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# GLOBAL SYMMETRIES &

# MACROSCOPIC QUANTUM SYSTEMS

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[QTIF.WEBLY.COM](http://QTIF.WEBLY.COM)

# SUMMARY

- ★ Macroscopic quantum systems
- ★ Global symmetries and Generalized Coherent States
- ★ Entangling dynamics of micro & macro
- ★ Measurement process

# MACROSCOPIC QUANTUM SYSTEMS

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MACROSCOPIC : large number  $N$  of quantum particles



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QUANTUM

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**MACROSCOPIC** : large number **N** of quantum particles

**QUANTUM** : finite dimension of the  
**EFFECTIVELY EXPLORED** Hilbert space

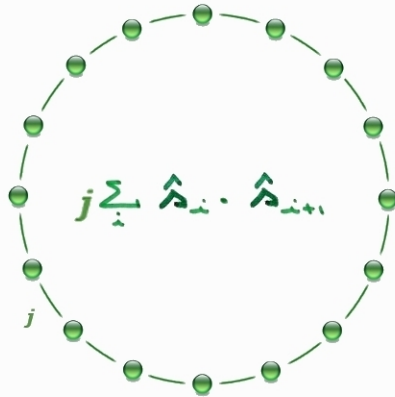
# MACROSCOPIC QUANTUM SYSTEMS

MACROSCOPIC

:

large number  $N$  of quantum particles

spin-ring



$$\hat{S} \equiv \sum_i \hat{S}_i$$

$$|\hat{S}|^2 = s(s+1)$$

QUANTUM

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finite dimension of the

EFFECTIVELY EXPLORED Hilbert space

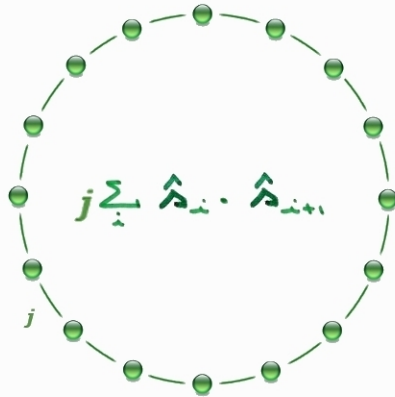
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QUANTUM

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finite dimension of the

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$S = \text{constant}$

:



$$S = 1/2$$

quantum

$$S = N/2$$

$\sim$  classical

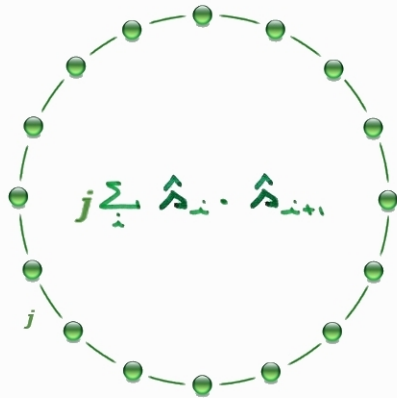
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A QUANTUM THEORY

$Q_N$

A QUANTUM THEORY with A GLOBAL SYMMETRY

$Q_N$

+

$X(N)$

A QUANTUM THEORY with A GLOBAL SYMMETRY

$$Q_N + X(N)$$

$$\hat{A}^N$$

GLOBAL PROPERTY / COLLECTIVE PHYSICAL QUANTITY

$$[\hat{A}^N, \psi] = 0 \quad : \quad \forall \psi \in X(N)$$



A QUANTUM THEORY with A GLOBAL SYMMETRY

$$Q_N + X(N)$$

$$\hat{A}^N$$

GLOBAL PROPERTY / COLLECTIVE PHYSICAL QUANTITY

$$[\hat{A}^N, \mathcal{U}] = 0 \quad : \quad \forall \mathcal{U} \in X(N)$$



$$\mathcal{U} |\equiv^l\rangle_N = |\equiv^i\rangle_N$$

$$\langle \equiv^l | \hat{A}^N | \equiv^l \rangle_N = \langle \equiv^i | \hat{A}^N | \equiv^i \rangle_N$$

$$\{ |\equiv^i\rangle_N \}_N$$



SYM-EQUIVALENT



A QUANTUM THEORY with A GLOBAL SYMMETRY

$Q_N$

+

$X(N)$

$\{ | \equiv \rangle \}_N \sim$

SYM-EQUIVALENT

A QUANTUM THEORY with A GLOBAL SYMMETRY

$Q_N$

+

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SYM-EQUIVALENT



$2 \rightarrow \infty$

A QUANTUM THEORY with A GLOBAL SYMMETRY

$$Q_N + X(N)$$

$$\{|\Xi^i\rangle_N\}_N$$

SOME SYM-EQUIVALENT



$N \rightarrow \infty$  BECOME

CLASSICALLY EQUIVALENT



$$\{|\Omega^i\rangle_N\}_N$$

$$: \lim_{N \rightarrow \infty} \langle \Omega^i | \hat{A}^N | \Omega^i \rangle_N = A(\Omega) < \infty$$

2 → 8

$$2 \rightarrow \infty$$

Q : Hilbert space, Lie algebra,  $\hat{H}$  & dynamical group  $G$ ,  $\langle \Omega | \hat{A} | \Omega \rangle$

$$N \rightarrow \infty$$

Q : Hilbert space, Lie algebra,  $\hat{H}$  & dynamical group  $G$ ,  $\langle \Omega | \hat{A} | \Omega \rangle$

C : Phase-space, Poisson brackets,  $h(\Omega)$ ,  $A(\Omega)$

$N \rightarrow \infty$

Q : Hilbert space, Lie algebra,  $\hat{H}$  & dynamical group  $G$ ,  $\langle \Omega | \hat{A} | \Omega \rangle$   
C : Phase-space, Poisson brackets,  $h(\Omega)$ ,  $A(\Omega)$



" CAN ONE FIND A CLASSICAL SYSTEM WHOSE DYNAMICS IS EQUIVALENT TO SOME  $N \rightarrow \infty$  LIMIT OF A GIVEN QUANTUM THEORY "



