Autumn Term, Week 17 Tutorial

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Read the following sections of University Physics and lectures.

Lorentz Transformations, Length Contraction, Time Dilation, Atmospheric Muons, Relativistic Doppler Effect, The Twin Paradox

Before you start solving Special Relativity questions, make sure you fully understand Lorentz transformation and can derive the formula for length contraction and time dilation from Lorentz transformation.

Brain teaser-A revised tunnel problem

A very fast train, of length L_0 (measured in its own frame), rushes through a station which has a platform of length $L(<L_0)$ in the rest frame of the station.

a. What is the speed v of the train such that the back of the train is opposite one end of the platform at exactly the same instant as the front of the train is opposite the other end, according to observers on the platform?

b. According to these observers, two porters standing at either end of the platform (a distance L apart) are foolish enough, but have quick enough reactions, to kick the train simultaneously as it passes, thereby making dents in it. When the train stops, the dents are found to be a distance L_0 apart.

Explain in words how the difference between L and L_0 is explained by
b1. an observer on the platform, and
b2. an observer travelling on the train *(Note: according to an observer on the train, what is the length of the platform or the distance between two porters on the platform, L′, in terms of L_0 and γ?)*

Lorentz transformation and the relativity of simultaneity

$$\Delta t' = \gamma \left(\Delta t - \frac{v\,\Delta x}{c^2} \right) , \qquad \Delta t = \gamma \left(\Delta t' + \frac{v\,\Delta x'}{c^2} \right) ,$$
$$\Delta x' = \gamma \left(\Delta x - v\,\Delta t \right) , \qquad \Delta x = \gamma \left(\Delta x' + v\,\Delta t' \right) .$$

Can you derive *the length contraction* and *time dilation* formula using the Lorentz transformation?

A revised exam question in the past PX148 paper
A spacecraft travels at a speed of 0.9c relative to Earth. An experiment onboard sends a particle at a speed of 0.5c (relative to the spacecraft) in the same direction as the spacecraft's motion over a distance of 120 m (as measured in the spacecraft).

1. *Calculate the duration of the particle's flight as measured within the spacecraft. [2]*
2. *Use Lorentz transformations to determine the duration of the particle's flight as measured by an observer on Earth. [5]*

Why Special Relativity?

The following questions will help you understand why we need Special Relativity rather than Newtonian.

- ✓ Can you derive the equation for transforming velocities between two inertial frames using Lorentz transformations?
- ✓ How are relative velocities calculated in Newtonian mechanics?
- ✓ Compare the Newtonian and Lorentz velocity transformations. What are the key differences?

The constancy of the speed of light is a fundamental principle of Special Relativity, setting it apart from Newtonian mechanics.

The Michelson-Morley experiment played a critical role in motivating Albert Einstein's development of relativistic theories. This experiment demonstrated the invariance of the speed of light, irrespective of the motion of the observer or the source, challenging the Newtonian framework.

For additional tests and experimental verifications of Special Relativity, refer to the Wikipedia page titled *"Tests of Special Relativity"*. This page provides a comprehensive overview of experiments that support the theory.

An exam question in the past PX148 paper

Maxwell showed that disturbances in electromagnetic fields propagate at the speed of light in a vacuum, independently of direction. Discuss why this is problematic if Gallilean invariance applies. [3]

Explain the concept of the "luminiferous aether." [2]