Autumn Term, Week 5 Tutorial
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Read the following sections of University Physics and lectures. Simple Harmonic Motion (SHM) SHM energy SHM Initial conditions Damped Oscillations

Q1.

(a) A mass m is supported from a light spring with spring constant k. The mass is displaced slightly from its equilibrium position. Show that it will undergo vertical oscillations of period $T = 2\pi\sqrt{m/k}$. Explain why the gravitational acceleration g does not enter into the expression for T.

(b) Describe the changes in the kinetic energy and the elastic and gravitational potential energies of the system as the mass oscillates up and down.

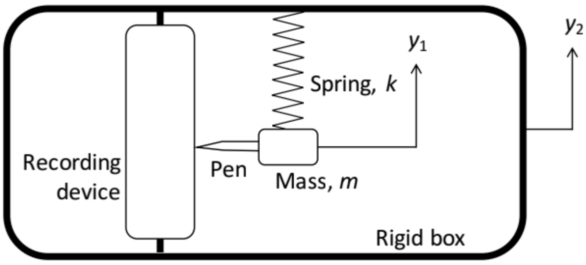
(c) What will the period become if the length of the spring is halved ?

(d) What will the period become if the two half-springs are used in parallel ?

(e) An astronaut on the surface of the moon weighs rock samples using a light spring balance. The balance, which was calibrated on earth, has a scale 100 mm long which reads from 0 to 1.0 kg. The astronaut observes that a certain rock gives a steady reading of 0.40 kg and, when disturbed, vibrates with a period of 1.0 s. What is the acceleration due to gravity on the moon?

Q2.

A seismograph allows the measurement of the amplitude of seismic waves of different frequencies. It can be modelled as a mass m suspended inside a rigid box by a massless spring of constant k, The motion of the mass is damped by a vertical viscous force which may be assumed to be of magnitude $b\,\dot{y}_1$, where y_1 is the height of the mass in the inertial frame of reference of the surface of the Earth (before any seismic wave is present) and b is a damping coefficient. A light frictionless pen records the motion of the mass relative to the box as shown in the figure below.



A seismic wave moves the box vertically so that its height as a function of time is given by $y_2 = a_s \cos(\omega t)$.
(a) Show the differential equation describing the motion of the mass is

$$\ddot{y}_1 + \frac{b}{m}\dot{y}_1 + \frac{k}{m}y_1 = \frac{k}{m}a_S \cos(\omega t).$$

(b) Show the steady state motion is the mass is of the form $y_1 = \mathrm{Re}\left(A(\omega)\mathrm{e}^{\mathrm{i}\Phi(\omega)}\mathrm{e}^{\mathrm{i}\omega t}\right)$, and find expressions for $A(\omega)$ and $\Phi(\omega)$.

(c) Sketch the amplitude of the response, $A(\omega)$, to seismic waves of fixed amplitude a_s as a function of the angular frequency ω for the case where b is small. Use this sketch to explain why seismographs are designed to work in the high-frequency domain.