# GENERALIZED COHERENT STATES

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$\mathcal{H}$

Hilbert space

$\hat{H}, G$

Dynamical group

$|R\rangle$

Reference state

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Dynamical group

 $|R\rangle$ 

Reference state

$$\mathbb{F} : \hat{f}|R\rangle = e^{i\varphi_f}|R\rangle \quad \forall \hat{f} \in \mathbb{F}, \quad G/\mathbb{F}, \quad \hat{\Omega} \in G/\mathbb{F}$$

$$|\Omega\rangle = \hat{\Omega}|R\rangle$$

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$\hat{H}, G$

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Hilbert space

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$$|\Omega\rangle = \hat{\Omega} |R\rangle$$

$$|\Omega\rangle \longleftrightarrow \hat{\Omega} \in G/F \longleftrightarrow \Omega \text{ in a manifold } \mathcal{M}$$

ONE-TO-ONE CORRESPONDENCE



$$\int_{\mathcal{M}} d\mu(\Omega) |\Omega\rangle \langle \Omega| = \mathbb{1}_{\mathcal{H}}$$

$d\mu(\Omega)$  invariant measure

$$\langle \Omega | \Omega' \rangle = \delta(\Omega - \Omega')$$

$Q_N$

$G_N$

$\Omega_N, \mathcal{H}_N$

$Q_N$ with  $X(N)$  symmetry $G_N$  $\Omega_N, \mathcal{M}_N$ sym-  
equivalent $\{\Omega_N^i\}_N$

$Q_N$

with  $X(N)$  symmetry

$G_N$

$\Omega_N, \mathcal{M}_N$

$\{\Omega_N^i\}_N$

sym-  
equivalent



$2 \rightarrow 8$

C

$Q_N$  with  $X(N)$  symmetry

$G_N$

$|\Omega\rangle_N, \mathcal{M}_N$

$\{|\Omega^i\rangle_N\}_N$

sym-equivalent



classically equivalent

$\{|\Omega^i\rangle_N\}_N$

${}_N\langle \Omega^i | \hat{A}_N | \Omega^i \rangle_N$

$N \rightarrow \infty$



$C$

phase-space

$\Omega$

$A(\Omega) < \infty$



$Q_N$

$Q_N$  with  $X(N)$  symmetry



$G_N$

$\mathcal{H}_N, \mathcal{K}_N$

$Q_N$

$Q_N$  with  $X(N)$  symmetry



$G_N$



$G_K$

$\mathcal{M}_N, |\Omega\rangle_N$

$\mathcal{M}_K, |\Omega\rangle_K$

$Q_N$

AND

$Q_k$

$Q_N$  with  $X(N)$  symmetry

$Q_k$  with  $k = \frac{1}{N}$



$G_N$



$G_k$

$\cong$

$\mathcal{M}_N, |\Omega_N|$

$\mathcal{M}_k, |\Omega_k|$

EXACT

$Q_N$

AND

EFFECTIVE

$Q_k$

$Q_N$  with  $X(N)$  symmetry

$Q_k$  with  $k = \frac{1}{N}$



$G_N$



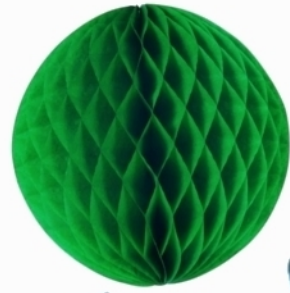
$G_k$

$\cong$

$\mathcal{M}_N, |\Omega| > N$

$\mathcal{M}_k, |\Omega| > k$

$\lim_{z \rightarrow \infty} Q_z$



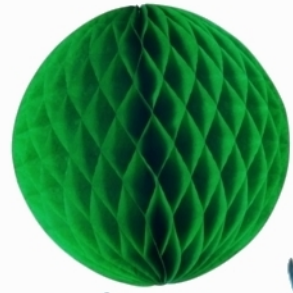
=

$\lim_{k \rightarrow 0} Q_k$



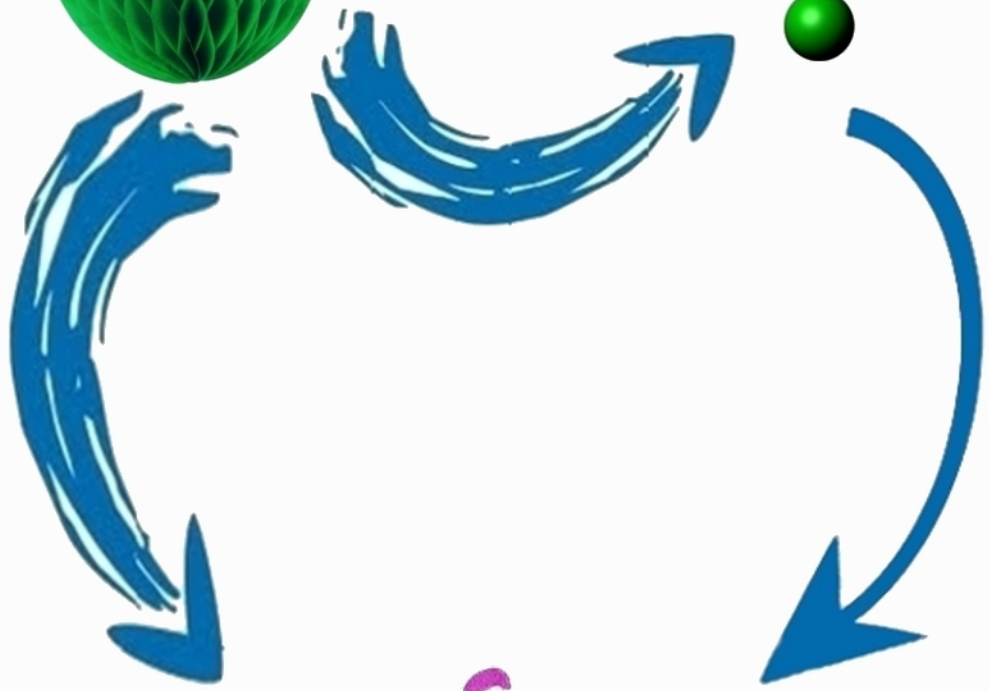
C

$\lim_{N \rightarrow \infty} Q_N$



=

$\lim_{k \rightarrow 0} Q_k$



C

$\mathcal{M}_N$



phase-space



$\mathcal{M}_k$

$\{|\Omega^i\rangle_N\}_N$



$\sim$   
classically  
equivalent



$|\Omega\rangle_k$

${}_N\langle \Omega^i | \hat{A}_N | \Omega^i \rangle_N$



$A(\Omega)$   
finite



${}_k\langle \Omega | \hat{A}_k | \Omega \rangle_k$

# ENTANGLING DYNAMICS OF MACRO with MICRO

MACRO  $\equiv$

with

MICRO  $\Gamma$

environment / apparatus

principal system

$Q_N$  with large  $N$



# ENTANGLING DYNAMICS OF MACRO with MICRO

MACRO  $\Xi$   
environment / apparatus

with

MICRO  $\Gamma$   
principal system

$Q_k$  with small  $k$



$|\Xi + \Gamma\rangle$



# ENTANGLING DYNAMICS OF MACRO with MICRO

MACRO  $\Xi$   
environment / apparatus

with

MICRO  $\Gamma$   
principal system

$Q_k$  with small  $k$



$$|\Xi + \Gamma\rangle$$

introduce GCS for  $\Xi$  only

$$\int_{\mathcal{X}} d\mu(\Omega) |\Omega\rangle \chi(\Omega) |\phi(\Omega)\rangle$$

