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#### TITLE: Irradiated Gaseous Discs Around White Dwarfs

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### Irradiated Gaseous Discs Around White Dwarfs

by

**Timothy Kinnear** 

Thesis

Submitted to the University of Warwick

for the degree of

Master of Science

### Physics

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# Contents

List of	Figures	iv
Acknow	vledgments	ix
Declara	ations	x
Abstra	ct	xi
Chapte	r 1 Background - White Dwarfs	1
1.1	Stellar Evolution - Stellar Life Resulting In White Dwarfs	1
1.2	DAZ & DBZ - White Dwarfs with Metal Contamination	2
1.3	Infrared Excess - White Dwarfs with Dust Discs	4
1.4	Call Lines - White Dwarfs with Gas and Dust Discs	5
Chapte	er 2 Background - Circumstellar Discs	10
2.1	Properties of Material	10
	2.1.1 Mass	10
	2.1.2 Composition	10
	2.1.3 Temperature	13
	2.1.4 LTE and NLTE Models	14
2.2	Possible Sources of the Material	15

	2.2.1	Post AGB Envelope	15				
	2.2.2	Interstellar Medium	16				
	2.2.3	Comets and Asteroids	16				
2.3	3 Survival of Planets/Planetesimals/Asteroids/Comets During Asymp-						
	totic (	Giant Branch	18				
2.4	Tidal	Disruption of Asteroids and Disc Formation	19				
Chapte	er3D	escription of Model	22				
3.1	Conce	pt of the Model	22				
3.2	Applic	ation of the Physical Situation to a Simulation	23				
	3.2.1	Geometry	23				
	3.2.2	Incident Radiation per Unit Area on Disc	24				
	3.2.3	Radial Temperature Variation	26				
	3.2.4	Total Incident Disc Radiation	28				
3.3	Introd	uction to the 'Cloudy' Photoionisation Code	29				
	3.3.1	Physics	30				
	3.3.2	Capabilities	32				
	3.3.3	Operation	32				
3.4	Simula	ation Setup	33				
	3.4.1	Overall Model Configuration	33				
	3.4.2	Disc Height	35				
	3.4.3	Application of the Physical Situation to the Model $\ldots$	37				
Chapte	er4R	esults and Analysis	40				
4.1	Single	Ring Models	40				
	4.1.1	Density	40				
	4.1.2	Luminosity	53				

	4.1.3	Radius	53			
	4.1.4	White Dwarf Temperature	55			
	4.1.5	Disc Height	60			
	4.1.6	Inclusion of Grains/Dust	62			
4.2	Disc N	/lodels	64			
	4.2.1	Density	64			
	4.2.2	Size	67			
	4.2.3	White Dwarf Temperature	68			
	4.2.4	SDSS 1228+1040	70			
Chapter 5 Discussion 74						
5.1	Summ	ation of Findings	74			
	5.1.1	Call H & K Lines	74			
	5.1.2	Call x, y & z Relative Strengths	75			
	5.1.3	White Dwarf Temperature Range	75			
	5.1.4	UV Spectrum	76			
	5.1.5	Neutral Lines	77			
5.2	_					
	Critiqu	Je of Method	((			
	Critiqu 5.2.1	Je of Method	77 77			
	Critiqu 5.2.1 5.2.2	Je of Method	77 77 79			

#### Chapter 6 Conclusions

# **List of Figures**

1.1	Comparison spectra of the first three gas disc white dwarfs discov-	
	ered. Graph courtesy of B. Gänsicke	8
3.1	Geometry of white dwarf and disc	23
3.2	Comparison of the radial disc temperature model by King [1997]	
	and the derivation presented here. The white dwarf temperature	
	was taken as 20,000 K	27
3.3	Layout of the model	34
3.4	Transforming the physical geometry to Cloudy equivalent	38
4.1	Simulated spectrum for a single ring at radius $4.25\times 10^{10}~{\rm cm}$ and	
	density $1.4\times 10^{-12}~{\rm g~cm^{-3}}$ around a standard white dwarf. $~$	42
4.2	Line fluxes for various density disc rings at a radius of $4.25\ \times$	
	$10^{10}$ cm, illuminated by a standard white dwarf. Vertical lines	
	indicate the densities of interest.	43

- 4.3 Line fluxes and lines ratios for various density disc rings at a radius of  $4.25 \times 10^{10}$  cm, illuminated by a standard white dwarf. Vertical lines indicate the densities of interest. Horizontal lines of a given colour indicate the observed ratio for SDSS 1228 for the line represented by the same colour to Call v.

