# Dmitri Krioukov (2012), *The proof of innocence*, arXiv:1204.0162

The article can be accessed at https://arxiv.org/abs/1204.0162 (you just need to click the PDF button in the download section). The paper is also available from the *Annals of Improbable Research* where you can find a whole host of other… interesting research.

You can also download the Cornell notes template for this paper (which includes the same questions) as a Word Document or PDF. Teachers, feel free to download this and forward it on to your students.

This week we're looking at a slightly more theoretical paper, but another paper where scientists show off their award winning sense of humour. We've discussed the arXiv in previous weeks, so this article has not gone through a thorough peer review. The reason for that, in this case, is that the author is not coming up with any new ideas, just using basic physics ideas to try and get out of paying a fine for a traffic violation in America.

We're also going to progress a little more with our notetaking. We won't provide any prompts during your reading, you can write these yourself. Do try and stick with the Cornell note style, even if its brief handwritten notes – we know some of you have been doing it by hand the whole time and it's how I prefer to note-take too. We're just going to give summary questions for each section. As this is a theory paper, you might want to alter your notetaking to thoroughly derive the equations as you go along and make sure they explain the physical scenario. I'll provide an example for how you might do this in section II.

To help this week, we have a series of tools that might be useful:

- We've made this applet to illustrate dynamically the two situations that the author describes. You can alter the acceleration of the car and the distance away the observer is to see (from the arrows) how the angular speed changes.
- You might want to use an online function plotter to try and plot some functions for yourself. Do email if you need any help.
- You also might want to check any integration and differentiation makes sense using an online tool.

With all of the tools above, don't rely on them to do the work for you. You should be able to attempt all of this plotting and calculus with pen and paper, but you should use them to check yourself.

## I. INTRODUCTION

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| Why does a stationary observer not measure the speed of an object? | Because of your perspective. Things further away look smaller and take up a smaller amount of your field of view. As they move, they don't traverse much of your field of view to begin with so seem to travel slowly. When an object is moving directly past you, it traverses a |

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| | lot of your field of view in a short time so seems to be moving faster. Consider it in terms of how much you need to turn your neck as an object moves past you left to right. Your neck has to turn most quickly when the object passes your position. |
| What scenarios is the author going to consider? | A car travelling at a constant speed (and hence driving through the stop sign without stopping). A car decelerating rapidly to a halt at the stop sign and accelerating away from the stop sign again rapidly (in the same direction as they were previously travelling). |

In the next section, the author will choose to use the point when the car is at the stop sign (when C is at S) as the origin of time – the point when $t = 0s$. Dealing with negative time is always a little confusing, but importantly for the argument running through this paper, all of the scenarios will be in some sense symmetrical about $t = 0s$. You can understand all of the maths by simply considering positive values for time (and this is sometimes beneficial).

## II. CONSTANT LINEAR SPEED

This link gives an example of how I would take notes and convince myself of all the steps in the working out through a paper. I'm annotating all of the steps, clarifying *why* things are the way they are and filling in the intermediate steps to check the maths.

If you want to see proof that $\frac{d}{dt}\arctan(t) = \frac{1}{1+t^2}$ this link shows how you can derive it.

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| Why is equation (2) not technically correct? | Because $x$ can't be negative – it's a length. If $t < 0$ then equation (2) would give negative values for $x$. Technically, for this situation we should probably write $x(t) = v_0\|t\|$ |
| What are the benefits of the author choosing the time $t = 0s$ to be the point when the car meets the stop sign? | It makes it easier to write symmetric functions around the origin. It's also the only real fixed point in time in the entire scenario we're looking at. All of the other 'moments' depend upon how fast the car is going – it doesn't start or end in a definite place. But the stop sign is a definite location and the observer's view of it gives a definite time point. |
| What rule of differentiation does the author use in Equation (6)? | Chain rule. $$\frac{d}{dt}g(f(t)) = \frac{dg}{df}\frac{df}{dt}$$ |
| Describe the graph shown in Figure 2. Link it to what is physically happening in the | When the car is approaching the stop sign, the angular speed is initially small as the car is far away. As the car approaches, the |

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| example of the car moving at a constant speed. | angular speed increases. The angular speed is maximum when the car reaches the stop sign (which is the point $t = 0s$). As the car moves away, the angular speed decreases. When the car is far away once again, the angular speed becomes small. |

## III. CONSTANT LINEAR DECELERATION AND ACCELERATION

Remember, we have made this applet to help you visualise the difference in motion.