PNAS 110 (2013) 6748

# ENTANGLING DYNAMICS OF MACRO with MICRO

MACRO  $\Xi$   
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$$|\phi^\Psi(\Omega)\rangle = \frac{1}{\chi^\Psi(\Omega)} \sum_{\mathcal{Y}} f_{\mathcal{Y}}(\Omega) |\mathcal{Y}\rangle \quad ; \quad f_{\mathcal{Y}}(\Omega) = \sum_{\mathcal{Z}} c_{\mathcal{Y}\mathcal{Z}} \langle \Omega | \Xi \rangle$$



$$\chi^\Psi(\Omega) = \sqrt{\sum_{\mathcal{Y}} |f_{\mathcal{Y}}(\Omega)|^2}$$

$\Omega$ -dependence  
iff  $|\Psi\rangle$  entangled

$$\int_{\mathcal{X}} d\mu(\Omega) (\chi^\Psi(\Omega))^2 = 1$$

normalized  
distribution on  $\mathcal{X}$

MEASURE-LIKE DYNAMICS (standard model)

$$\hat{H}_\psi = g \hat{O}_\psi \otimes \hat{D}_{\underline{0}} + \hat{H}_{\underline{0}}$$

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$$\hat{H}_\psi = g \hat{O}_\psi \otimes \hat{D}_\equiv + \hat{H}_{0\equiv}$$

$$\sum_\alpha c_\alpha |\alpha\rangle | \equiv \rangle \longrightarrow \sum_\alpha c_\alpha |\alpha\rangle e^{-\frac{i\hat{H}_\psi t}{\hbar}} |R\rangle$$

$$\hat{H}_\alpha = g\omega_\alpha \hat{D}_\equiv + \hat{H}_{0\equiv}$$

MEASURE-LIKE DYNAMICS (standard model)

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$$\hat{H}_\alpha = g\omega_\alpha \hat{D}_\Xi + \hat{H}_{0\Xi}$$

$$\int_{\mathcal{X}} d\mu(\Omega) |\Omega\rangle \chi(\Omega) |\phi(\Omega)\rangle$$

$$x_t^2(\Omega) = \sum_\alpha |c_\alpha|^2 |\langle \Omega | R_t^\alpha \rangle|^2 = \sum_\alpha |c_\alpha|^2 h_t^\alpha(\Omega)$$

# MEASURE-LIKE DYNAMICS (standard model)

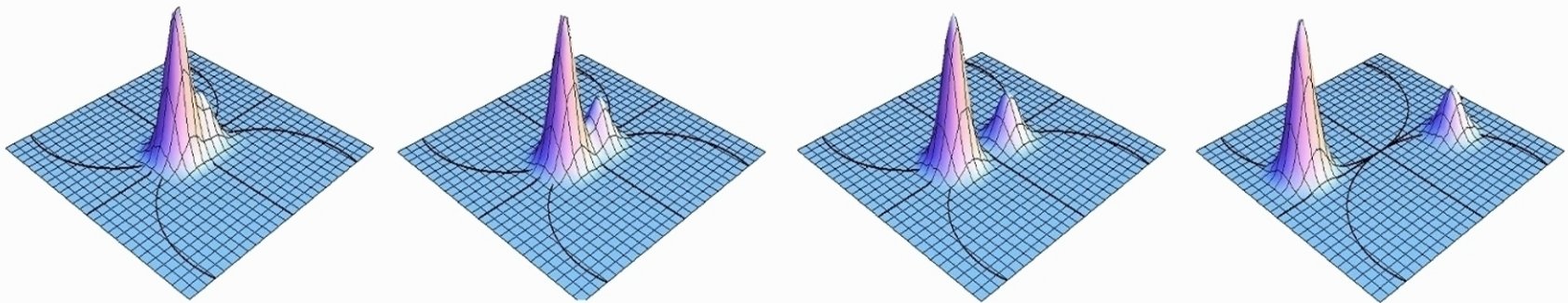
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$$\hat{H}_{qb} = \nu \hat{b}^\dagger \hat{b} + g\sqrt{E} \hat{\sigma}^z (\hat{b} + \hat{b}^\dagger) \quad ; \quad [b, b^\dagger] = \hbar \quad ; \quad \mathcal{X} \text{ complex plane}$$

$\hat{\sigma}^z$  qubit (MICRO)

$b, b^\dagger$  field (MACRO)



MEASURE-LIKE DYNAMICS (standard model)

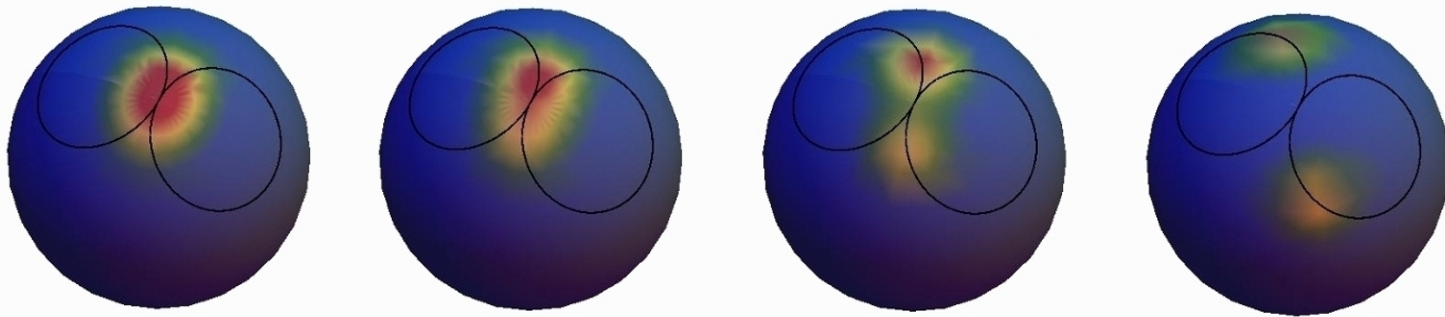
$$\hat{H}_\psi = g \hat{O}_\alpha \otimes \hat{D}_\alpha + \hat{H}_{O_\alpha}$$

