

Read the following sections of University Physics and lectures.

Gauss's Law and Electrical Potential

The equation for Gauss's Law in the context of the gravitational field is:

$$\oint_S \mathbf{g} \cdot d\mathbf{A} = -\frac{4\pi G M_{enc}}{c^2}$$

This equation states that the total gravitational flux through any closed surface is proportional to the total mass inside the surface. It simplifies the calculation of gravitational fields when dealing with symmetric mass distributions.

Similarly, the equation for Gauss's Law in the context of the electric field is:

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

This equation tells us that the electric flux through a closed surface is proportional to the total charge enclosed within that surface. It is particularly useful for calculating electric fields in situations with high symmetry, such as spherical, cylindrical, or planar charge distributions.

Consider the following conditions and use words to describe the field's strength at different locations and the charge distribution on the surface of a conductor.

1. Using Gauss's Law, estimate the strength of the electric field at a distance  $d$  from a point charge  $Q$ , and similarly, estimate the strength of the gravitational field at a distance  $d$  from a point mass  $M$ .
2. Consider a closed surface of any shape that does not enclose the charge  $Q$ . Describe the strength of the electric field across this surface. Does your answer align with Gauss's Law? Explain why or why not.
3. Using Gauss's Law, estimate the strength of the electric field at a distance  $d$  from the centre of a uniformly charged solid sphere with total charge  $Q$ . What if the sphere has a non-uniform charge distribution?
4. Use Gauss's Law to estimate the strength of the electric field inside a hollow sphere of charge  $Q$ , assuming the charge is uniformly distributed across the sphere.
5. Consider a single charge placed at the centre of an isolated hollow spherical conductor. How would the charge be distributed on the surface of the conductor? What happens if the conductor is earthed?

A past exam problem for PX157

- d) A hollow, cylindrical conducting shell has an inner radius  $a$  and an outer radius  $b$  with zero net charge. A line of charges with linear charge density  $\lambda$  is at the axis of the shell (see figure below).
- (i) Find the linear charge density on the inner and outer surface of the conducting shell. {2}
- (ii) Using results from part c), find the potential  $V(r)$  as a function of distance from the axis,  $r$ . {4}