43

- 4.5 Transmitted spectrum for a density of  $3.4 \times 10^{-12}$  g cm<sup>-3</sup> with the disc rings at radius  $4.25 \times 10^{10}$  cm around a standard white dwarf. 45
- 4.6 Vertical temperature profiles for selected several densities with the disc rings at radius  $4.25 \times 10^{10}$  cm around a standard white dwarf. 48
- 4.7 Vertical ionisation profiles of Calcium for selected density disc rings at a radius of  $4.25 \times 10^{10}$  cm, illuminated by a standard white dwarf. 49
- 4.8 Comparison of several gas parameters in a ring of density  $3.4 \times 10^{-12}$  g cm<sup>-3</sup> at a radius of  $4.25 \times 10^{10}$  cm, illuminated by a standard white dwarf. Depth scales are the same in each graph, showing the link between the temperature profile (top) and the ionisation levels of Calcium and Sodium (middle and bottom) . . 51
- 4.9 Comparison of several gas parameters in a ring of density  $3.4 \times 10^{-12}$  g cm<sup>-3</sup> at a radius of  $4.25 \times 10^{10}$  cm, illuminated by a standard white dwarf. Depth scales are the same in each graph, showing the link between the temperature profile (top) and the ionisation levels of Magnesium and Iron (middle and bottom)  $\ldots$  52

v

- 4.10 Line fluxes for disc rings illuminated by a white dwarf of varying luminosity only, using the standard white dwarf SED. The disc rings were at a radius of  $4.25 \times 10^{10}$  cm and had a density of  $1.7 \times 10^{-12}$  g cm<sup>-3</sup>.
- 4.11 Line fluxes for disc rings at a range of radii from  $\approx 6.3 \times 10^9$  cm to  $\approx 1.2 \times 10^{11}$  cm  $(0.09 1.7R_{\odot})$ . The rings had a density of  $1.7 \times 10^{-12}$  g cm<sup>-3</sup> and were illuminated by a standard white dwarf. 55
- 4.13 Line fluxes for disc rings illuminated by white dwarfs of varying temperature (and associated luminosity). The disc rings were at a radius of  $4.25 \times 10^{10}$  cm and had a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup>.

58

- 4.15 Line fluxes for a range of disc heights. Disc height is given in terms of the multiplier used to modify it from the theoretical disc height as described in 3.4.2. The disc rings were at a radius of  $4.25 \times 10^{10}$  cm, had a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup> and were illuminated by a standard white dwarf.

4.16	Line ratios against the Call 'y' line for a range of disc heights. Disc	
	height is given in terms of the multiplier used to modify it from the	
	theoretical disc height as described in 3.4.2. The disc rings were	
	at a radius of $4.25\times 10^{10}~{\rm cm},$ had a density of $5.4\times 10^{-12}~{\rm g~cm^{-3}}$	
	and were illuminated by a standard white dwarf.	61

- 4.17 Total spectra (white dwarf plus disc) for discs of several different densities, plotted with the observed spectrum for SDSS1228. The simulated discs span radii of  $1.4 \times 10^{10}$  to  $6.3 \times 10^{10}$  cm (0.2 to 0.9  $R_{\odot}$ ) and were illuminated by a standard white dwarf. . . . . . 66
- 4.18 Total line flux for each of 57 rings at radii between  $\approx 6.3 \times 10^9$  cm to  $\approx 1.2 \times 10^{11}$  cm  $(0.09 - 1.7R_{\odot})$ . Illumination was from a
- 4.20 Graphs of average Calcium triplet line strength, normalised to the same proportions as in Figure 4.14. Simulated points are in black, points from SDSS data are in red and points from WHT data are in green. The discs spanned radii between  $1.4 \times 10^{10}$  cm and  $6.4 \times 10^{10}$  cm and had a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup>. . . . . . 70
- 4.21 Total spectra (black) and disc spectra only (red) for disc systems illuminated by white dwarfs of varying temperatures and associated luminosity. The simulated discs span radii of  $1.4 \times 10^{10}$  to  $6.3 \times 10^{10}$  cm (0.2 to 0.9  $R_{\odot}$ ) and had a density of  $1.71 \times 10^{-12}$  g cm<sup>-3</sup>. 71

4.22	Spectrum of the simulated version of SDSS $1228+1040$ . Simulated	
	disc spectrum is in blue, total spectrum (disc $+$ white dwarf) in	
	black, and the observational data in red. Inset is a zoomed in view	
	of the Call x, y and z lines. $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$	73

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# Declarations

This thesis is the sole work of Timothy M. Kinnear, all other works and contributions are acknowledged. This work has not been submitted to any other university or for the purpose of any other degree or qualification.