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| Explain the steps the author takes between equation (9) and (12) to prove the relationship given in equation (8) | Equation (9) gives the velocity in a situation of constant acceleration in a situation where $v = 0$ when $t = 0$. Equation (10) defines velocity as the time differential of position. This can be rearranged, in equation (11), as a derivative can be understood loosely as a fraction of $\frac{dx}{dt} = \frac{little\ bit\ of\ distance}{little\ bit\ of\ time}$ and so $dx$ and $dt$ can be manipulated as in normal algebra. Equation (12) starts by stating that position is the integral of lots of small bits of position up to the point we care about $x = \int_0^x dx$. You'll notice the limits here are between 0 and $x$, allowing us to not have to think about negative time. By replacing $dx$ with equation (11) and changing the limits to reflect that we're now looking at the time equivalents ($t = 0$ when $x = 0$ and by definition $t = t$ when $x = x$). As we're working with constant acceleration, we can replace $v$ with that in equation (9). Performing the integration gives us the last step. |
| Using the explanation given after equation (12), can you derive the relationship given in equation (13)? | $$\alpha(t) = \arctan\frac{x(t)}{r_0}$$ $$\alpha(t) = \arctan\frac{\frac{1}{2}a_0t^2}{r_0} = \arctan\frac{a_0t^2}{2r_0}$$ We can write $f(t) = \frac{a_0t^2}{2r_0}$ $$\omega(t) = \frac{d\alpha(t)}{dt} = \frac{d}{df}(\arctan(f))\frac{d}{dt}\left(\frac{a_0t^2}{2r_0}\right)$$ $$\omega(t) = \frac{d\alpha(t)}{dt} = \left(\frac{1}{1+f^2}\right)\left(\frac{2a_0t}{2r_0}\right)$$ $$\omega(t) = \frac{d\alpha(t)}{dt} = \left(\frac{1}{1+\left(\frac{a_0t^2}{2r_0}\right)^2}\right)\left(\frac{a_0t}{r_0}\right)$$ |

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| | $\omega(t) = \left( \frac{\frac{a_0 t}{r_0}}{1 + \left(\frac{{a_0}^2}{4{r_0}^2}\right) t^4} \right)$ |
| Describe the graph shown in Figure 3. Link it to what is physically happening in the example of decelerating/accelerating car. | When the car is far away and approaching, the angular speed is small. As the car approaches, the angular speed increases. As the car approaches, though, it is also decelerating – this means that the maximum angular speed is reached before the car passes by. Once the maximum angular speed is reached, the angular speed decreases to zero. At the point when the angular speed is zero, the car has stopped at the stop sign.<br><br>The car then accelerates rapidly, and the angular speed increases rapidly. This is balanced, though, by the fact that the car is also moving further away, which has a diminishing effect on the angular speed. These effects combine so that the angular speed reaches a maximum before decreasing. When the car is far away (and still accelerating), the angular speed is small again. |

## IV. BRIEF OBSTRUCTION OF VIEW AROUND $t = 0s$

Remember, we have made this applet to help you visualise the difference in motion.

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| What are $x_f$ and $x_p$? | $x_f$ is the distance over which the author's car is fully obscured by the longer car. $x_p$ is the distance over which the larger car at least partially obscures part of the author's car. |
| What is the biggest assumption made in this section? | That the car can achieve an acceleration of $10\text{m/s}^2$. |
| What is the time $t'$? | The time taken for the angular speed to reach its maximum in the situation when the car has stopped at the stop sign and then accelerated quickly. |
| What conclusion does the author draw from the fact that $t_p > t'$? | That his car could have been partially obstructed by another for longer than it took for him to accelerate to a |

## V. CONCLUSION

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| Describe how the three bullet points in the conclusion link to Figure 5. | By measuring angular speed, your observation is strongly focussed around the moment when an object passes you to understand how it is moving – if an |

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| | obstruction happens in this moment, your brain is forced to extrapolate. In both scenarios shown in figure 5, the curves are changing most rapidly around t=0s and are very similar at t>2s and t<-2s.<br><br>By decelerating and accelerating quickly, the angular speed over time is significantly altered, forming a peak either side of t=0s – shown by the blue line in Fig. 5. But, given the abrupt nature of the acceleration and deceleration, these peaks are fairly close to one another, so that it does bare some resemblance to a constant velocity scenario – shown by the red line.<br><br>By estimating the window over which the view of the observer was at least partially obstructed – shown by the black dotted lines - the author has been able to show that such an obstruction, combined with such a sharp deceleration/acceleration could be misinterpreted easily. This observer misinterpretation is shown by the dotted red line. |

**APPENDIX 2: ARE THERE ANY FLAWS IN THIS ARGUMENT?**

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| This question is going to be a challenge and set as an optional summary question. The author states that a "Yaris accelerates to 100 km/h in 15.7 s", but claims that an acceleration of $10m/s^2$ in the early stages may be justified. I have also seen that a Toyota Yaris moves from 50mph to 70mph in around 8s. BE CAREFUL WITH ALL THE DIFFERENT UNITS. Do you think it's possible that an acceleration of $10m/s^2$ is achievable for a Toyota Yaris, given the manufacturer's data? HINT: Graphs. |

**SUMMARY QUESTIONS (submit these, along with your SKIM-READ answers to thomas.millichamp@warwick.ac.uk)**

Explain why an object moving at a constant speed does not necessarily appear that way to an observer as it moves towards, past and then away from a stationary observer.

In a similar way to my notes for section II. Constant Linear Speed, send in your notes on how you arrived at the solution for section III. Constant Linear

Deceleration And Acceleration. You can submit this as a photo of your hand-written notes if that's easier – however suits you best.

As budding physicists, do you think that this paper presents an adequate defence for the author's position that he did actually stop at the stop sign?

Optional summary question from the appendix.

**FURTHER READING**

This week, we don't have further reading tied to the topic, just a completely separate (and short) article from Physics Today entitled *Transportable clocks achieve atomic precision* about using portable atomic clocks to measure gravitational redshift. This is an excellent opportunity to practice writing your own skim-read and summary questions. Feel free to send in your work!