$$\sum_\alpha c_\alpha |\alpha\rangle |E\rangle \longrightarrow \sum_\alpha c_\alpha |\alpha\rangle e^{-\frac{i\hat{H}_\alpha t}{\hbar}} |R\rangle$$

$$\hat{H}_\alpha = g\omega_\alpha \hat{D}_\alpha + \hat{H}_{O_\alpha}$$

$$\int_{\mathcal{R}} d\mu(\Omega) |\Omega\rangle \chi(\Omega) |\phi(\Omega)\rangle$$

$$\chi_\pm^2(\Omega) = \sum_\alpha |c_\alpha|^2 |\langle \Omega | R_\alpha \rangle|^2 = \sum_\alpha |c_\alpha|^2 h_\pm^\alpha(\Omega)$$

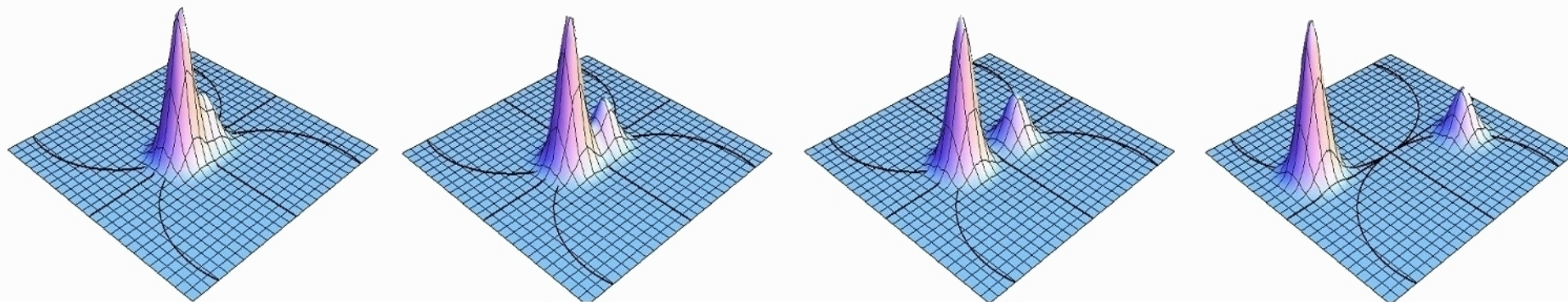


$$\hat{H}_{q\beta} = \hbar \hat{J}_z + g \hat{\sigma}^z (\hat{J}_x)$$

$\hat{\sigma}^z$  qubit (MICRO)

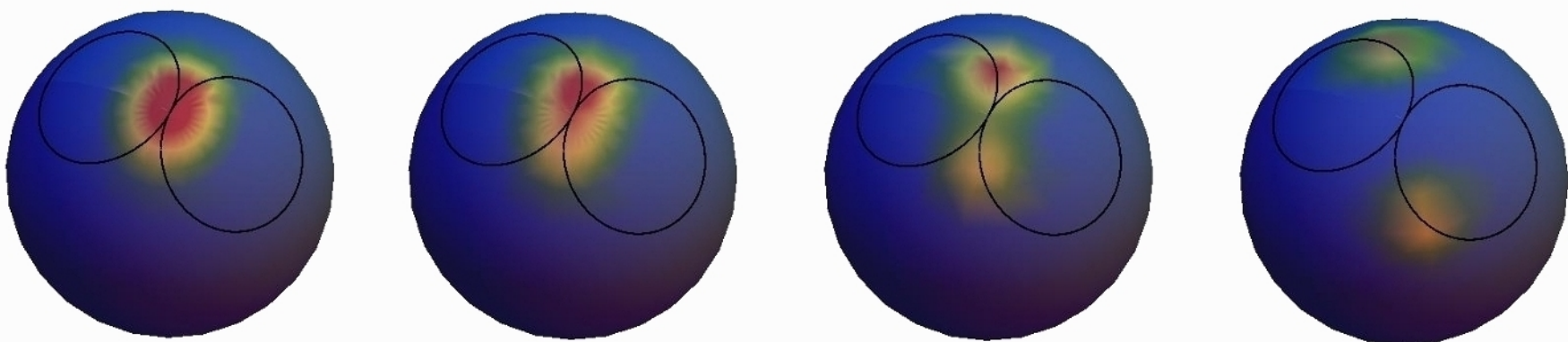
$$; [\hat{J}_\alpha, \hat{J}_\beta] = i\epsilon^{\alpha\beta\delta} \hat{J}_\delta ; \text{ of unit 2-sphere}$$

$\hat{J}^\alpha$  magnet (MACRO)



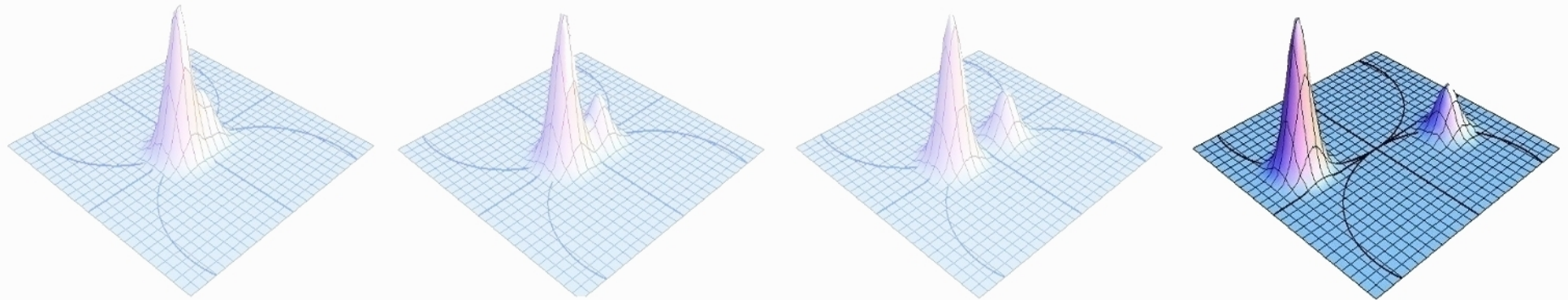
$$\chi_t^2(\Omega) = \sum_r |c_r|^2 h_t^r(\Omega) \quad \text{normalized distributions on } \mathcal{M} \text{ with } \varepsilon\text{-support}$$

$$\mathcal{S}_t^\varepsilon : h_t^r(\Omega) > \varepsilon \text{ for } \Omega \in \mathcal{S}_t^\varepsilon$$