### Abstract

Since 1987 the number of known white dwarfs with metal contamination and an IR excess resulting from a circumstellar dust disc has risen to 14 objects, constituting 50% of metal contaminated white dwarfs with implied accretion rates larger than  $3 \times 10^8$  g s<sup>-1</sup>. Since 2006 5 objects have been found possessing Near IR Call emission lines in the form of a triplet at approximately 8600 Å with Doppler profiles asserting their source as being a circumstellar disc. Of these 5 objects, 3 have been confirmed to also possess an IR excess suggesting that the disc line emission and IR disc emission are part of the same phenomenon. The material is suspected to be the debris of tidally disrupted asteroids, and the phenomenon may potentially be related to metal contamination in white dwarfs as a whole.

The aim of this thesis was to create a simulation which is as self consistent and physically accurate as was possible within the constraints of the timescale of an MSc. The model utilises the photoionisation code 'Cloudy' to simulate the illumination of a circumstellar disc by modeling it as radially sequential discrete ring segments. The results of the investigation illustrates that the model successfully replicates the observed Call triplet. In addition it favourably simulates the Call 3950 Å H and K lines, without exhibiting the problem of these lines being excessively strong relative to the observed Call triplet displayed by previous LTE and NLTE models. Gas densities between  $\approx 10^{-12}$  and  $\approx 10^{-11}$  g cm<sup>-3</sup> appear to provide the best Call triplet strengths without introducing large numbers of unobserved neutral lines associated with higher densities. This agrees with previous estimates which place the density in the same region. The results also appear to suggest that it is plausible for the gas and dust to exist in a layered vertical structure. Gas at the surfaces of the disc could exist at temperatures of around 4,000 K, while the high optical depth in the UV of the gas protects dust settled in the midplane from reaching sublimation temperatures. Additionally, results agree with the range of white dwarf temperatures over which Call emission has been observed to this date. Simulated Call triplet emission exists over the range of beyond 22,000 K falling away towards 14,000 K at which point the emission strength has decreased by one order of magnitude, but would still likely be observable. Below this temperature emission line strength continues to fall away, decreasing to 2 orders of magnitude by 8,000 K. One feature of this model which does not appear to reflect the observational data is the presence of an Nal emission line at 5892 Å in the simulations. This line is consistently about as strong as the Call triplet emission and no modification to the parameters investigated seemed to significantly reduce its strength. Certain conditions also appear to introduce further neutral lines which are not observed. The UV region of the spectrum could not be successfully simulated from the raw Cloudy output due to the difficulty in accurately simulating highly optically thick material. However, looking at other output properties, lines which may be found in the UV include Na, Ca, Mg and Al; these being the elements most strongly contributing to heating of the gas through UV absorption and ionisation.

### Chapter 1

# **Background - White Dwarfs**

#### 1.1 Stellar Evolution - Stellar Life Resulting In White Dwarfs

Main sequence stars with masses between  $\approx 1$  and  $\approx 10 M_{\odot}$  are expected to follow a set evolution resulting in a white dwarf. Main sequence stars in this mass range will typically live for about 20 Myr - 12 Gyr, at which point the hydrogen fueling the fusion processes begins to be expended. This decrease in hydrogen results in the fusion processes slowing and the star beginning to cool. Since all stars exist as an equilibrium between their internal fusion powered radiation causing thermal pressure outwards, and the gravitation force attempting to collapse the star inwards, the star begins to shrink. As the material falls inwards, pressures in the centre of the star reach levels which permit fusion burning of helium. Once this helium burning begins, the core heats up again and the outward radiative force quickly increases. The outer material previously falling inwards is pushed outwards at a rate greater than that with which gravitational collapse can compete. This results in an expansion of the star's outer radius. This outer material is not subject to helium burning and, with decreasing pressure and density due to expansion, becomes fairly cool (typically a few thousand Kelvin). This cool, expanding cloud of a star is a Red Giant, and the process signals the star's transition into and along the 'Asymptotic Giant Branch' (AGB) of stellar evolution from main sequence stars of the sizes described.

A significant portion of the material expanding away from the star does so at a rate which means that it will not be recovered by gravitational forces (varying from losing around 50% by mass for solar mass stars up to  $\approx$ 85% for 8 -10  $M_{\odot}$  stars). The core of the star burns away the helium into carbon and oxygen, with some of the nearby remnant of the envelope of hydrogen(/helium) falling back onto its surface. The dense core, plus the surface hydrogen/helium material compacts itself into a hot sphere approximately the size of the Earth. This newly formed white dwarf will have a high temperature due to its prior internal energy and its subsequent compaction, but is no longer producing energy through fusion. The conditions are extreme enough that the material in the core is degenerate. Radiation stems purely from thermal emission of its latent energy.

