Understanding Stellar Explosions

When the Sun stops creating energy in its core, there is no support, and it collapses downward on a free-fall timescale. The acceleration of the outer layer is given by

$$g = \frac{GM_\odot}{R_\odot^2}. \quad (1)$$

1. What is the surface gravity of the Sun, mass, $M_\odot = 2 \times 10^{30}$ kg, radius $R_\odot = 7 \times 10^8$ m (G=6.67 $\times 10^{-11}$ m^3 kg^{-1} s^{-2})?

   Surface grav of sun is 272.245 m s$^-2$

2. How long would it take an object dropped from the surface of the Sun to reach the centre (you can assume all the mass in in the core, and there is no atmospheric resistance)?

   Since $s = ut + 1/2at^2$ *and* $u = 0$ *we can calculate this by re-arranging for* $t = \sqrt{2s/a} = 2200s.$

3. How fast would the object be travelling when it reached the centre?

   As v= u + at this would be 616624 m/s.

In practice, the Sun won't explode, but lets look at a more massive star, 10 times the mass of the sun. The star collapses until it hits the hard surface of the neutron star that has just been formed.

4. If 99% of the kinetic energy is given to only 1% of the mass what is the resulting velocity of the outgoing shock wave? You can assume the infall velocity at the bounce is the same as you just calculated above.

$T = 1/2mv^2$*, so the total kinetic energy = 3.8* $\times 10^{42}$ *J. We can use conservation of energy, if we assume there is no potential energy, so just compare* $T = 1/2m_1v_1^2 = 1/2m_2v_2^2$*, so that* $v_2 = \sqrt{m_1v_1^2/m_2}$ *and since* $m_1/m_2 \approx 100$ *then* $v_2 \sim 10v_1$*. The velocity of the outgoing shock wave is then 6000 km/s.*

This sort of system makes more "normal" supernovae that we observe, but it doesn't explain the most powerful explosions in nature. In these events more energy continues to be put into the explosion from the neutron star or black hole that has been left behind. The energy of a spinning sphere is given by

$$E = \frac{1}{2}I\omega^2, \quad (2)$$

where I is the moment of inertia and is given by $I = \frac{2}{5}MR^2$.

The Sun is currently rotating with a period of 24.5 days, and massive stars may have similar spin periods. When they collapse they conserve *angular momentum*, this is similar to linear momentum ($p = mv$) but accounts for rotating bodies, so that the angular momentum $L = mvr = I\omega$. In order to conserve angular momentum as an object gets smaller, the angular velocity must increase. Hence neutron stars spin much faster than other stars.

5. Calculate the spin rate of a neutron star formed from a collapsing star with the same mass and rotation period as the sun. You may assume the collapse is from 1$R_\odot$ to 10 km, and that the mass is unchanged?

   The moment of inertia scales with the square of the radius, hence the ratio of radii is 70000 and the ratio of moments of inertia is 4.9×10^9*. To conserve angular momentum the angular frequency must increase by this much so 24 days becomes (24 days / 70000*^{2} *= 0.4 ms).*

6. Calculate the energy stored in a neutron star with 1.0 M$_\odot$ and spinning at this period.

   The energy is just $E = 1/2I\omega^2 = \frac{1}{5}MR^2\omega^2 = \frac{4\pi^2MR^2}{5T^2} \sim 8 \times 10^{45}$ *J.*

An alternative way to get energy out is to accrete material onto the neutron star or black hole. In this case the energy is

$$E = \epsilon mc^2, \quad (3)$$

where ϵ is an efficiency factor = 0.01.

7. How much mass would you need to accrete to get the same energy as the spinning neutron star?

   $m = E/\epsilon c^2 = 5\ M_\odot$.

8. How do these energies compare with the kinetic energy in the 1% of the ejecta?

   These energies are actually substantially larger than the initial kinetic energy, and so can drive a more powerful explosion, although the details we have looked at here aren't entirey accurate and there is probably more ejecta, making the total kinetic energy and engine energy more comparable.

What is actually really important for making the biggest explosions in nature, is that this energy comes out in different ways to the kinetic energy released on core collapse. In particular, it might come out in a jet, or at a much later time, giving the ejected material a push, in order to make the explosion brighter.