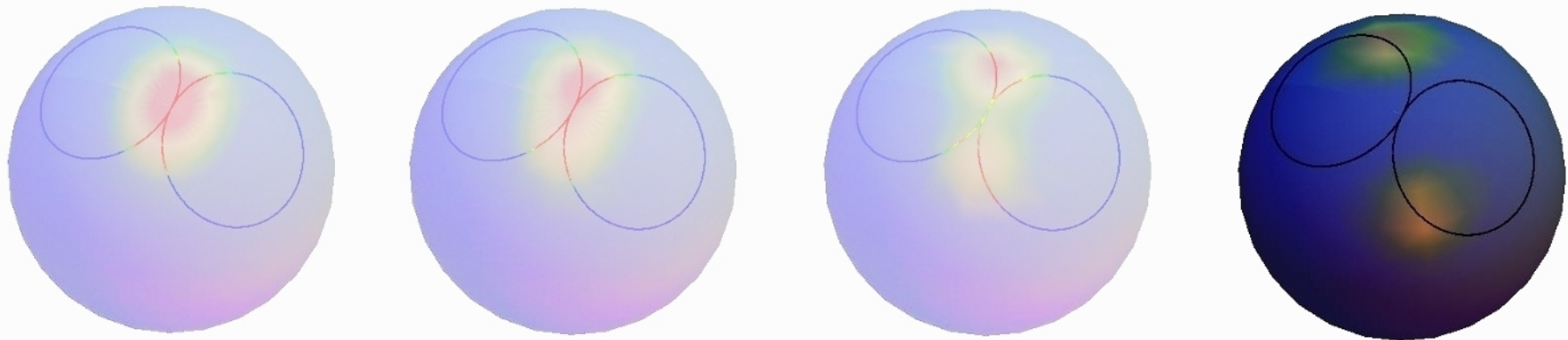




# DECOHERENCE

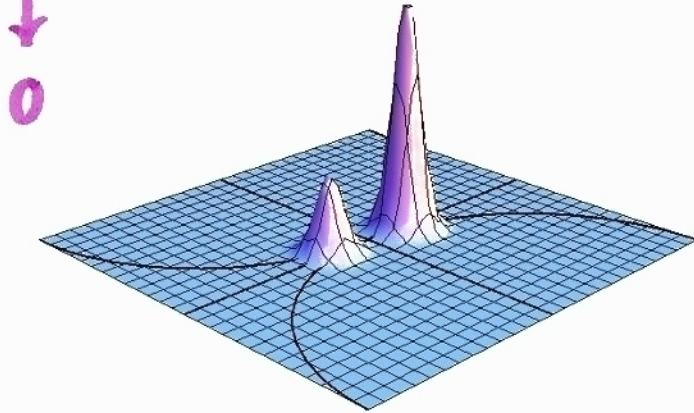
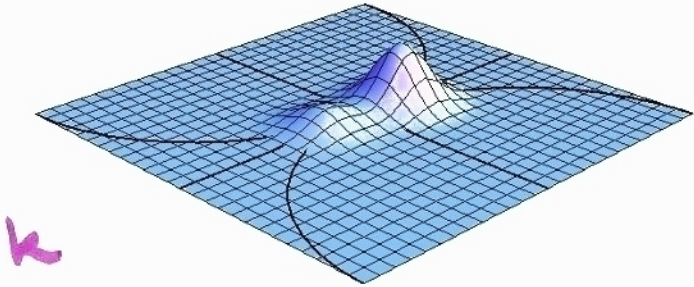


for each  $\gamma$  a different trajectory on  $\mathcal{M}_k$



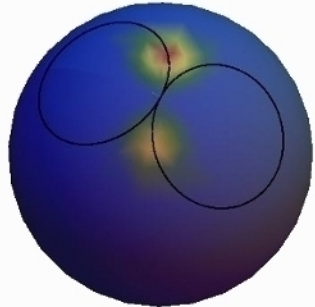
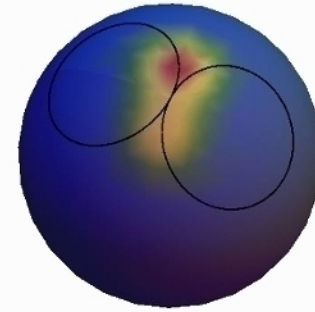
after decoherence has occurred  $\Omega \in S_t^\gamma$  well defined for different  $\gamma$