#### 1.2 DAZ & DBZ - White Dwarfs with Metal Contamination

It is estimated that approximately 10% of stars in the Galaxy by mass are white dwarfs, of the order of  $10^{10}$  by number [Napiwotzki, 2009]. These white dwarfs are subdivided based on features in their spectra indicative of their outer layer composition. The main two categories are those with strong hydrogen absorption lines, indicating a primarily hydrogen atmosphere (labelled 'DA'); and those with strong helium absorption lines, indicating a helium dominated atmosphere (labelled 'DB'). The creation of each type dependant on the manner in which the atmospheres settle. For cool white dwarfs ( $T_{\rm WD} < 25,000$  K), the DB to DA ratio

varies between approximately 0.25 at the hotter end of the range, up to 0.4 at the cooler end [Tremblay and Bergeron, 2008; Davis et al., 2009]. However, this ratio varies with both temperature and stellar environment [Davis et al., 2009]. Pulses of energy from subsurface fusion of helium (so called 'helium shell flashes') affect the top layer of hydrogen, sufficiently intense bursts of energy will deplete the hydrogen. The extent to which the hydrogen gets depleted in this state is related to the length of time before the final pulses occur. The radial structure of a white dwarf envelope is highly layered due to strong gravitational settling of elements. This results in the lightest elements populating the outer layers.

An additional sub-class, into which about 25% of cool DA white dwarfs fall [Zuckerman et al., 2003], is that of white dwarfs which possess a number of absorption lines corresponding to various metallic elements, these are designated with an additional 'Z' after their label (ie, DBZ, for a helium dominated white dwarf with metal contamination). Due to the strong layering expected of white dwarfs these elements are expected to settle deeper into the structure of the white dwarf. Atmospheric settling times are dependent on temperature and atmospheric composition. Particularly cool white dwarfs (generally less than 15,000 K depending on other parameters) can potentially have settling times of hundreds to thousands of years. However, with increased effective temperature this very rapidly decreases to less than a year or even of the order of a day. For calcium for example, a DA white dwarf of 6,000 K can have a settling time of about 1,000 years for calcium, a white dwarf of 10,000 K will have a settling time of 70 years, and at 15,000 K it becomes just 3 days [Koester and Wilken, 2006]. The settling times also depend on surface gravity in addition to temperature, for the example of Calcium settling log g = 8.25 was used (matching the estimated surface gravity of SDSS 1228 of g = 8.24 [Gänsicke et al., 2006]). In the case of non-DA white dwarfs settling times are orders of magnitude larger. In the case of a 6,000 K DB white dwarf, the theoretical settling time would be as much as 3 Myr [Koester, 2009].

Nevertheless, even given the longest settling times, when compared to the active lifetime of a cool white dwarf of  $\approx 1$  GYr, these elements are unlikely be an intrinsic part of the white dwarf itself. The settling times mean that these elements have to be actively accreted into the atmosphere of the white dwarf, else they would rapidly disappear and not be detected. Detected elements tend to be refractory metals, calcium, sodium, iron, magnesium; though the ratios of abundances of these materials are found to vary from star to star [Wolff et al., 2002].

#### 1.3 Infrared Excess - White Dwarfs with Dust Discs

Some time after the detection of metal contamination in the form of absorption lines, another unusual feature began to be observed in some white dwarfs. Several white dwarfs known to have metal contamination were found to possess a flux excess in the infrared. The first of these was G29-38, by Zuckerman and Becklin [1987], which was initially suspected to be an indication of an orbiting brown dwarf. This was refuted by analysis of pulsation timing between the white dwarf and its spectra (the white dwarf is a ZZ Ceti variable), and the corresponding pulsations expected to occur due to a disc being illuminated as such [Graham et al., 1990]. This showed that a dust disc was the most likely source of the excess, with the light curves corresponding well to such a model, as there is no reason to believe that the same effect should be seen by a brown dwarf companion or a shell of material. The amount of dust that could be contained in such discs is substantial, the dust disc around G29-38 is responsible for 3% of the total flux detected from

the white dwarf [Reach et al., 2005], further corroborating the idea of the source being a dust disc, as it would be unlikely that a brown dwarf could account for such emission [Graham et al., 1990].

Analyses preformed on several examples of this dust suggest dust temperatures in the region of 300 - 1,000 K, and occupying a region between 0.1 and 1  $R_{\odot}$  [Farihi et al., 2008].

A recurring aspect of these IR excesses is an emission at 10  $\mu$ m corresponding to a known silicate emission feature, showing the presence of appreciable amounts of silicate dust as a component of the disc composition[Reach et al., 2005].

In recent years the number of known white dwarfs with an IR excess has increased rapidly, with better observation platforms such as the Spitzer Space Telescope [Farihi et al., 2008; Brinkworth et al., 2009; Jura et al., 2007]. To date 14 such objects have been identified amongst the highest metal contamination and white dwarf effective temperatures ranging from 7,400 K up to 22,020 K [Farihi et al., 2009].

