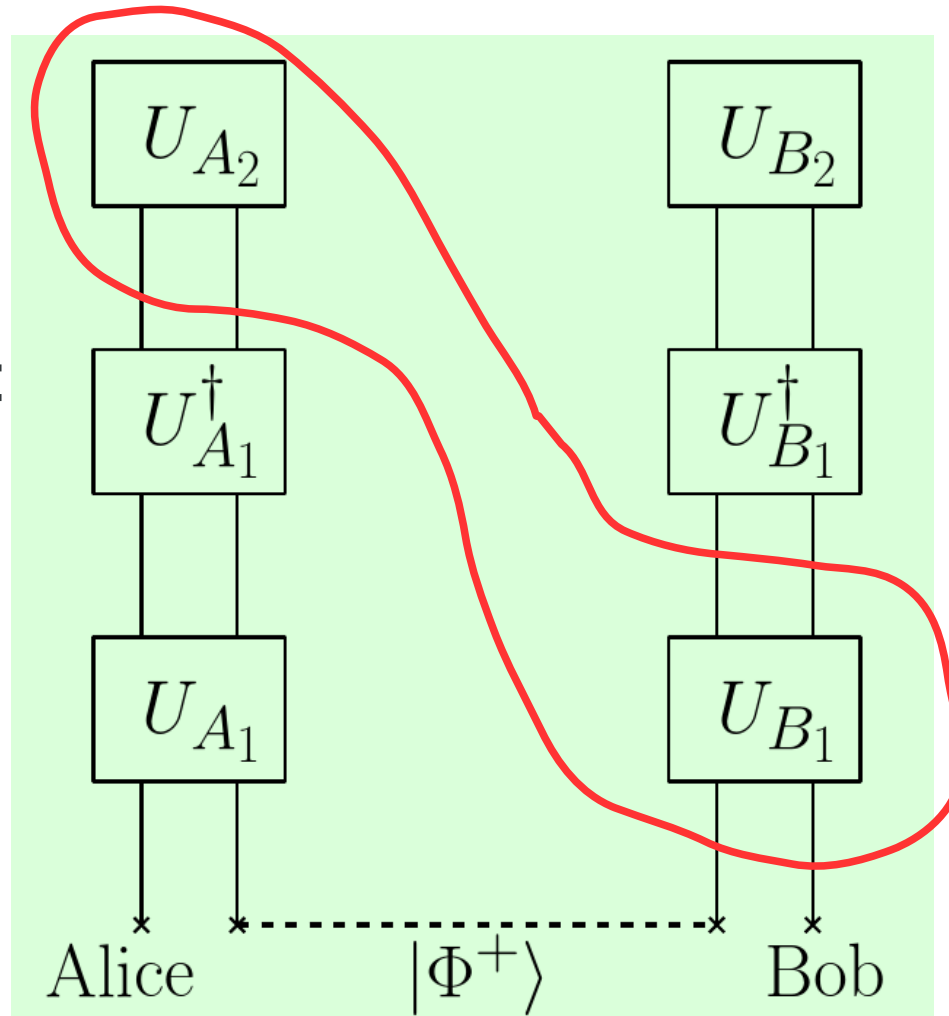


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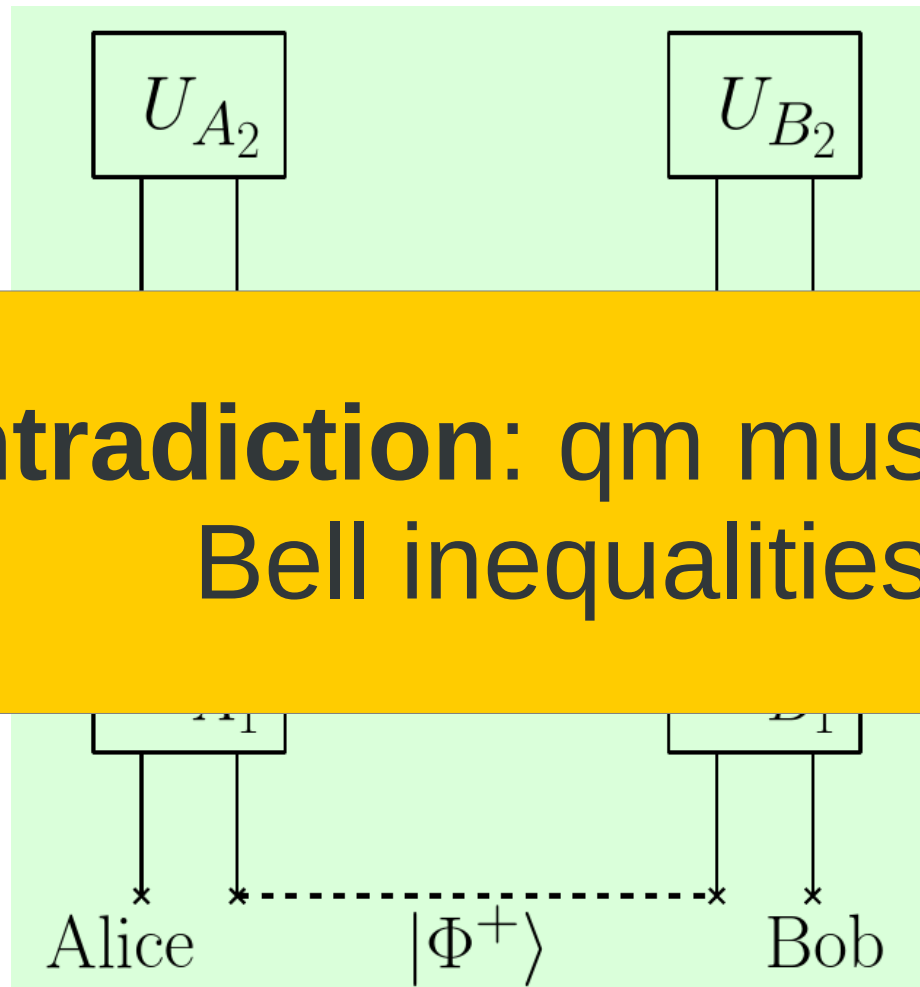