# BACK TO $Q_N$ VIA CLASSICAL EQUIVALENCE



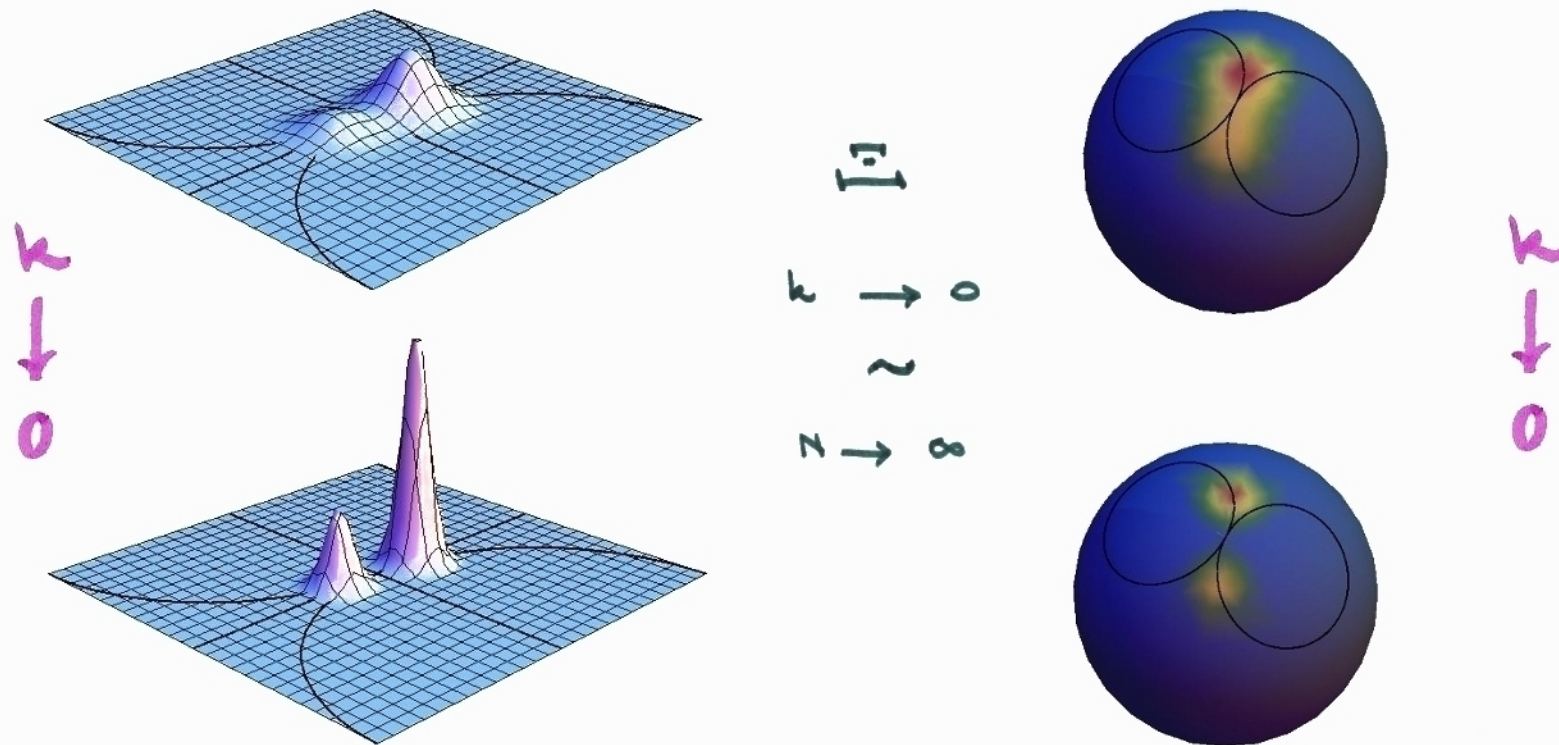
$k \rightarrow 0$

$l \rightarrow 0$   
 $z \rightarrow 8$



$k \rightarrow 0$

# BACK TO $Q_N$ VIA CLASSICAL EQUIVALENCE

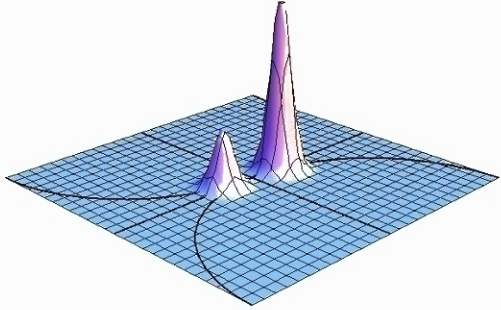


LET  $k \rightarrow 0$  IN  $Q_k$  AND SEE  $Q_N$  FOR  $N \rightarrow \infty$

OTHERWISE  
NOT-ACCESSIBLE

# BACK TO $Q_N$

DIFFERENT  $\gamma$  ARE NOT CLASSICALLY EQUIVALENT





# BACK TO $Q_N$

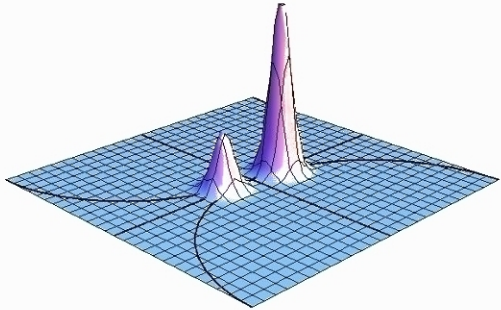
DIFFERENT  $\gamma$  ARE NOT CLASSICALLY EQUIVALENT

BUT

THEY ALL HAVE THE SAME ENERGY

$$\langle \Omega | \hat{H}^N | \Omega \rangle_N = \langle \tilde{\Omega} | \hat{H}^N | \tilde{\Omega} \rangle_N$$

AS FAR AS  $X(N)$  HOLDS



# BACK TO $Q_N$

DIFFERENT  $\gamma$  ARE NOT CLASSICALLY EQUIVALENT

BUT

THEY ALL HAVE THE SAME ENERGY

$$\langle \Omega | \hat{H}^N | \Omega \rangle_N = \langle \tilde{\Omega} | \hat{H}^N | \tilde{\Omega} \rangle_N$$

AS FAR AS  $X(N)$  HOLDS

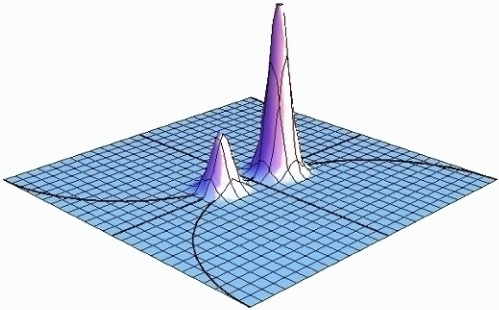


$$|\Omega_t\rangle_N = e^{-it(\hat{H}_N^\alpha - \hat{H}_N^{\alpha'})} |\Omega'_t\rangle_N$$

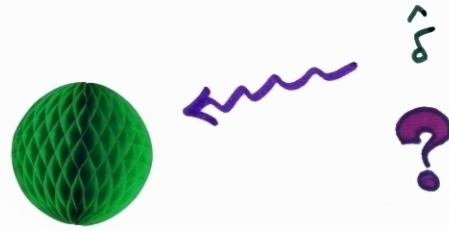
$\in \{ |\Omega^i\rangle_N \}_N^\alpha$                        $\in \{ |\Omega^i\rangle_N \}_N^{\alpha'}$