#### 1.4 Call Lines - White Dwarfs with Gas and Dust Discs

Recently a white dwarf in the Sloan Digital Sky Survey (SDSS) was found to have Call line emission in the Visible/Near Infrared (SDSS1228). Since then, four more white dwarfs with such emission lines have been found, with the first three having been confirmed to also possess the same IR excess attributed to dust as described in Section 1.3. 'SDSS J122859.93+104032.9' by Gänsicke et al. [2006] (also first of the Call emission line objects to additionally be shown to possess an IR excess, by Brinkworth et al. [2009]), 'SDSSJ104341.53+085558.2' [Gänsicke et al., 2007]

being the first two, both DAZ, with the third being the first DBZ equivalent to be found, 'SDSSJ084539.17+225728.0' [Gänsicke et al., 2008]. These three comprise the set of objects determined to have both the Call emission feature, as well as an IR excess. Shown in Table 1.1 are the tabulated properties of these objects including the type of white dwarf, the effective white dwarf temperature  $(T_{eff})$ , the estimated white dwarf mass  $(M_{\rm wd})$  and the equivalent width of the Call IR triplet lines (EW). For the EW measurements, the value on the main line for the object is from the original SDSS data, with the value on the next line from follow up WHT observations of the object. SDSS J122859.93+104032.9 (SDSS 1228) will be used as the main comparison with the models, as its spectra possesses the strongest and most well defined lines of the set. Shown in Figure 1.1 are the SDSS spectra of the three published objects. Most notable is the presence of the Ca-II triplet at  $\approx 8600$  Å, as shown in the cut-away, which is uniformly stronger than any other spectral emission or absorption lines in these objects (this triplet will be referred to as 'x, y and z', in order of increasing wavelength). The only line not attributable to calcium is an Fell line at 5169 Å. The observed lines are strongest in SDSS 1228, with clear differences in strength between the lines of the triplet, with y being the strongest, then z at approximately 0.85-0.9 of the strength of y, and x at approximately 0.75-0.8 the strength of y.

The profile of these lines is a double-peaked shape consistent with Doppler broadening due to Keplerian rotation [Horne and Marsh, 1986]. This double peaked shape is the best evidence so far for the material being a disc rather than a shell. While indicating a disc as the source, the double peaked shape is  $\sin i$  dependant (where *i* is the inclination of the disc relative to the direction of observation). This means that it is difficult to constrain the radii of the gas emitting region. Models suggest that these lines come from an emitting region with inner radii in the range

 $pprox 0.3R_{\odot}$  to  $0.5R_{\odot}$  and outer radii in the region of  $pprox 1R_{\odot}$ , and viewed at a fairly high inclination (i.e., more edge-on than face-on to the disc,  $\approx$   $70^\circ$  in the case of SDSS 1228). This would appear to match the estimated radial extents of the dust discs, further suggesting that they are part of the same phenomenon. The asymmetry present in the double peaked shape in Figure 1.1 is likely the result of an eccentricity in the disc. Potential reasons could include an asymmetry to the emission itself, possibly one area of the disc which is hotter or denser; or alternatively a physical asymmetry with the disc eccentricity. The former, asymmetric emission on a physically circular disc, could result from anything from spiral shocks observed in many different forms of discs; to some form of pocket of high density as a result of the evolution of the remains of the tidally disrupted asteroid; to some unusual asymmetry in the illumination of the material. The latter possibility, physical asymmetry of the orbit of the disc, could result from incomplete circularisation of the initially elliptical orbit of a tidally disrupted asteroid, or potentially gravitational distortion due to some orbiting planet. Some combination of these hypotheses is also possible.

In the time between the first observations and the follow up observations with the William Herschel Telescope (WHT), the direction of the asymmetry has changed. This suggests precession with a maximum period of the order of several years. As only two instances of observation are available, it is impossible to determine how many times it precessed in the gap between the first and second observations, and hence the actual period. Another interesting difference between the two observations is that the line strengths in several of the cases appear to vary. The source of this variability, as with the asymmetry, is unknown. Both of these unknowns could be remedied with more observations over a time sufficient to show a time-resolved entire precession.



