

THE UNIVERSITY OF WARWICK

FIRST YEAR EXAMINATION: Summer 2021

FILES-BASED OPEN BOOK ASSESSMENT

ELECTRICITY AND MAGNETISM

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ANSWER ALL QUESTIONS

Time Allowed: **3 hours**

- *Read all instructions carefully* — and read through the entire paper at least once before you start writing.
- Hand-write your answers on plain or lined A4 paper (on one side only), preferably in dark blue or black ink. Add your student ID and page number on each page, at the top right. Make clear, in the left margin, which question you are answering.
- Photograph or scan your answers and then upload them using the AEP, preferably as a single PDF. *It is your responsibility to ensure that your work is legible* in the electronic version while avoiding unnecessarily large file sizes.
- Add your student number to all uploaded files.
- You are permitted to access module materials, notes, resources, references and the internet during the online assessment.
- You must not communicate with any other candidate during the assessment period.
- By starting this assessment, you are declaring yourself fit to undertake it. You are expected to make a reasonable attempt at the assessment by answering the questions in the paper.

Please note that

- You have **3 hours** from the scheduled start time of the exam to complete the assessment. You are granted an additional 45 minutes beyond this stated duration to allow for downloading the assessment and uploading your solutions and handle technical delays.
- *Late submissions are not accepted.* Once started, you must complete the assessment. It is your responsibility to upload within the allowed timeframe (3 hours+45min).
- Students with approved Alternative Exam Arrangements (Reasonable Adjustments) that permit extra time and/or rest breaks will have this time added on to the stated duration.

*The nominal mark assigned to each part of a question is indicated by means of a bold figure enclosed by curly brackets, e.g. {2} , immediately following that part. This mark scheme is for guidance only and may be adjusted by the examiner.*

The following information may be used:

Constants:

Permittivity of free space  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ kg}^{-1} \text{ m}^{-3} \text{ s}^2$

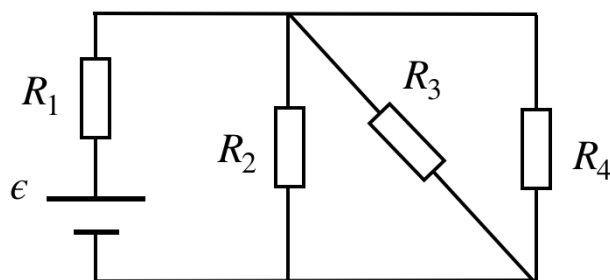
Permeability of free space  $\mu_0 = 4\pi \times 10^{-7} \text{ C}^{-2} \text{ kg m}$

**Electricity and Magnetism**

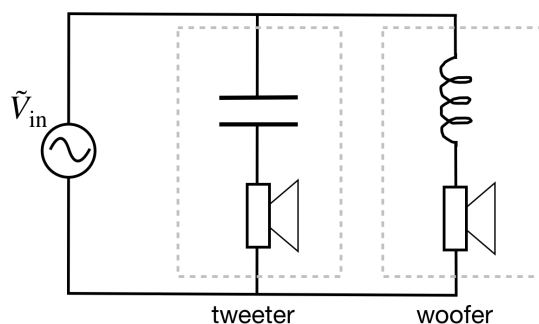
Time allowed: 3 hours

**ANSWER ALL QUESTIONS**

1. a) The DC circuit below contains one emf with value  $\epsilon$ , and four resistors labelled  $R_1, \dots, R_4$  with equal resistance  $R$ . By applying Maxwell loops:
- (i) Find the currents flowing through each of the resistors. {6}
  - (ii) Determine the magnitude of the potential drop across each resistor. {3}



