Autumn Term, Week 3 Tutorial
Jiachen Jiang

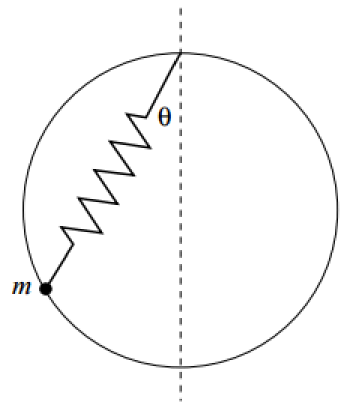
Read the following sections of University Physics and lectures. 1. Vectors 2. Forces 3. blocks on the slope

Q1. A brave swimmer attempts to cross the Thames River, which has a constant flow velocity $\boldsymbol{w}$ parallel to its banks. The swimmer can maintain a constant swimming velocity $\boldsymbol{v}$ relative to the water. Assume that the riverbanks are straight, parallel lines, and the width of the river is L.

1. **Shortest Time:** What is the shortest time it will take for the swimmer to reach the opposite bank of the river?
2. **Reaching a Specific Point:** If the swimmer wants to reach a point directly opposite to his starting position (i.e., the displacement is perpendicular to the riverbanks), in what direction relative to the water should the swimmer swim? Under what condition will this be possible?
3. **Minimum Displacement Along the Bank:** If it is not possible for the swimmer to reach the exact opposite point, what is the minimum distance he can end up from the opposite point (in the direction parallel to the bank) when he reaches the other side?

Q2.

A small ring of mass m slides without friction round a circular hoop of radius r in the vertical plane. As shown in the diagram, the mass is connected to the top of the hoop by a spring which has a natural length r, and spring constant k.



(a) Draw a diagram showing the directions of the forces on the ring due to its weight and the tension in the spring.

(b) Show that when the spring makes an angle θ to the vertical, the length of the spring is $2r\cos\theta$.

(c) In addition to the weight of the ring and the tension in the spring, what other force must be acting on the ring for it to be in equilibrium? Show this force on your diagram. (Remember that there is no frictional force between the ring and the hoop.)

(d) By resolving the components of the weight of the ring and the tension in the spring *in one appropriate direction only*, show that in static equilibrium the angle the spring makes to the vertical is θ_{eq}, where

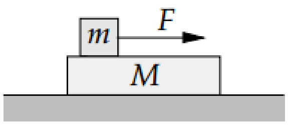
$$\cos\theta_{eq} = \frac{1}{2}\frac{1}{(1-(mg/(kr)))}.$$

[You may need the trigonometric identity that $\sin 2\theta = 2\sin\theta\cos\theta$.]

(e) What happens if $mg/(kr) > 0.5$?

Q3.

A slab of mass M rests on a frictionless floor. A block of mass m rests on top of the slab as shown in the diagram. The coefficients of static friction and dynamic friction between the block and the slab are μ_s and μ_d respectively and $\mu_d < \mu_s$. The block is pulled by a horizontal force of magnitude F.



(a) Using Newton's 3rd law, draw free-body diagrams for both the block and the slab to show all the forces acting on them; note in particular that the direction of the frictional force exerted by the slab on the block is determined by considering the direction of the block's impending acceleration.

(b) If the block and slab accelerate together without slipping with a common acceleration, deduce a value for this acceleration. By considering your free-body diagrams, show that the maximum acceleration the slab can have is given by $\mu_s mg/M$. Hence explain why the block and slab will only accelerate at the same rate if F is below a limiting value F_{lim}; show that $F_{lim} = (M+m)\mu_s mg/M$, where g is the acceleration due to gravity.

(c) For $F > F_{lim}$, find expressions for the accelerations of the slab and the block in terms of M, m, μ_d, F and g.