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**contradiction: qm must violate  
Bell inequalities!**



way out:

if we erase the memory, the  
measurement hasn't happened

(remember the importance of the memory in a measurement)

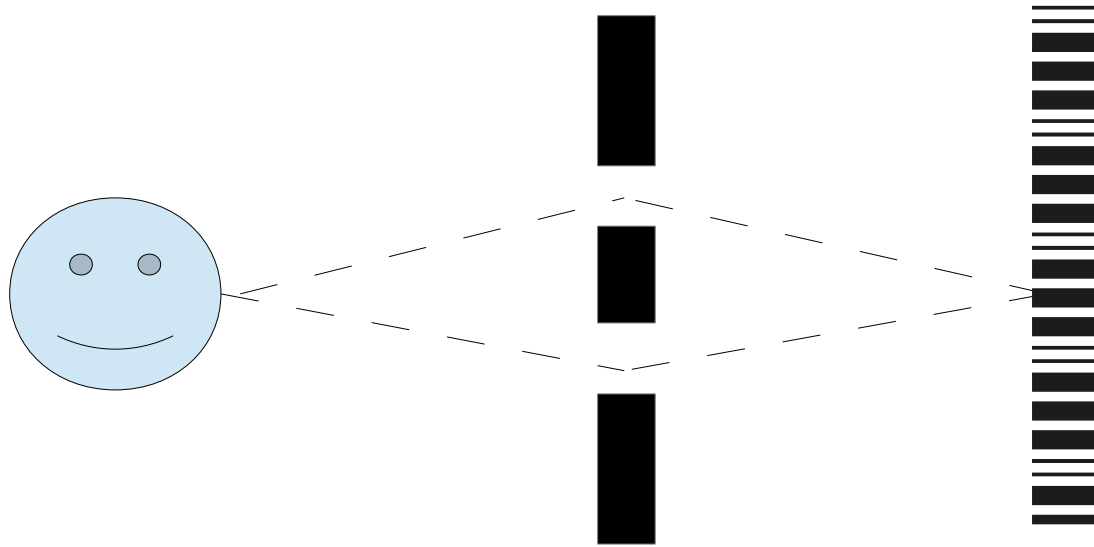


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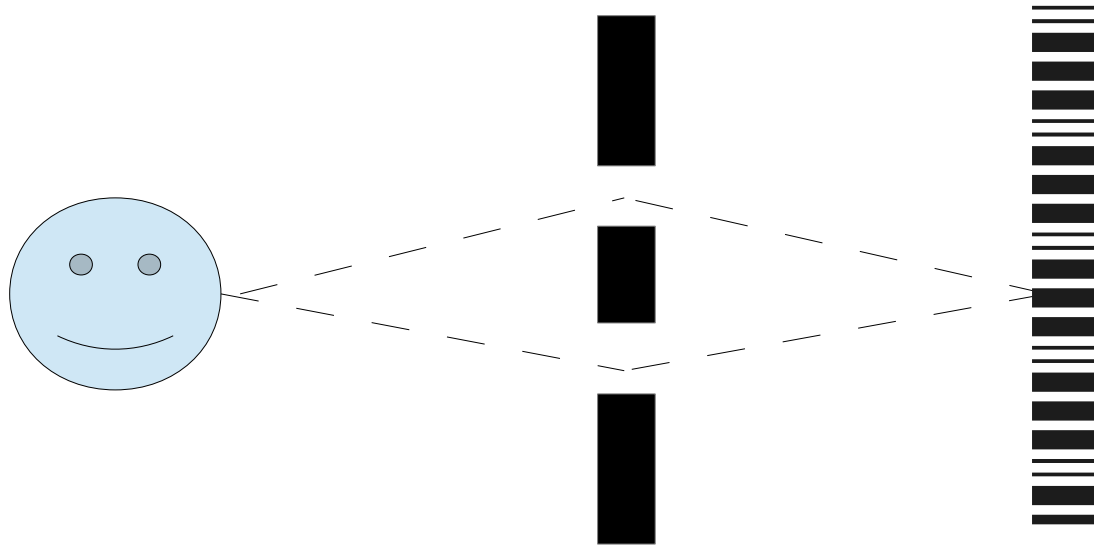


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If he remembers where he passed, no interference. If he doesn't, can we say that he passed through **one** slit? If so, which one?

