Autumn Term, Week 4 Tutorial

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Read the following sections of University Physics and lectures. Accelerations (Time Dependent, Velocity Dependent) Work and Kinetic Energy Conservative Forces and Potential Energy Deriving Force from Potential Gravitational Potential Energy Using Energy Conservation

Q1.

A diver of mass m begins a descent from a 10-m diving board with zero initial speed.

(a) Calculate the speed v_0 on impact with the water and the approximate elapsed time from dive until impact.

(b) Assuming that under the water the downwards gravity force is exactly balanced by the upwards buoyancy force and $-cv^2$ is the drag force due to the water, the equation of motion for the vertical descent of the diver through the water is given by:

$$ma = -cv^2,$$

where a is the downwards acceleration of the diver. Hence show that the speed v varies with depth x as

$$m\frac{\mathrm{d}v}{\mathrm{d}x} = -cv.$$

(You will need to use the chain rule to find an expression for a in terms of v and x rather than v and t.)

(c) Show by substitution that $v = A\exp(-\alpha x)$ is a solution to this equation and find α; then, by imposing the condition that $v = v_0$ at $x = 0$, find A.

(d) If $c/m = 0.4\ \mathrm{m}^{-1}$, estimate the depth at which $v = 0.1v_0$.

(e) The relationship between time t under water and depth x is given by

$$t = \frac{1}{\alpha A}\left(\mathrm{e}^{\alpha x} - 1\right).$$

By differentiating both sides of the equation with respect to t (implicit differentiation), or otherwise, show that this is consistent with the expression for v in (c). How long does it take for the diver to reach the bottom of a 5-m deep pool?

Q2.

This is a question about using potential energy functions to find equilibrium positions.

(a) The potential energy of an object which can only move along the x-axis is given by

$$U(x) = 2x^2 - x^4.$$

Find all the equilibrium positions and determine whether they are stable or unstable. Sketch the function $U(x)$.

(b) The interatomic interaction between two atoms a distance r apart in a simple molecule is described by the following potential energy function:

$$U(r) = U_0 \left(\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6} \right),$$

where a and U_0 are constants.

(i) Find expressions for the force, $F(r)$, between the atoms, their equilibrium separation and their potential energy at this separation.

(ii) To what values does $U(r)$ tend when $r \to \infty$ and $r \to 0$? What can you deduce from these about the stability of the equilibrium?

(iii) Draw clearly labelled sketches of $U(r)$ and $F(r)$ as functions of r.

Q3.

The body of a rocket has a mass m_b and also carries initially a mass m_f of fuel.

(a) Use the method described in the lectures to show that the speed u of the rocket after all the fuel is burnt is given by

$$u = u_0 \ln\left(1 + \frac{m_f}{m_b}\right),$$

where u_0 is the speed of the ejected gases (assumed constant) relative to the body of the rocket. Be careful about the signs in your derivation, showing clearly how you arrive at the above result.

(b) A rocket of body mass 90 kg carries an initial fuel mass of 300 kg, and is to be used to lift vertically a payload of mass 10 kg to a height of 100 km above the Earth's surface. On the assumption that all the fuel is burnt in a very short time relative to the time of flight of the rocket, what must be the speed of the ejected gases relative to the rocket?

(c) The rocket described in (a) is now divided into two stages, each with body mass $m_b/2$ and initial fuel mass $m_f/2$. The first stage is discarded when all its fuel has been consumed. What is now the final speed of the second stage? Explain why this is larger than your answer to part (a).