[Journal Club - Week 9]

Aim: To show the car first decelerated at a constant rate to come to a stop at S before accelerating away with the same constant acceleration denoted by a_0.

Steps: Derive a new equation for $x(t)$
Find angular velocity $w(t)$ that O observes
Conclude

$x(t)$ a displacement as a function of time

Given a_0 is constant acceleration/deceleration we know

$$v(t) = a_0 t$$

velocity as a function of time

$$[v] = LT^{-1}$$
$$[a] = LT^{-2}$$
$$[t] = T$$

$$LT^{-1} = LT^{-2} \times T$$

dimentionally consistent

using knowledge of differentiation

$$v(t) = \frac{dx}{dt}$$

multiplying by dt

$$dt\, v(t) = dx$$

Integrating dx with respect to time gives us an equation for $x(t)$

$$\int_0^x dx = \int_0^t v(t) \times \int_0^t t\,dt$$

$$= a_0 \times \frac{1}{2} t^2$$

$$x(t) = \frac{1}{2} a_0 t^2$$

Subbing into $\alpha(t) = \arctan \frac{x(t)}{r_0}$

gives $\alpha(t) = \arctan \frac{\frac{1}{2} a_0 t^2}{r_0}$

when $r_0 = |OS|$

and α = angle between OS and OC.

Differentiating according to

$$w(t) = \frac{d\alpha}{dt}$$

$$\Rightarrow w(t) = \frac{d}{dt} \arctan \frac{\frac{1}{2} a_0 t^2}{r_0}$$

Using the chain rule

$$g(f) = \arctan f(t)$$

$$f(t) = \frac{\frac{1}{2} a_0 t^2}{r_0}$$

$$\frac{dg}{df} = \frac{d}{df} = \frac{d}{df} \arctan f(t) = \frac{1}{1+f^2}$$

subbing in $f(t) = \dfrac{\frac{1}{2} a_0 t^2}{r_0}$ gives

$$\frac{d}{df} \arctan f(t) = \frac{1}{1 + \left(\frac{\frac{1}{2} a_0 t^2}{r_0}\right)^2}$$

simplify

$$= \frac{1}{1 + \frac{1}{4}\left(\frac{a_0}{r_0}\right)^2 t^4}$$

we know $\dfrac{d}{dt} \arctan f(t) = \dfrac{1}{1+f^2} \dfrac{df}{dt}$

so $\dfrac{df}{dt} = \dfrac{a_0 t}{r_0}$

$$\therefore \quad w(t) = \frac{1}{1 + \frac{1}{4}\left(\frac{a_0}{r_0}\right)^2 t^4} \times \frac{a_0 t}{r_0}$$

$$\Rightarrow \frac{\frac{a_0}{r_0} t}{1 + \frac{1}{4}\left(\frac{a_0}{r_0}\right)^2 t^4}$$ as required.

This shows when $a_0 = 10\,ms^{-2}$ (which is what was suggested) and $r_0 = 10m$ then at $t = 0$ when the car reached the stop sign the angular speed is zero which means the car came to a complete stop at the stop sign proving the innocence of the author.