Autumn Term, Week 7 Tutorial

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Read the following sections of University Physics and lectures. Torque Moments of Inertia for Continuous Rigid Bodies

A gyroscope wheel is at one end of an axle of length d. The other end of the axle is suspended from a string of length s, that makes a fixed angle θ with the vertical. The wheel is set into motion so that it executes uniform precession in the horizontal plane. The wheel has mass M and moment of inertia about its centre of mass I. Its angular speed about the axle through its centre is ω. Neglect the mass of the axle and the mass of the string and assume that $|\omega| \gg |\Omega|$. What is the direction and magnitude of the angular velocity of precession?

Hints:

1. *Watch the following Youtube video if you struggle with this question. Be very careful with the direction in which the gyroscope spins and processes.* *https://youtube.com/shorts/jhwvCKrUq9U?si=2qPK2HVKG1P-Qzny*
2. *Draw a diagram to show the following vectors: the torque on the wheel by the string, all the forces on the wheel, the direction of the precession, and the direction of the wheel's rotation around the axle. Don't forget the right-hand rule when calculating* $\underline{G} = I\,\underline{\Omega} \times \underline{\omega}$.

17 Summary: Linear - Rotational Equivalencies

We can identify quantities in linear mechanics and rotational mechanics which behave in equivalent fashions. If you are not sure what to do in a rotational problem, think what you would do in the equivalent linear problem and then use the table below.

Linear		Rotational	
mass	m	moment of inertia	I
small displacement	$\mathrm{d}\underline{r}$	small angular displacement	$\mathrm{d}\underline{\theta}$
velocity	$\dot{\underline{r}} = \underline{v} = \frac{\mathrm{d}\underline{r}}{\mathrm{d}t}$	angular velocity	$\dot{\underline{\theta}} = \underline{\omega} = \frac{\mathrm{d}\underline{\theta}}{\mathrm{d}t}$
acceleration	$\ddot{\underline{r}} = \underline{a} = \frac{\mathrm{d}^2\underline{r}}{\mathrm{d}t^2}$	angular acceleration	$\ddot{\underline{\theta}} = \frac{\mathrm{d}\underline{\omega}}{\mathrm{d}t} = \frac{\mathrm{d}^2\underline{\theta}}{\mathrm{d}t^2}$
linear momentum	$\underline{p} = m\dot{\underline{r}} = m\underline{v}$	angular momentum	$\underline{L} = I\dot{\underline{\theta}} = I\underline{\omega}$
force	$\underline{F} = \frac{\mathrm{d}\underline{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m\underline{v})}{\mathrm{d}t}$	Moment / torque	$\underline{G} = \frac{\mathrm{d}\underline{L}}{\mathrm{d}t} = \frac{\mathrm{d}(I\underline{\omega})}{\mathrm{d}t}$
linear kinetic energy	$K.E = \frac{mv^2}{2}$	rotational kinetic energy	$R.K.E = \frac{I\omega^2}{2}$
work done	$\mathrm{d}W = \underline{F} \cdot \mathrm{d}\underline{r}$	work done	$\mathrm{d}W = \underline{G} \cdot \mathrm{d}\underline{\theta}$
linear impulse	$\underline{F}\mathrm{d}t = \mathrm{d}\underline{p}$	angular impulse	$\underline{G}\mathrm{d}t = \mathrm{d}\underline{L}$