$\forall \gamma, \gamma'$

(because they have the same initial state)

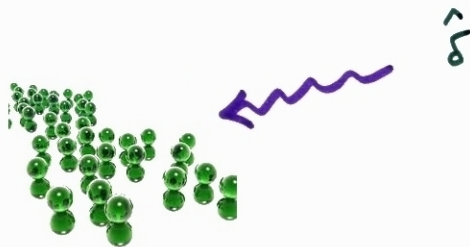


# BREAKING $\mathbb{X}(N)$ BY A LOCAL PERTURBATION $\delta$



$$[\hat{\delta}, \alpha] \neq 0 \quad \alpha \in \mathbb{X}(N)$$

# BREAKING $\chi(N)$ BY A LOCAL PERTURBATION $\delta$



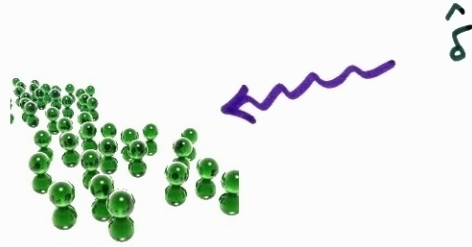
degeneracy of different trajectories  
IS REMOVED

$$\langle \Omega | \hat{\delta}^N | \Omega \rangle_N \neq \langle \tilde{\Omega} | \hat{\delta}^N | \tilde{\Omega} \rangle_N$$

for  $|\Omega\rangle_N \in \{|\Omega_i\rangle_N\}_N^{\chi}$  and  $|\tilde{\Omega}\rangle_N \in \{|\tilde{\Omega}_i\rangle_N\}_N^{\chi'}$   $\chi \neq \chi'$



# BREAKING $\chi(N)$ BY A LOCAL PERTURBATION $\hat{\delta}$



degeneracy of different trajectories  
IS REMOVED

$$\langle \Omega | \hat{\delta}^N | \Omega \rangle_N \neq \langle \tilde{\Omega} | \hat{\delta}^N | \tilde{\Omega} \rangle_N$$

for  $|\Omega\rangle_N \in \{|\Omega_i\rangle_N\}_N^{\chi}$  and  $|\tilde{\Omega}\rangle_N \in \{|\tilde{\Omega}_i\rangle_N\}_N^{\chi'}$   $\chi \neq \chi'$

$$N \rightarrow \infty$$

$\hat{\delta}$  removes the degeneracy of different trajectories  
but does not change classical equivalence

$$\bigcup_{\delta} \{ \Omega_{\delta} \}$$

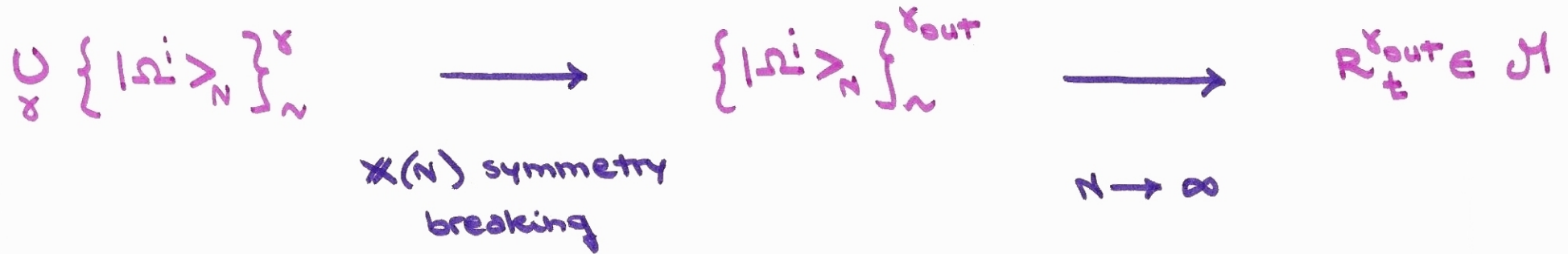
# OUTPUT PRODUCTION

$$\bigcup_{\gamma} \{ |\Omega_i| > N \}_{\sim}^{\gamma} \longrightarrow \{ |\Omega_i| > N \}_{\sim}^{\gamma_{\text{out}}}$$