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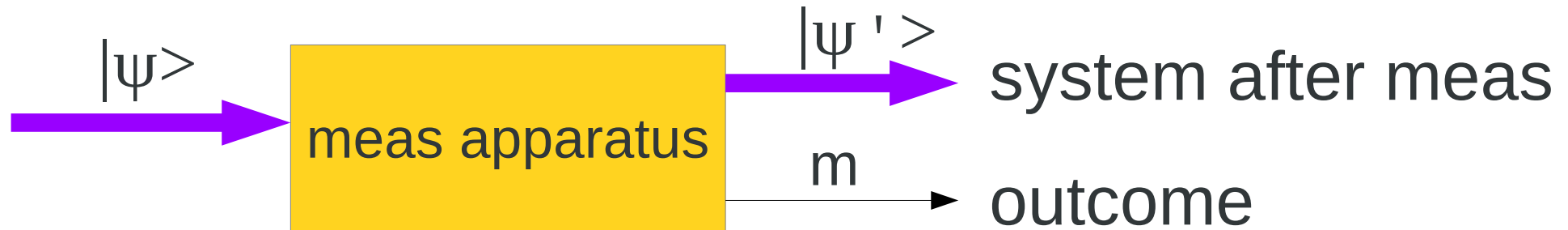
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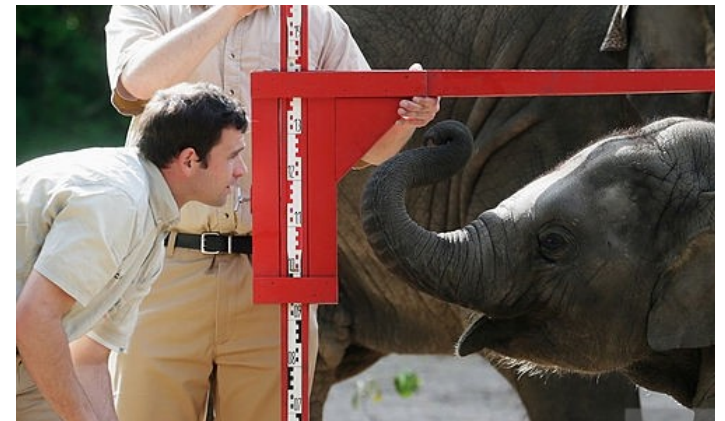
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we'll keep unmodified qm for the rest of the talk

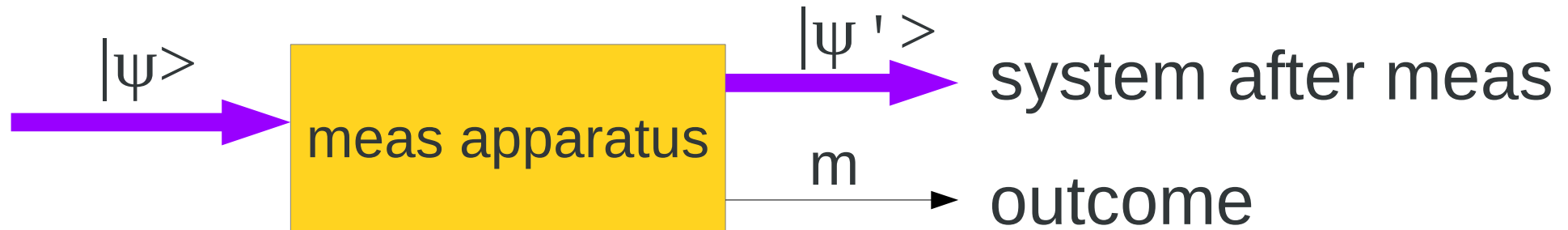
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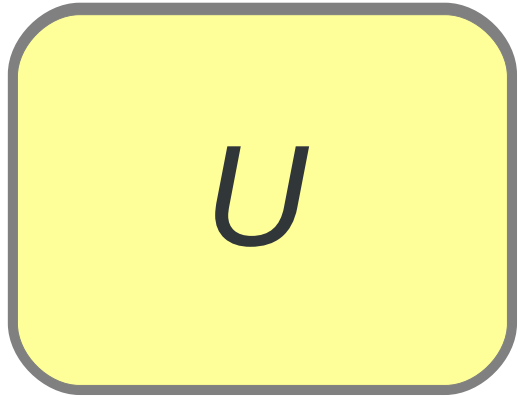
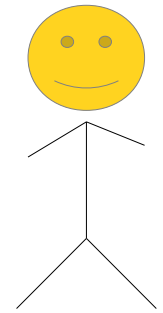
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**Yes** (von Neumann)  
use 2 stages

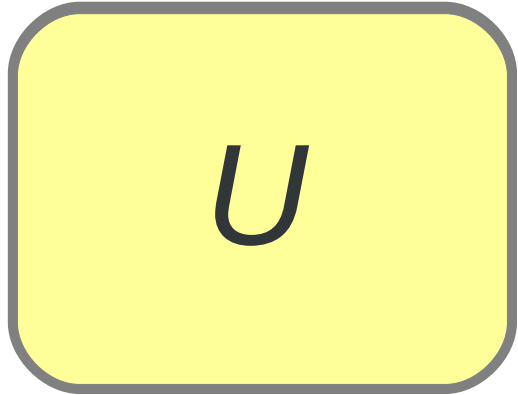
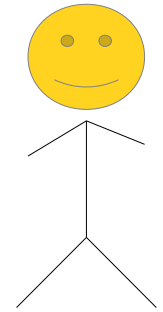


**stage 1:** pre-measurement  
interaction





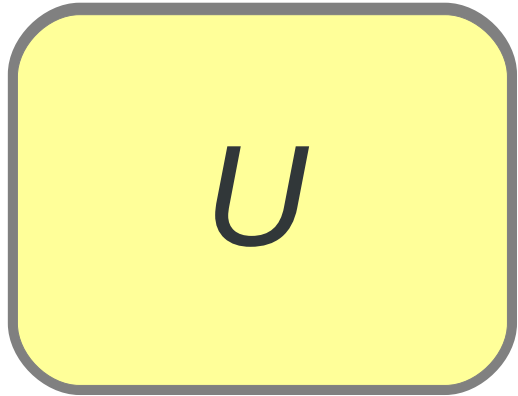
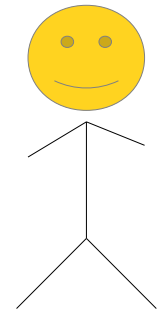
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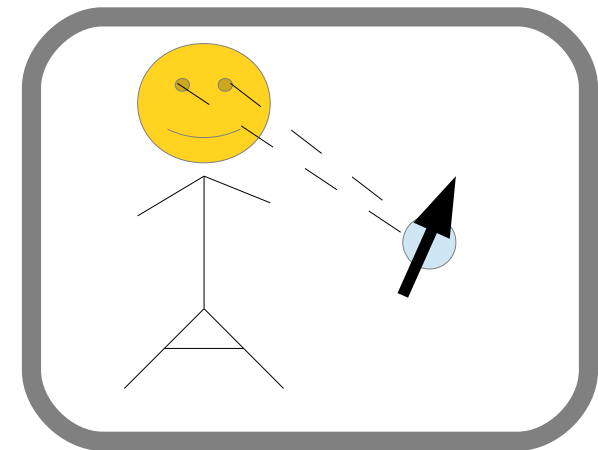