Figure 1.1: Comparison spectra of the first three gas disc white dwarfs discovered. Graph courtesy of B. Gänsicke

SDSS Name	Туре	$T_{\mathrm{eff}}$ (K)	$M_{ m wd}$ ( $M_{\odot}$ )	EW (Å)	Reference
J122859.93+104032.9	DAZ	22,020±200	0.77 ±0.02	$62.56\pm1.74$	[1]
				$69.69\pm0.24$	
J104341.53+085558.2	DAZ	$18,330{\pm}523$	0.67 ±0.07	$21.71\pm4.16$	[2]
				$20.32\pm0.62$	
J084539.17+225728.0	DBZ	$18,600{\pm}500$	$\approx 0.7$	$21.52\pm1.11$	[3]
				$13.81\pm0.41$	
J1617	DAZ	pprox 13,500	$\approx 0.63$	$18.59\pm1.06$	[4]
				$5.61\pm0.62$	
J0738	DBZ	pprox 15,000		$10.34\pm4.12$	[4]
				$7.32\pm4.12$	

[1] Gänsicke et al. [2006]; [2] Gänsicke et al. [2007]; [3] Gänsicke et al. [2008]; [4] Private Communication with B. Gänsicke

For EW data, the initial line is from SDSS spectra, and the second line is from follow up WHT spectra

Table 1.1: Table of key properties of the five white dwarf objects with circumstellar calcium-II emission lines

### **Chapter 2**

# **Background - Circumstellar Discs**

#### 2.1 Properties of Material

#### 2.1.1 Mass

Based on a combination of line strengths and theoretical settling times, it is possible to estimate the rate at which material is deposited onto the surface of the white dwarf. This method of analysis has been performed by Jura [2003] as well as others, with results being of the order of above  $10^8$  g s<sup>-1</sup>. This has led to a wide range of expected total masses contributing to the accretion of  $10^{23}$  to  $10^{24}$  g for optically thick emission, or comparable to the order of the mass of Ceres [Jura et al., 2007; Jura, 2008]; to as low as of the order of  $10^{19}$  g for optically thin emission by Reach et al. [2009].

#### 2.1.2 Composition

The composition of the circumstellar material can be examined from observations of each of the three categories of metal possessing white dwarfs. Firstly, by looking at which absorption lines are present in the white dwarf spectra of DAZ white dwarfs with dust, this will show which elements and in what ratios are being accreted to the atmosphere of the white dwarf, which can be assumed to be the same as present in the discs themselves. This is subject to numerical boundaries resulting from the length of the diffusion timescales. Warm DAZs are ideal, as the diffusion timescales are short and the difference in the diffusion timescales between different elements are minimal. The next method, models for dust can be used to determine the likely approximate species of dust present in discs by comparison with the IR excess in the spectra of white dwarfs. Finally, the emission lines in the white dwarfs described in Section 1.4 should indicate which elements are present in their gas phase in the circumstellar disc.

#### **Absorption Lines**

There are a number of studies regarding the absorption lines in metal-rich white dwarfs; Zuckerman et al. [2007] speculates the contamination as being from asteroids or planetesimals. It may be expected that it is only possible to use DAZ white dwarfs with an IR excess as the objects to study for this purpose. However, line ratios in DAZ white dwarfs appearing to be externally polluted show themselves to be significantly carbon deficient in comparison to Solar values [Jura, 2006]. This is suggestive of the material source being similarly carbon-poor, which appears to be a good indication that asteroids or other rocky objects are also the source of this accretion [Jura, 2006]. Furthermore, Farihi et al. [2009] shows from the distribution of contaminated white dwarfs, that 50% with accretion rates beyond  $3 \times 10^8$  g s<sup>-1</sup> display IR excess, suggesting that it may be the case that many of the white dwarfs with contamination but no IR excess, may be due to the same material source as those with an IR excess, but under circumstances where an emitting disc has either not been able to form, or rapidly collisionally dissipated into gas.

Under the assumption that the material of the disc has the same composition as the material absorbed into the white dwarf atmosphere and detected as additional absorption lines, the relative abundances of the atmospheric metals should give an idea of the bulk make up of the disc. This analysis was performed by Zuckerman et al. [2007] for the star GD 362 (a metal-rich white dwarf with an IR excess indicating a dusty disc) in order to provide the approximate relative abundances of a putative extra solar minor planet. It is these abundances which will be used for this project. The results of the analysis are shown in Table 2.1.

#### Dust

Models by Jura [2003] using blackbody models of dust, and in more detail using computational grain models by Reach et al. [2005] estimate the type and amounts of dust present in the circumstellar disc of G29-38. The IR excess seems generally to be composed of two components, the thermal blackbody excess, plus an additional feature with a peak at 10  $\mu$ m. This emission feature is indicative of silicates. Analysis of the spectral energy distribution of the excess by Reach et al. [2005] shows similarity with zodiacal light, more so than with cometary dust or the interstellar medium. More detailed models of emission from specific grains by Reach et al. [2009] shows high proportions of silicate species. Specific species thought to be present include amorphous carbon (or other materials with featureless emission spectra), both amorphous and crystalline silicates, as well as components of ice (in some cases) and metal sulphides. These results show the disc at G29-38 to be very carbon deficient compared to Solar averages. This corresponds well to known carbon deficiencies of solar asteroids and the Earth itself, supporting orbital rocky material as the source of the material [Jura et al., 2009].

