Understanding Stellar Explosions

When the Sun stops creating energy in its core, there is no support, and it collapses downward on a free-fall timescale. The acceleration of the outer layer is given by

$$g = \frac{GM_\odot}{R_\odot^2}.$$

1. What is the surface gravity of the Sun, mass, $M_\odot = 2 \times 10^{30}$ kg, radius $R_\odot = 7 \times 10^8$ m ($G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2})?
2. How long would it take an object dropped from the surface of the Sun to reach the centre (you can assume all the mass in in the core, and there is no atmospheric resistance)?
3. How fast would the object be travelling when it reached the centre?

In practice, the Sun won't explode, but lets look at a more massive star, 10 times the mass of the sun. The star collapses until it hits the hard surface of the neutron star that has just been formed. It then bounces off this surface, and the light, outer layers of the star are expelled into space.

4. If all of the kinetic energy gained during the collapse is given to only 1% of the mass what is the resulting velocity of the outgoing shock wave? You can assume the infall velocity at the bounce is the same as you calculated in Q3 above.

This sort of system makes more "normal" supernovae that we observe, but it doesn't explain the most powerful explosions in nature. In these events more energy continues to be put into the explosion from the neutron star or black hole that has been left behind. The energy of a spinning sphere is given by

$$E = \frac{1}{2} I \omega^2,$$

where I is the moment of inertia and is given by $I = \frac{2}{5} MR^2$.

The Sun is currently rotating with a period of 24.5 days, and massive stars may have similar spin periods. When they collapse they conserve *angular momentum*, this is similar to linear momentum ($p = mv$) but accounts for rotating bodies, so that the angular momentum $L = mvr = I\omega$. In order to conserve angular momentum as an object gets smaller, the angular velocity must increase. Hence neutron stars spin much faster than other stars.

5. Calculate the spin rate of a neutron star formed from a collapsing star with the same mass and rotation period as the sun. You may assume the collapse is from $1R_\odot$ to 10 km and the mass is unchanged.
6. Calculate the energy stored in a neutron star with 1.0 M$_\odot$ and spinning at this period.

An alternative way to get energy out is to accrete material onto the neutron star or black hole. In this case the energy is

$$E = \epsilon mc^2,$$

where ϵ is an efficiency factor = 0.01.

7. How much mass would you need to accrete to get the same energy as the spinning neutron star?
8. How do these energies compare with the kinetic energy in the 1% of the ejecta?

What is actually really important for making the biggest explosions in nature, is that this energy comes out in different ways to the kinetic energy released on core collapse. In particular, it might come out in a jet, or at a much later time, giving the ejected material a push, in order to make the explosion brighter.

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