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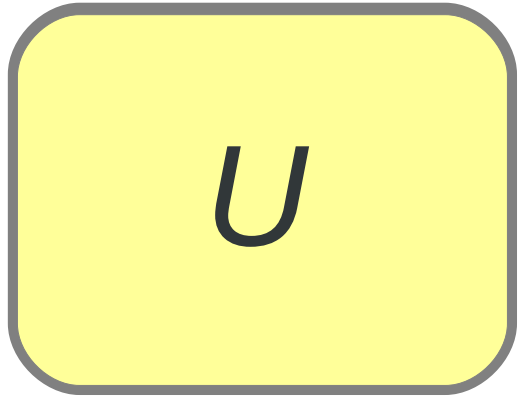
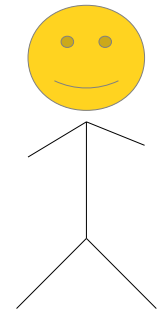
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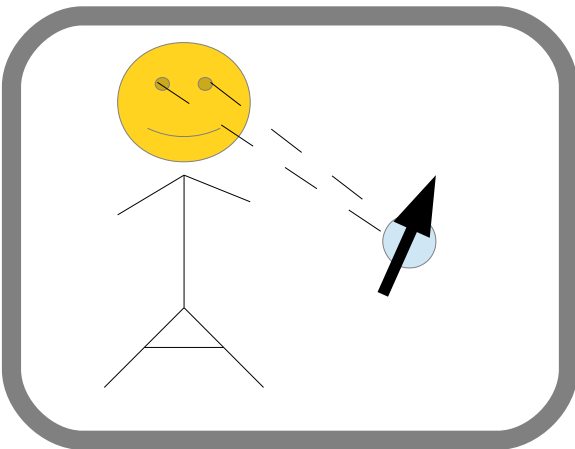


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$$\text{Outcome } a, p(a) = |\psi_a|^2$$

when/where does stage 2 happen?





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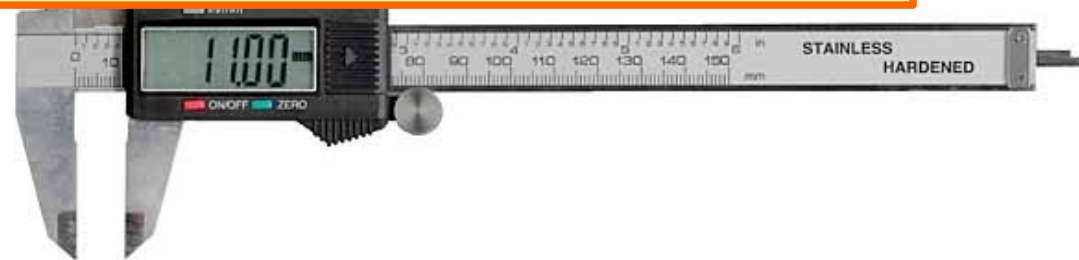
**YES!!!!**

connection measurement-time problem Ask for details...

Quantum measurements: only  
foundational relevance?

No! Practical aspects of  
q. technologies:

**Quantum METROLOGY**



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**Usually:**  $\sqrt{N}$  enhancement: the error goes as  $1/N$