Element	$Log[\mathrm{N}_{\mathrm{element}}/\mathrm{N}_{\mathrm{H}}]$	Element	$ m Log[N_{element}/N_{H}]$
He	$1.14\pm0.10$	Ti	$\textbf{-6.81}\pm0.10$
С	≪-4.50	V	$-7.60 \pm 0.30$
Ν	≪-3.00	Cr	$\textbf{-6.27}\pm0.10$
0	≪-4.00	Mn	$\textbf{-6.33}\pm0.10$
Na	$\textbf{-6.65}\pm0.20$	Fe	$-4.51\pm0.10$
Mg	$-4.84\pm0.25$	Со	$-7.36 \pm 0.40$
Al	$\textbf{-5.26} \pm \textbf{0.20}$	Ni	$\textbf{-5.93}\pm0.15$
Si	$-4.70\pm0.30$	Cu	$-8.06\pm0.40$
Ca	$-5.10\pm0.10$	Sr	$\textbf{-9.28}\pm0.30$
Sc	$-9.05\pm0.30$		

Table 2.1: Atmospheric relative elemental abundances by number for GD 362, as determined by Zuckerman et al. [2007], to be used as the baseline for disc abundances.

#### **Emission Lines**

While Call emission is strongest, Fell emission at 5169 Å is also present for both SDSS J122859.93+104032.9 and SDSS J084539.17+225728.0. In addition to these emission features, in an analysis performed by subtracting a simulated pure white dwarf spectra, two further Ca-II lines are found in the optical (3930 Å and 3970 Å) of the same order of strength as the lines of the triplet; the Fraunhofer 'H' and 'K' lines.

#### 2.1.3 Temperature

From the thermal emission of dust, determining approximate temperatures through matching a Planck function to the observed spectra provides an approximate measure of the dust temperature. This can be complicated by a source having a range of temperatures, rather than a single average temperature. This complication can be overcome through a blackbody based model, and has been used in a number of cases, finding temperatures averaging in the region of minimum temperatures in a disc of around 300 to maximum temperatures in a disc of just over 1,000 K

[Farihi et al., 2008]. In addition to this, a broader constraint can be placed on temperatures of any dust due to known sublimation temperatures of dust species. The majority of dust species will sublimate before reaching 1,500 K.

The gas component of the disc must therefore be the same temperature if in thermal equilibrium, or some setup such that the two can exist out of thermal equilibrium. This could either be through being physically separate, or that the system is at a sufficiently low density that it does not settle into thermal equilibrium. Existing analyses suggest that one of the latter options is likely. Previous LTE and NLTE models of the gas component of the disc have required significant temperatures throughout an optically thin disc, around 5,000 or 6,000 K, in order to recreate the observed emission [Werner et al., 2009]. This would be incompatible with the dust sublimation temperatures unless the system is out of thermal equilibrium.

#### 2.1.4 LTE and NLTE Models

Constituting the main work performed to date on the gas component of the discs, it is worth examining some of the aspects of the LTE and NLTE models mentioned. The LTE modeling was performed using simple, fixed size, isothermal, isobaric, slab of material, designed to simulate Call emission for the chosen temperature and density of the material [Dosanjh, 2008]. This method is simplistic in comparison to the known properties of the disc, not accounting for any temperature variation, nor specifying any heating source. While it was possible to find parameters with this model replicating the observed Call IR triplet, it would consistently produce Call H & K emission lines several orders of magnitude stronger than the triplet lines. The NLTE model by Werner et al. [2009] was more sophisticated, including a number of aspects not considered in the LTE model, such as temperature and

density profiles with disc height. However, this model was based on the concept of an accreting  $\alpha$  disc. This meant that all heating was provided by means of artificially controlling the accretion rate, with no effects of illumination by the white dwarf taken into account. As a result, in order to obtain good agreement between the model and the Call triplet, the accretion rate was made artificially high, being of the order of  $10^{18}$  g s<sup>-1</sup>. This is many orders of magnitude larger than inferred accretion rates from the observations, with even the highest accretion rates not being more than  $10^{11}$  g s<sup>-1</sup> [Koester and Wilken, 2006].