- b) A loudspeaker system has a tweeter for high-frequency sound and a woofer for low-frequency sound. Its AC circuit is shown below. The tweeter consists of a capacitor with value  $C$  and a resistor with value  $R$  in series. The woofer consists of an inductance with value  $L$  and a resistor with value  $R$  in series.
- (i) Find the equivalent complex impedances of the tweeter and the woofer? {3}
  - (ii) Find expressions for the currents flowing through the tweeter and the woofer. Give also their magnitudes. {4}
  - (iii) Calculate the frequency  $\omega$  where tweeter and woofer draw equal currents. {3}
  - (iv) Sketch the magnitudes of the two currents as a function of  $\omega$ , noting the behaviour at  $\omega = 0$  and  $\omega \rightarrow \infty$ . {6}

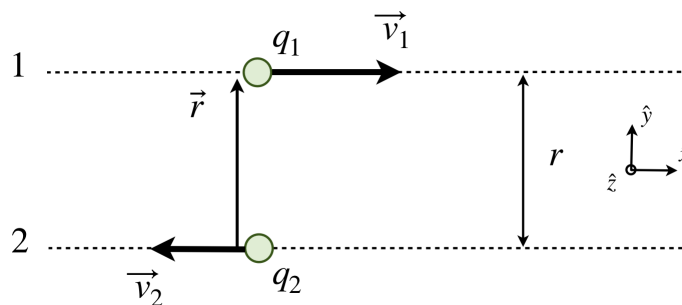


2. a) A positive charge  $Q$  is uniformly distributed over the surface of a thin spherical shell of radius  $a$ . Using Gauss's Law, show that the electric field as a function of radial distance from the centre of the shell,  $r$ , is given by:

$$\mathbf{E}(r) = \begin{cases} \mathbf{0} & r < a \\ \frac{Q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} & r \geq a \end{cases} . \quad \{\mathbf{8}\}$$

- b) Calculate the electrostatic potential,  $V(r)$ , inside and outside of this spherical shell. Plot the potential as a function of distance  $r$ . Clearly state the position of the reference point. {7}
- c) A second thin spherical shell of radius  $b > a$  and uniform charge  $-Q$  is added centred on the first shell. Show how the electric field and electrostatic potential are changed. {5}
- d) If the system described in part c) were to include a dielectric material with relative permittivity  $\epsilon_r$  between the two shells, calculate the induced electric field  $\mathbf{E}_i$ . Express the result in terms of the surface density of free charge  $\sigma_a$  at  $r = a$ . {5}

3. a) At time  $t$  two charges are located as shown in the figure below. Charge 1 has charge  $q_1$  and moves at speed  $v_1$  in the  $x$ -direction. Charge 2 has charge  $q_2$  and moves at speed  $v_2$  in the  $-x$ -direction.



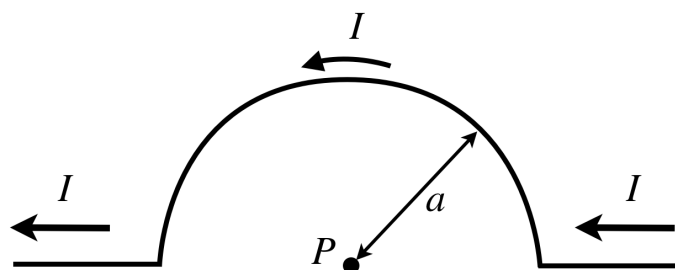
- (i) Use the Biot–Savart Law to calculate the magnetic field at the location of each charge due to the movement of the other charge at time  $t$ , i.e.  $\mathbf{B}_2(\text{at } 1)$  and  $\mathbf{B}_1(\text{at } 2)$ . {3}
- (ii) Calculate the magnetic Lorentz force  $\mathbf{F}_B$  acting on each charge due to its motion through this magnetic field. {3}
- (iii) Calculate the Coulomb force  $\mathbf{F}_E$  acting on each charge. {3}
- (iv) Illustrate your answers to parts (i), (ii) and (iii) in a figure. {4}
- (v) Demonstrate that for each charge the ratio of the magnitudes of the forces is equal to:

$$\frac{|\mathbf{F}_B|}{|\mathbf{F}_E|} = \left(\frac{v_1 v_2}{c}\right)^2,$$

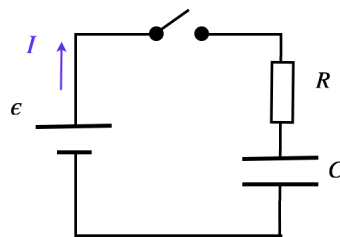
where  $c = 1/\sqrt{\epsilon_0 \mu_0}$  is the speed of light. {3}