# Example: quantum metrology

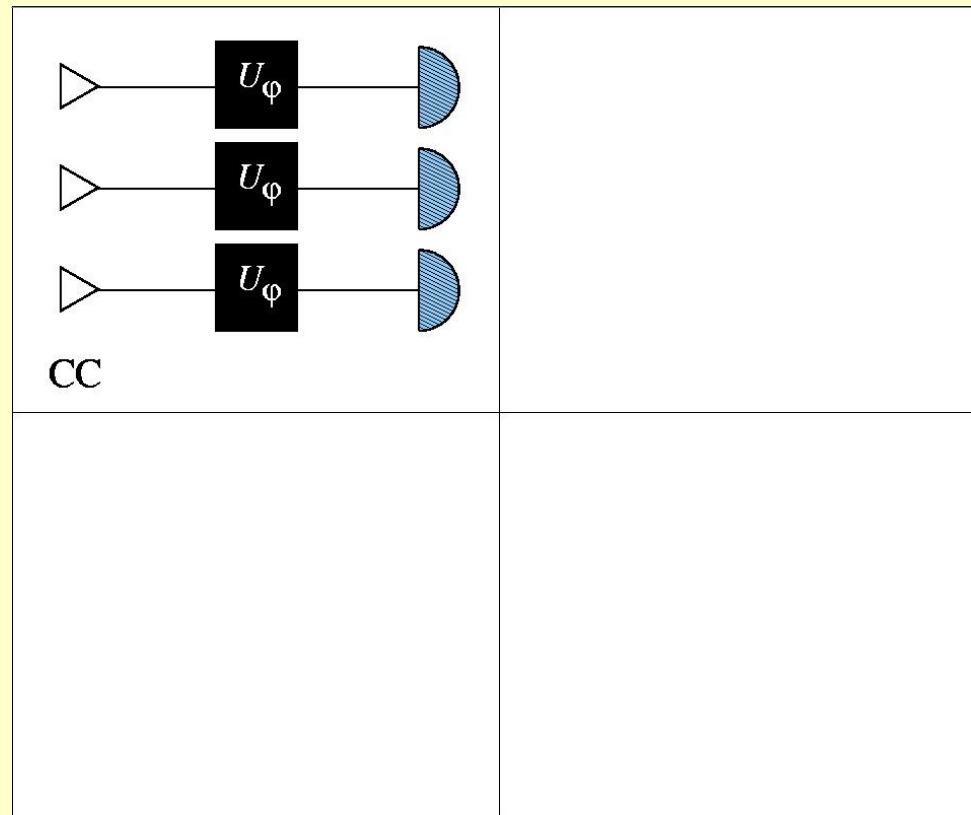


Phys. Rev. Lett. **96**, 010401 (2006)

