Journey to the Centre of the Earth

Introduction

The novel *Voyage au centre de la Terre* was written by Jules Verne and published in 1864 (1871 in English translation). With its mix of high adventure, scientific discovery and familiar themes (such as surviving dinosoars, hollow earths and the uncertain battle to return home), it rapidly proved popular. The novel has been adapted many times for film, television, radio and other media. Perhaps the most well known adaptation in the 1959 film version staring James Mason and Pat Boone.

In the original, and most adapted versions, the narrative follows an academic professor, his assistant and one or more other companions as they descend into a volcano in the far North, discover routes into underground caverns, encounter an array of prehistoric creatures, find an underground body of water, and eventually emerge (via a volcano, whirlpool or other vent) far South of their starting point.

The basic premise of *Journey to the Centre of the Earth* shares problems with the category of Hollow Earth stories more generally, but with its own variants on the theme. Unusually for fiction of this kind, it actually tackles the majority of those problems in the text, explicitly stating the laws of Physics as understood at the time of writing and allowing the characters to discuss their implications. Much of this discussion can be found in Chapter 25 of the text, although references are scattered throughout

The depth to which the explorers descend is unclear, and appears to vary from version to version. In the original text, the depth of 40 miles is mentioned, together with the implication that their journey is following a shallow chord remaining within the crust. In other adaptations, it is implied that they travel almost directly through the centre of the planet, emerging in the southern hemisphere. While they encounter large caverns, the Earth is clearly described as being mostly solid rather than entirely hollow.

There are, of course, several areas which can effectively be explored using undergraduate physics:

- The variation of temperature with depth
- The variation of air pressure with depth
- The variation of gravity with depth

Temperature and Air Pressure

> *'Forty miles?' I shouted.*
> *'In all probability.'*
> *'But that's the extreme limit that science has ascribed to the thickness of the Earth's crust.'*
> *'I will not contradict you.'*
> *'And here, according to the law of increasing temperature, there should be a temperature of over 1,500°.'*
> *' "Should be", my boy.'* *- Professor Lidenbrock and Alec*

As Verne noted in the above section, the geothermal gradient is likely to be a significant factor for explorers venturing deep below the surface of the Earth. Deep mines, for example the diamond mines of South Africa, can reach significant temperatures, with measured rock temperatures of $\sim$60° at a 4 km depth. This heat is radiating outwards from the centre of the planet and arises both from the residual gravitational potential released when the planet formed and from radioactive decay.

Temperature (T) will be proportional to the amount of energy being provided to each unit volume of rock (unit area for a shell of fixed thickness) divided by its thermal heat capacity. Assuming near-uniform composition in the crust, and that heat primarily comes from below rather than being generated in the crust itself, $T(r) \propto \mathrm{heat\ flow}/4\pi r^2$ where r is measured radius from the centre of the Earth. So:

$$T(d) \propto (R-d)^{-2} \propto (1-(d/R))^{-2} \approx 1 + 2d/R$$

where R is the radius of the Earth, depth $d = (R - r)$ and d/R is small, i.e. temperature increases linearly with depth. The resulting geothermal gradient varies with location and proximity to tectonic features, but modern estimates average around 25 °C/km in the crust, consistent with that quoted by Professor Lidenbrock.

While Professor Lidenbrock's party found little evidence for increasing temperature, they did measure an increase in air pressure. Chapter 25 of the original novel discusses the significance of this, while most adaptations overlook it. Lidenbrock asks *'Have you noticed how intensely the sound is propagated?'* and notes that when the air density reaches that of water *'we will just have to put stones in our pockets'*. The atmospheric density at sea level derives from the weight of air rising in a column above it. Thus, it is perhaps unsurprising that Verne inferred that an increased column would lead to higher pressure. The bulk of our atmosphere is contained within the Troposphere (0-12 km above sea level). Therefore a naive calculation might imply that to encounter air with the density of water (a pressure of ~780 atm), our explorers might have to descend ~8000 km in depth (rather further than the centre of the Earth, R=6400 km!). Having done so, they would indeed encounter intense propagation of sound, since the speed of sound $v = \sqrt{K/\rho}$ where K is the bulk modulus of a given material and ρ its density. At high densities, pressure waves (like sound) travel more quickly.

However, such calculations neglect a rather important effect - that of...