- (vi) How do the result of parts (i), (ii) and (iii) change if we transform to a frame of reference that moves with charge 1. {3}

- b) Use the Biot–Savart law applied to a wire to calculate the magnitude and direction of the magnetic field at point  $P$  due to the current in a wire, shown in the figure, that includes a semicircular section of radius  $a$  centred on  $P$ . [Hint: make use of  $d\hat{\mathbf{R}} = \hat{\phi} d\phi$ ] {6}



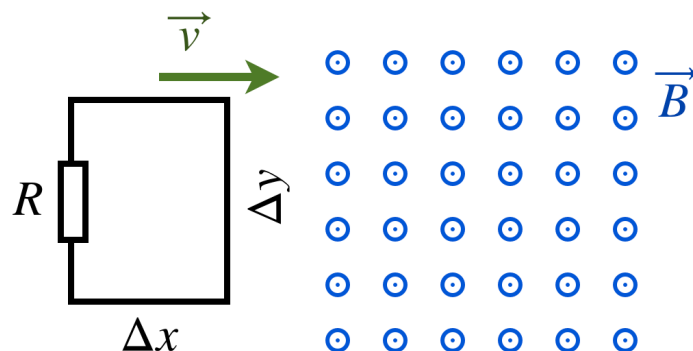
4. a) A circuit has an source of emf  $\epsilon$ , resistor  $R$  and capacitor  $C$  in series (see figure). The capacitor has two plates of area  $A$  that are separated by a distance  $d$  and which are initially uncharged.



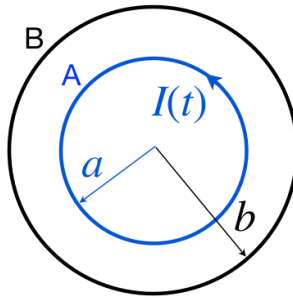
When the switch is closed at  $t = 0$ , a current flows along the circuit wires of the form

$$I(t) = \frac{V}{R} e^{-t/RC} .$$

- (i) Calculate the surface charge density on the capacitor plates as a function of time. {3}
- (ii) Calculate the displacement current  $I_D$  between the two plates of the capacitor. {3}
- b) A rectangular circuit of dimensions  $\Delta x \times \Delta y$  moves at a constant velocity  $\mathbf{v}$  into, through, and then out of a region of uniform magnetic field, as shown in the figure. The magnetic-field region is considerably wider than the circuit. The circuit has a resistance  $R$ .



- (i) Find the magnitude and direction (clockwise or counterclockwise) of the current induced in the circuit as it is going (a) into the magnetic field; (b) totally within the magnetic field, but still moving; (c) and moving out of the field. {7}
- (ii) Sketch a graph of the current in this circuit as a function of time, including the preceding three cases. {4}
- c) Two, long, coaxial solenoids are arranged as shown in the figure below. Solenoid A, with radius  $a$ , length  $\ell$  and  $N_a$  windings, carries an anti-clockwise time-varying current  $I(t)$ . A second solenoid B, with radius  $b > a$ , length  $\ell$ ,  $N_b$  windings and resistance  $R$ . The self-inductances in both solenoids are ignored.



- (i) Calculate the magnitude and direction of the induced current  $I_b$  in solenoid B due to solenoid A. {5}
- (ii) Calculate the magnitude and direction of the induced emf in solenoid A due to solenoid B. {3}