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## Parallel strategies

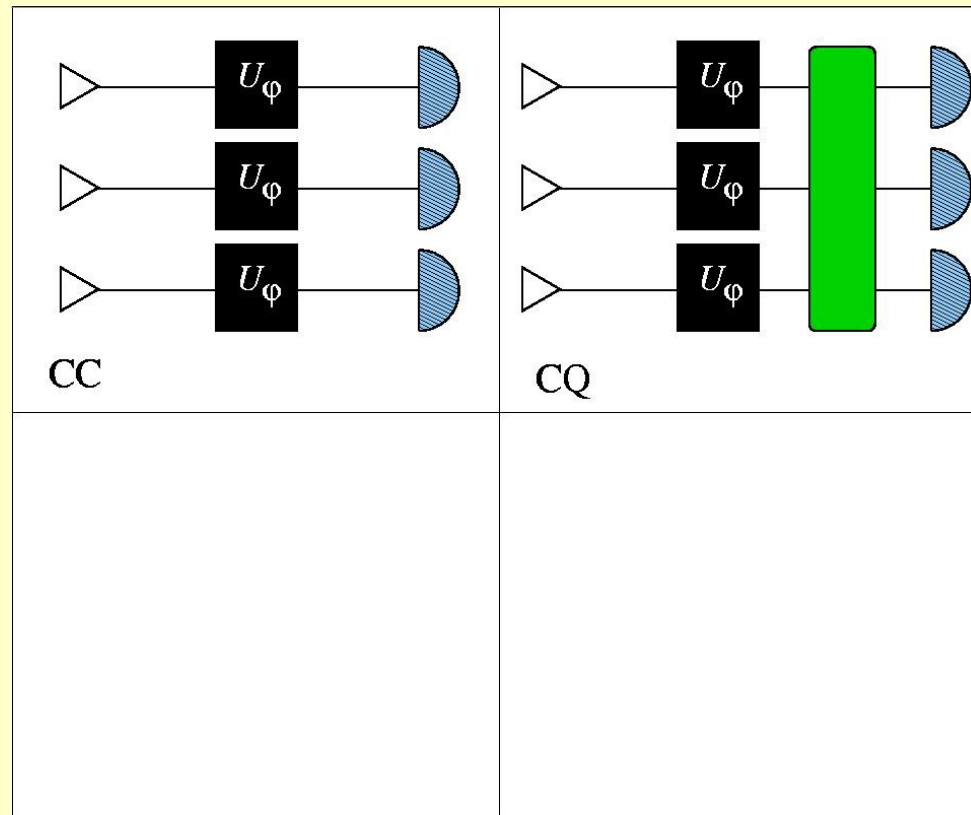
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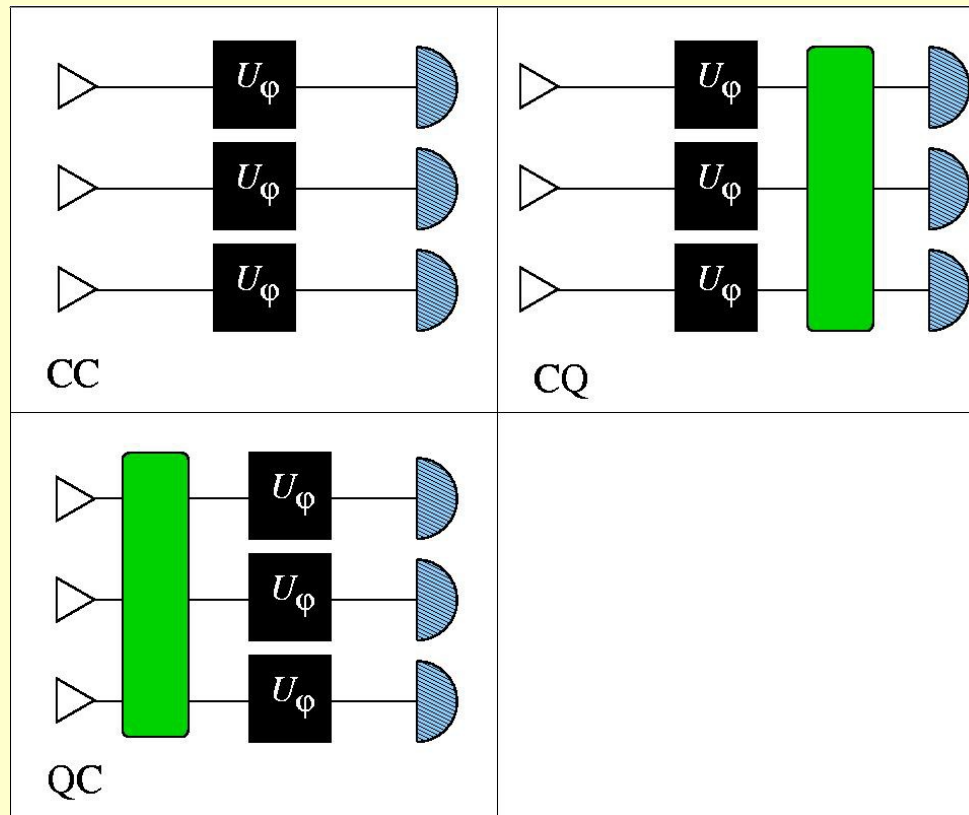
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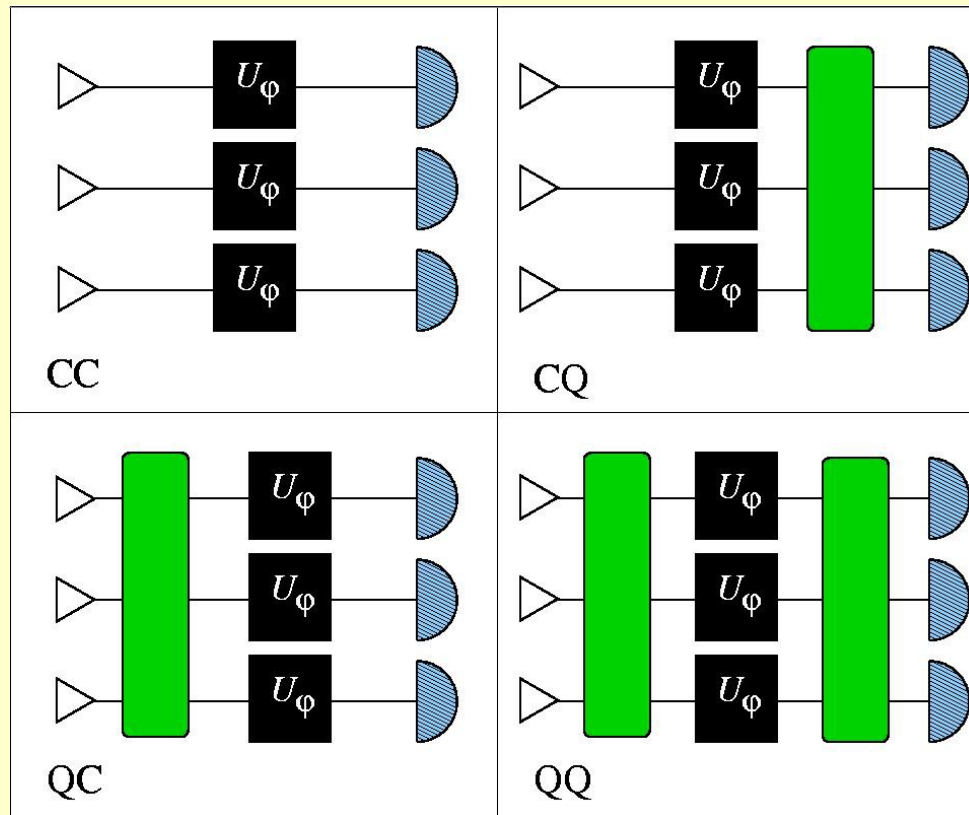
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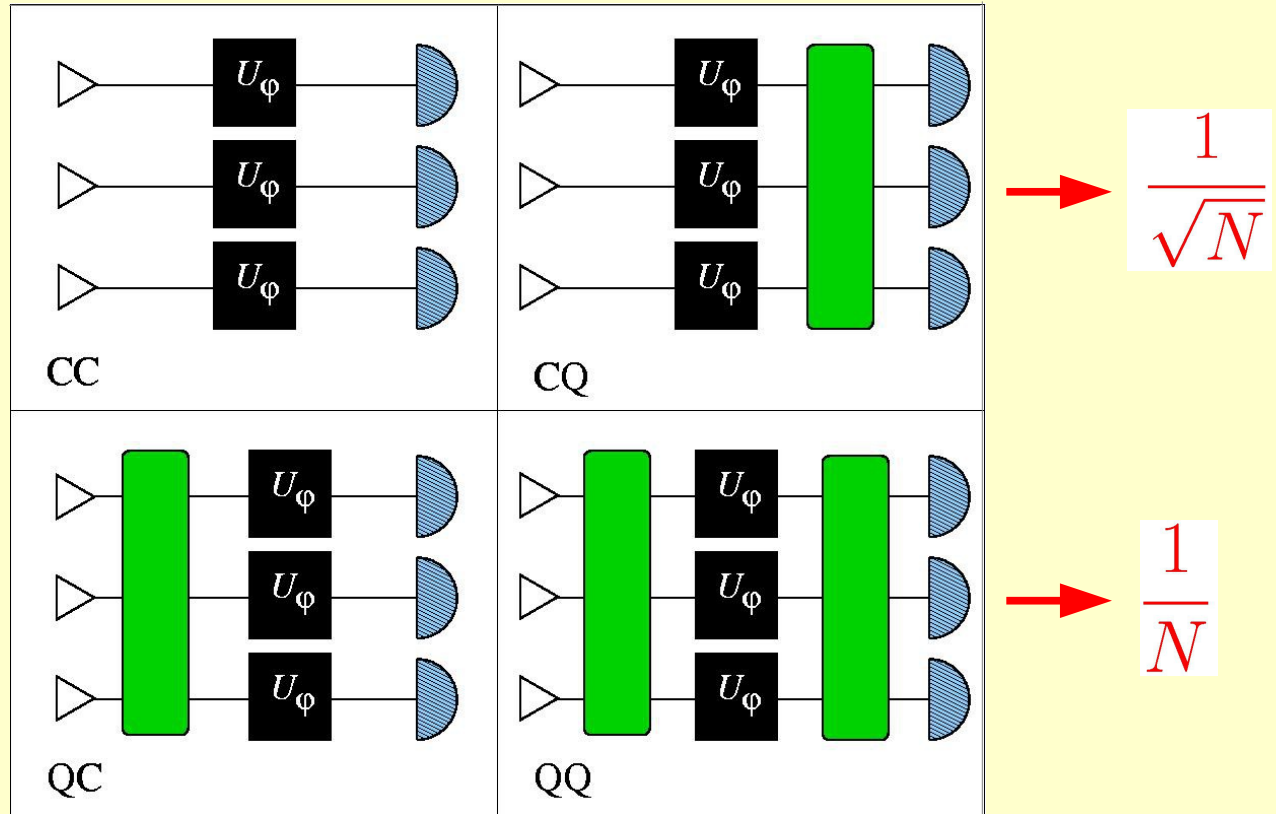