$X(N)$  symmetry  
breaking

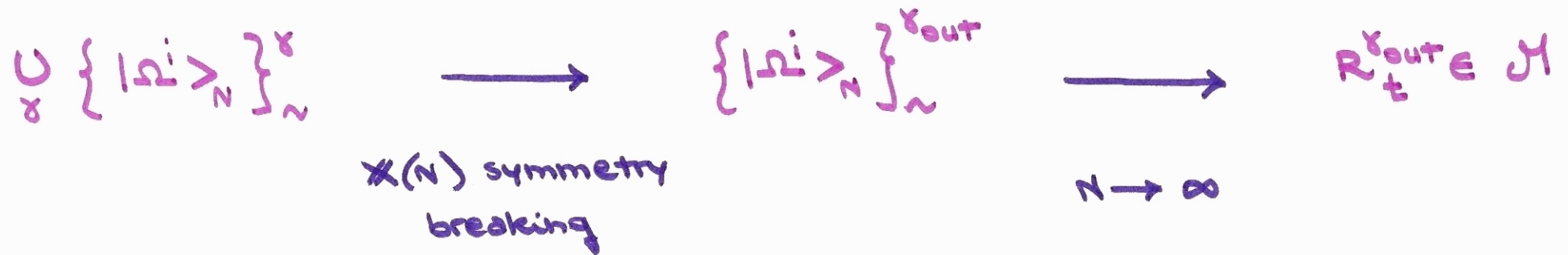
$\gamma_{\text{out}}$  is selected

# OUTPUT PRODUCTION & EMERGENCE OF CLASSICALITY



$\gamma_{\text{out}}$  is selected

# OUTPUT PRODUCTION & EMERGENCE OF CLASSICALITY



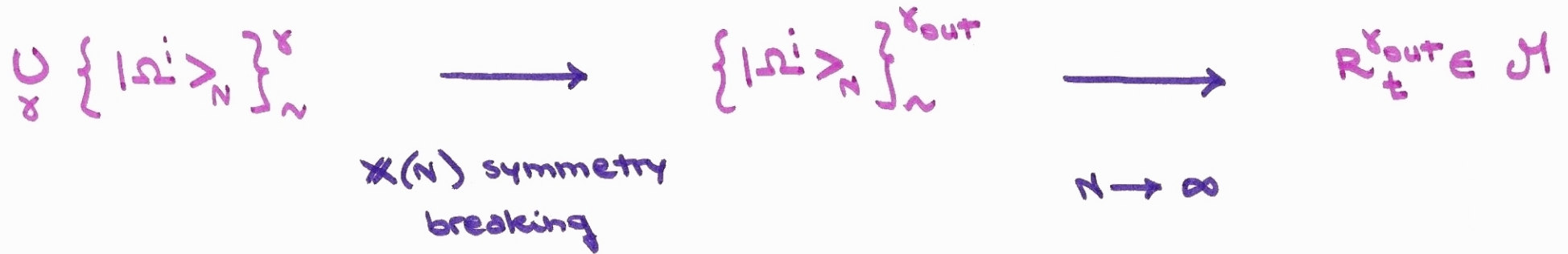
$\gamma_{\text{out}}$  is selected

EPL 111 (2015) 40008

BORN'S RULE

with probability  $|c_{\gamma}|^2$

# OUTPUT PRODUCTION & EMERGENCE OF CLASSICALITY



$\gamma_{\text{out}}$  is selected

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BORN'S RULE

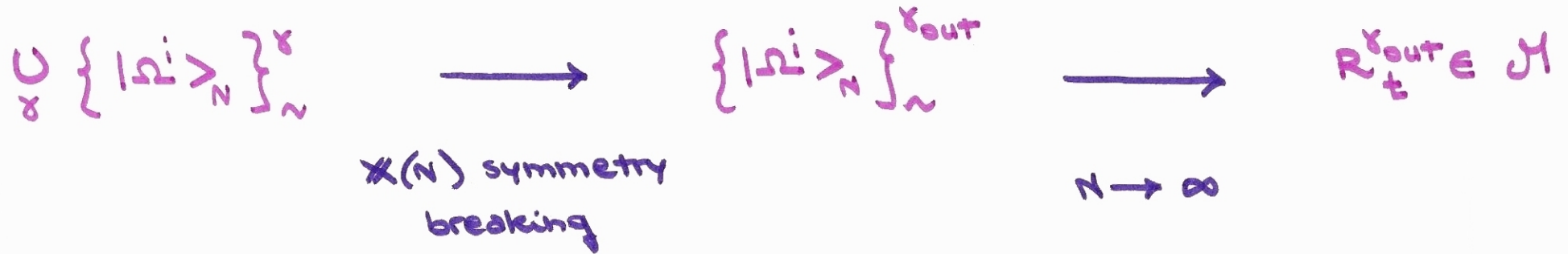
with probability  $|c_{\gamma}|^2$

final state of  $\Omega$

$$\int_{\mathcal{M}} d\mu(\Omega) \chi_{\pm}(\Omega) |\Omega\rangle |\phi_{\pm}(\Omega)\rangle$$

$$|\phi_{\pm}(\Omega)\rangle = \frac{1}{\chi_{\pm}(\Omega)} \sum_{\gamma} c_{\gamma} \langle \Omega | \mathbb{E} \rangle |\gamma\rangle = \sum_{\gamma} \frac{e^{i\phi_{\pm}^{\gamma}}}{\left( 1 + \sum_{\gamma' \neq \gamma} \frac{|c_{\gamma'}|^2 W_{\pm}^{\gamma'}(\Omega)}{|c_{\gamma}|^2 W_{\pm}^{\gamma}(\Omega)} \right)^{1/2}} |\gamma\rangle$$

# OUTPUT PRODUCTION & EMERGENCE OF CLASSICALITY



$\gamma_{\text{out}}$  is selected

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BORN'S RULE

with probability  $|c_{\gamma}|^2$

final state of  $\Omega$

$$\int_{\mathcal{M}} d\mu(\Omega) \chi_{\pm}(\Omega) |\Omega\rangle |\phi_{\pm}(\Omega)\rangle$$

$$|\phi_{\pm}(\Omega)\rangle = \frac{1}{\chi_{\pm}(\Omega)} \sum_{\gamma} c_{\gamma} \langle \Omega | \Xi \rangle |\gamma\rangle = \sum_{\gamma} \frac{e^{i\phi_{\pm}^{\gamma}}}{\left( 1 + \sum_{\gamma' \neq \gamma} \frac{|c_{\gamma'}|^2 W_{\pm}^{\gamma'}(\Omega)}{|c_{\gamma}|^2 W_{\pm}^{\gamma}(\Omega)} \right)^{1/2}} |\gamma\rangle$$

STATE REDUCTION

$$|\phi(\Omega)\rangle \longrightarrow |\gamma^{\text{out}}\rangle$$

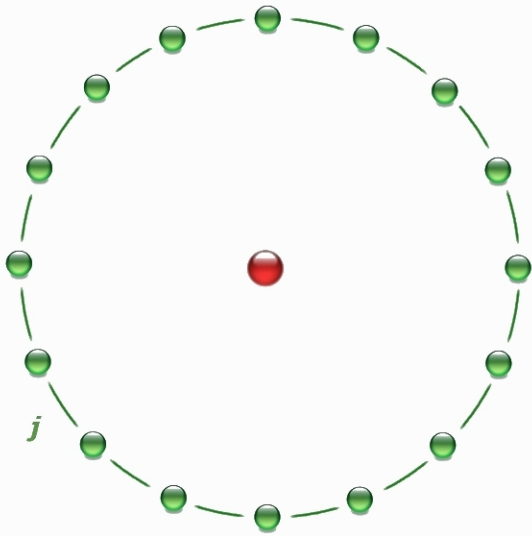


# COMMENTS

- no hidden variables
- all components are in genuinely quantum superpositions as far as  $\chi(N)$  holds
- very unlikely that a local perturbation  $\hat{\delta}$  does not enter the game  
that's why macro-quantum states are not "usual"
- the  $N \rightarrow \infty$  description holds for whatever dynamics (not only measure-like) as far as some  $\chi(N)$  exists
- experimental test possible by designing "ad hoc" local  $\hat{\delta}$

WORK IN PROGRESS

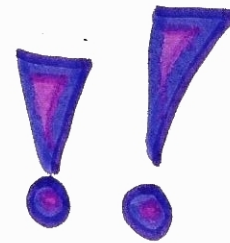




EXP



THANKS



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