Autumn Term, Week 16 Tutorial
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Read the following sections of University Physics and lectures.

Lorentz Transformations, Length Contraction, Time Dilation, Atmospheric Muons, Relativistic Doppler Effect, The Twin Paradox

Before you start solving Special Relativity questions, make sure you fully understand Loretz transformation and can derive the formula for length contraction and time dilation from Lorentz transformation.

Apply the Lorentz transformation to the intervals to find the unknown values. If S′ is the frame moving at speed v parallel to, and in the direction of, the positive x axis of frame S, then the Lorentz transformation of the interval, $(\Delta x, \Delta y, \Delta z, \Delta t)$, between two events as observed in the S frame, and the interval $(\Delta x', \Delta y', \Delta z', \Delta t')$ between the same two events as observed in the S′ frame is:

$$\Delta t' = \gamma\left(\Delta t - {v\Delta x}/{c^2}\right); \qquad \Delta t = \gamma\left(\Delta t' + {v\Delta x'}/{c^2}\right)$$
$$\Delta x' = \gamma(\Delta x - v\Delta t); \qquad \Delta x = \gamma(\Delta x' + v\Delta t')$$
$$\Delta y' = \Delta y; \qquad \Delta y = \Delta y'$$
$$\Delta z' = \Delta z; \qquad \Delta z = \Delta z'$$

Where $\gamma = \sqrt{\frac{1}{(1-v^2/c^2)}}$

Or using 4-vectors and matrices:

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v/c & 0 & 0 \\ -\gamma v\,/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

A spaceship sets off from Earth for a distant destination, travelling in a straight line at a uniform speed of $3c/5$. Ten years later, as measured on the Earth, a second spaceship sets off in the same direction with a speed of $4c/5$. The captains of the two vessels are twins.

(a) For how long, in the Earth's frame, do each of the spaceships travel before the second spaceship catches up with the first?

Consider three events: (A) the slower spaceship leaves the Earth; (B) the faster spaceship leaves the Earth; and (C) the faster spaceship catches up with the slower spaceship.

(b) If the event A has coordinates $x = 0, t = 0$ in the Earth's frame, what are the coordinates of the other two events, also as observed in the Earth's frame?

(c) By transforming these events to frames moving with the slower and faster spaceships respectively, determine which of the twins is older, and by how much, when the faster spaceship catches up with the slower spaceship.

{ (a) 40 years, 30 years; (b) $x = 0, t = 10$ years and
$x = 24$ light years, $t = 40$ years;
(c) the first captain is older by 4 years }