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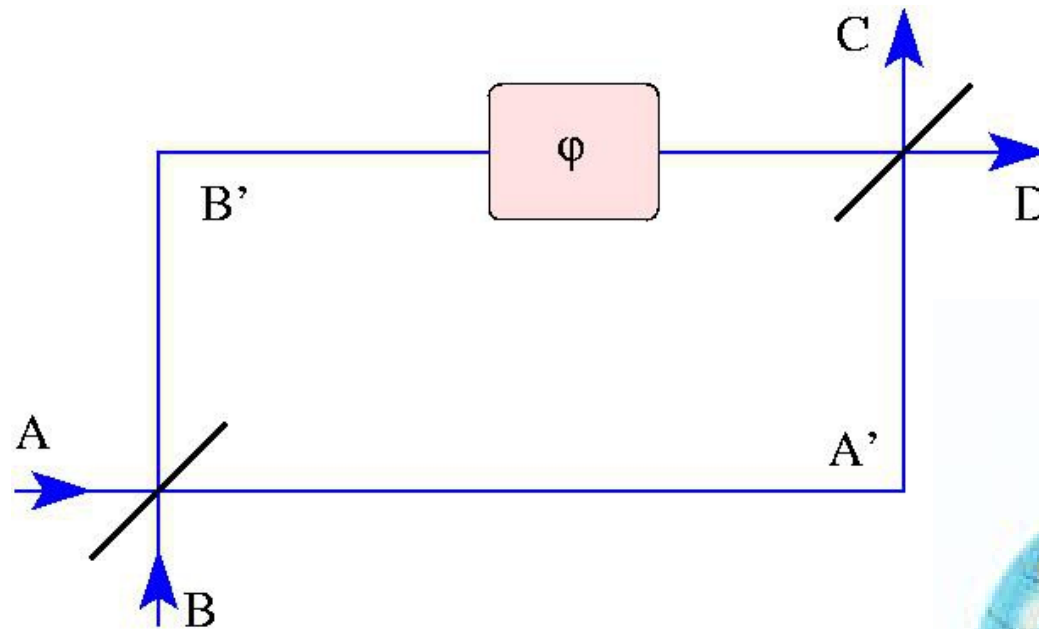
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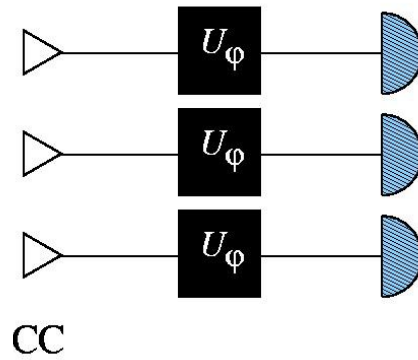


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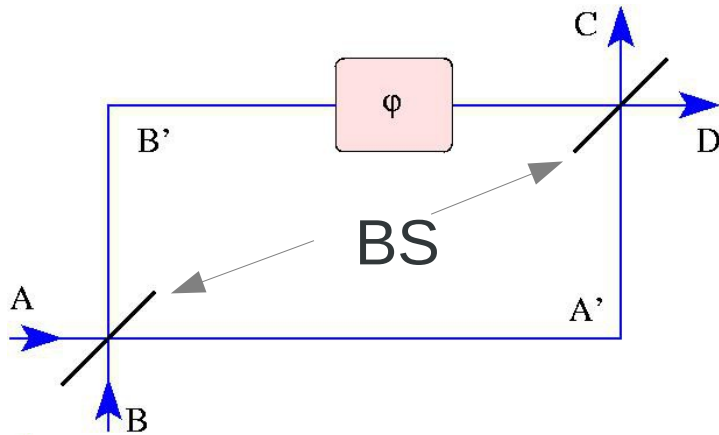
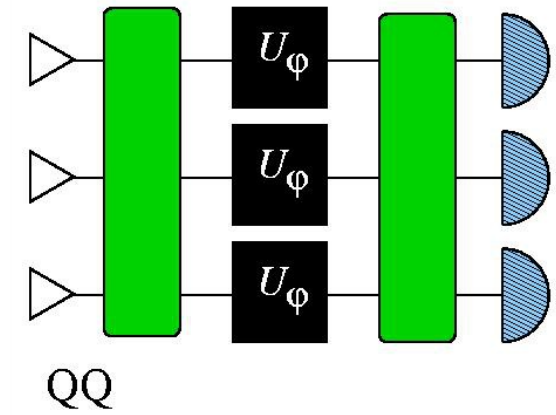
Measuring the phase in  
an optical interferometer