Gravity

> *'It is true that the force of gravity will decrease in proportion to our descent. You know that it is at the surface itself of the Earth that its action is most strongly felt, and that objects no longer have any weight at the centre of the globe.'* *- Professor Lidenbrock*

It is difficult to fault Verne for this assertion, which is straightforward to prove, given application of Gauss's theorem: $\oint \vec{g} \cdot d\vec{A} = -4\pi GM$, where the integral is taken over a closed surface, and M is the mass enclosed by that surface. Taking our Gaussian surface to be an imaginary sphere at radius r from the centre of the sphere, the surface vector is radial at all points, and so $\oint \vec{g} \cdot d\vec{A}$ is simply $g(r).4\pi r^2$. Therefore, if $M(r) = V(r) \times M_\mathrm{T}/V_\mathrm{T}$ is the mass enclosed within radius r, then:

$$g(r) = \frac{-4\pi GM(r)}{4\pi r^2} = -\frac{G}{r^2} M_\mathrm{T} \frac{r^3}{R^3} \quad \propto \quad r = (R-d)$$

So gravity does indeed decrease in proportion to increasing depth and will be zero at the Earth's core (where no mass is enclosed by our imaginary surface). At 40 miles depth, only 1% of the distance to the Earth's core, the acceleration due to gravity will itself only have decreased by 1% - scarcely noticeable to our explorers. If anything, given muscles trained in higher gravity, they will find the going easier! However on a journey to the *centre* of the Earth, any expedition would certainly encounter weightlessness amongst their many other problems.

Returning to the question of air pressure, we can now incorporate our new understanding of gravity. The pressure exerted by an enclosed volume of air is given by the product of the kinetic energy carried by each molecule with the number of such molecules. The atmosphere (above ground!) is often approximated as isothermal - at least in its densest lower layers. This assumption is predicated on the fact that, when free to move, the molecules of a perfect gas will redistribute thermal energy amongst themselves and settle into a Maxwell-Boltzmann distribution of velocities.

The equipartition of energy then states that each molecule will carry comparable kinetic energy, which must balance it in turn against gravitational energy. The resulting distribution of particle density is $N(h) \propto \exp(-mgh/kT)$ where mgh is the gravitational potential energy of particles with mass m at height h assuming a constant acceleration due to gravity g. Since $PV = nRT$ and $N(h) = n(h)/V$, pressure is proportional to the same factor at constant temperature.

As we've already seen, the constant temperature assumption is... shaky... with increasing depth. However if we consider the "Verne-verse" Professor Lidenbrock's party occupies, we can take their observations and assume the temperature variation is small. So $P(r) \propto \exp(-rg(r))$ and since $g(r) \propto r$, $P(r) \propto \exp(-r^2)$. So pressure scales as $exp(-2$d) with increasing depth d - at 40 miles depth, the air pressure would be a mere 2% higher than that at sea level - rather lower than the highest barometer reading recorded on Earth's surface (108.57 kPa, 7% above one standard atmosphere).

Abandoning the "Verne-verse" assumption, for a 40 km depth, actually makes relatively little difference to the air pressure. We can combine all our previous work:

$$P(r) \propto \frac{nR}{V}T(r) \propto T(r)\exp(\frac{rg(r)}{T(r)}) \propto r^{-2}\exp(\frac{r^2}{r^{-2}}) \propto 4r^{-2}\exp(r) \approx 1.02\,P_0 \text{ for } r = 0.99\,R$$

Concluding Thoughts

> *'Who would ever have thought that there could be a real ocean inside the Earths crust, with its own ebb and flow, its own sea breezes and storms!'*
> *'And why not? Is there some physical reason to prevent it?'*
> *'Not that I can see, if we abandoned the theory of heat at the centre.'*
> *- Professor Lidenbrock and Alec*

Unfortunately, Lidenbrock's *if* is a big one. If the temperature variation with depth is less severe than believed, then neither gravity nor air pressure present an insurmountable problem to his expeditions. Unfortunately, here the "Verne-verse" and our own diverge and we are unlikely ever to explore deep into the crust of our planet!

References and Further Reading

http://www.ibiblio.org/julesverne/books/jce%20revd%20edn.pdf - Full text of the novel with introduction and notes, translated by William Butcher, Oxford University Press, 1992

http://www.astrobio.net/amee/March/retrospections_02.htm - An article on Verne's expression of the scientific revolution in our world-view

http://www.youtube.com/watch?v=7K07q5AiGbM - Trailer for the 1959 movie adaptation