Classical strategy:



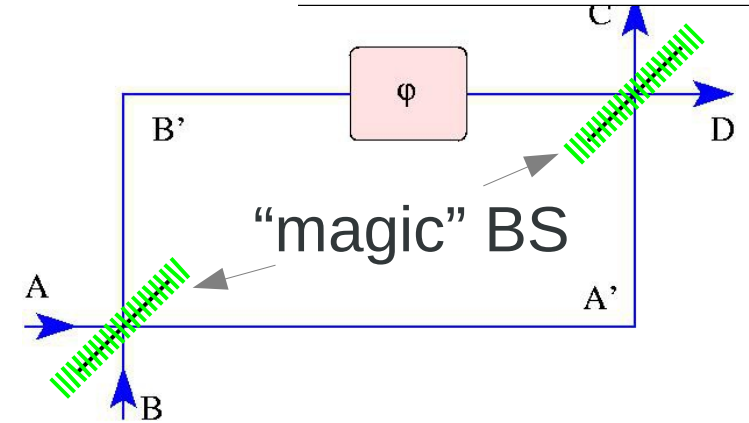
Quantum strategy:



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \times N$$

each of the  $N$  photons is treated independently in the interferometer

$$\Rightarrow \Delta\varphi = 1/\sqrt{N}$$



$$\frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + e^{iN\varphi}|1\rangle^{\otimes N})$$

the  $N$  photons are “collectively” employed (i.e. they are entangled) in the interferometer

$$\Rightarrow \Delta\varphi = 1/N$$

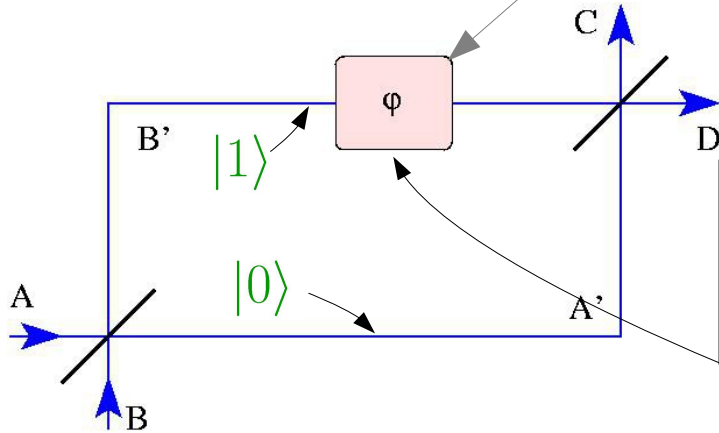


# Example



## estimation of a phase $\varphi$ from a Mach-Zehnder interferometer

[or a Ramsey interferometer or a discrete FT, see Rosetta stone: J. Mod. Opt. 49, 2325 (2002)].



$N$  photons at the input  $A \rightarrow NP_\varphi$  at output  $C$  and  $N(1 - P_\varphi)$  at output  $D$ . What's  $\varphi$ ?

Beam splitter:

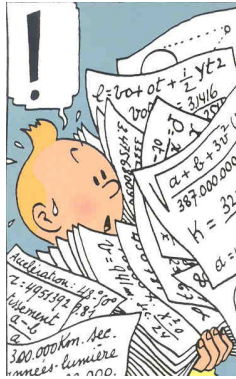
$$|0\rangle \leftrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|1\rangle \leftrightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Phase shift:

$$|0\rangle \rightarrow |0\rangle$$

$$|1\rangle \rightarrow e^{i\varphi}|1\rangle$$



$$\Rightarrow P_\varphi = |\langle \Psi_{in} | \Psi_{out} \rangle|^2 = \frac{1}{4} |(\langle 0| + \langle 1|)(|0\rangle + e^{i\varphi}|1\rangle)|^2 = \cos^2(\varphi/2)$$

and the variance of the mean (from the variance of a binomial):  $\Delta^2 P_\varphi = P_\varphi(1 - P_\varphi)/N$

so that from error propagation:

$$\Delta\varphi = \frac{\Delta P_\varphi}{|\partial P_\varphi / \partial \varphi|} = \frac{\sqrt{\cos^2 \frac{\varphi}{2} \sin^2 \frac{\varphi}{2}}}{\sqrt{N} |\cos \frac{\varphi}{2} \sin \frac{\varphi}{2}|} = \frac{1}{\sqrt{N}}$$

# ENTANGLEMENT does it better!!

Instead of using  $N$  times the state  $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$  in the interferometer, use ONCE the  $N$  photon state:

$$\frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}) \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + e^{iN\varphi}|1\rangle^{\otimes N})$$

Now, the overlap between input and output states is:

a  
n  
d

$$\Delta^2 P'_\varphi = P'_\varphi(1 - P'_\varphi)$$

$$\Rightarrow \Delta\varphi = \frac{\Delta P'_\varphi}{|\partial P'_\varphi / \partial \varphi|} = \frac{\sqrt{\cos^2 \frac{N\varphi}{2} \sin^2 \frac{N\varphi}{2}}}{N |\cos \frac{N\varphi}{2} \sin \frac{N\varphi}{2}|} = \frac{1}{N}$$

$A\sqrt{N}$  gain over the classical strategy!!!!





N.b. We are using a “modified” MZ: the necessary state cannot be obtained from a conventional beam splitter.

# Quantum measurement


- What is it? —————> role of memory
- Measurement problem
- Subjectivity of the Born rule
- What is the role of time?
- Quantum metrology



Take home message



Measurement:  
correlation between  
q system and **memory**



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