#### 2.2 Possible Sources of the Material

#### 2.2.1 Post AGB Envelope

As described in Section 1.1, the stellar evolution resulting in a white dwarf also results in a large envelope of material in the space surrounding the white dwarf. This material would be expected to be composed largely of hydrogen and helium. However, there is no reason to expect that such an envelope would be particularly carbon deficient with respect to the Sun. This would be expected to be reflected in large amounts of graphite type dust compared to silicates, if present in discs of white dwarfs, which is not observed.

For the dust, this points to a non-stellar origin of the material. More conclusive, however, are the gas emission lines. Considering the emission alone, the presence of metals without hydrogen and helium, strongly weighs against the gas being stellar material. There is no obvious mechanism by which such a disc could be preferentially depleted of such quantities of volatile elements.

When considering that a strong possibility is the dust and gas are part of essentially the same phenomenon, it seems unlikely that such material could be formed out of the post AGB envelope.

#### 2.2.2 Interstellar Medium

A historical suggestion for the source of contamination resulting in metal-rich white dwarfs is the Interstellar Medium (ISM). This could conceivably be extended to hypothesise the ISM as the source of material for the observed discs, both dust and dust + gas. However, this presents a number of problems. Firstly, just in terms of its plausibility as the source of material for metal-rich white dwarfs. Work by Kilic and Redfield [2007] showed insufficient correlation between observed density of the ISM and observed numerical density of DAZ white dwarfs compared to DA white dwarfs. Also, it is questionable whether observed ISM densities could provide enough material to create contamination of the scale observed.

Futhermore, neither modeling, nor observations, of the IR excess from white dwarfs can be replicated by ISM abundances.

In addition to this, modelling of the IR excesses caused by the dust discs, as well as spectra/photometry of observed excesses, do not appear to match up with modelled and observed IR profiles of ISM dust [Reach et al., 2005].

In addition to this, as described in Section 2.1.2, evidence appears to show that contamination is carbon poor, which would be inconsistent with ISM dust.

#### 2.2.3 Comets and Asteroids

Suggested by Debes and Sigurdsson [2002], another possible candidate for the source of the contamination are comets. The aforementioned paper refers specifically to the contamination of the atmosphere of white dwarfs, however the concept can be extended to include this as a source of material for the discs which could conceivably then feed into the atmosphere of the white dwarfs.

Originally it was suspected that the IR excess of G29-38 was caused by a brown dwarf companion to the white dwarf. This was refuted by Graham et al. [1990], suggesting that it was instead a dust disc. The idea of a dust disc was later discussed by Jura [2003] with the hypothesis that the source of the material could be an asteroid, falling from a previously circular orbits at several AU or beyond through gravitational interactionDebes and Sigurdsson [2002]. If the asteroids pass within the Roche limit of the white dwarf then the resulting debris will form a disc in a similar manner to Saturn's rings, as described by Dones [1991].

The similarity of the IR excess to zodiacal light in Reach et al. [2005] and composition as modeled in Reach et al. [2009] both strongly support the idea that the material is likely to be of types associated closely with volatile-poor bodies such as asteroids. Combined with the detected silicates[Jura et al., 2009], the evidence appears to most strongly back asteroids or planetary/planetesimal debris as being the source of the material.

While comets could create the material for a circumstellar debris disc, the average solar system asteroid is of significantly greater mass than an average comet, and would be more likely to be capable of providing the amounts of material theorised to make up these discs [Jura, 2003].

Jura [2008] also posited that, rather than a single event of an asteroid being disrupted, continual disruption of small asteroids could be the source of disc material. A system such as the Main Belt of the Solar System could provide a very extensive pool of potential mass, assuming it survives the AGB phase. Were multiple asteroids to be the source, the likely scenario would be that, after any initial event, subsequent asteroids falling inwards within the tidal disruption radius of the white dwarf would likely be in a slightly different plane to the existing disc. The intersection of the discs would result in high rates of collisions, until the less massive of the discs became incorporated into a common plane and most of the grains destroyed [Dones, 1991].

### 2.3 Survival of Planets/Planetesimals/Asteroids/Comets During Asymptotic Giant Branch

A promising candidate is some form of asteroid. Such a source relies on two things. A mechanism by which they can be perturbed from their orbits, and importantly, whether it is plausible for them to have survived the AGB phase of the star.

Numerical simulations have shown that for AGB mass loss rates, the outward migration of planets do not necessarily cause instabilities on a short timescale [Debes and Sigurdsson, 2002; Villaver and Livio, 2007]. While it is possible that a planetary system will rapidly become unstable, there should be a high enough survival rate of planetary systems into the white dwarf phase of stars that such a system should not be uncommon.

Similar techniques have been applied to our own Solar System [Duncan and Lissauer, 1998]. The gas giants are expected to survive 'tens of billions of years subsequent to the Sun's death'. Many modelled asteroids were observed to survive (those being in resonant orbits being particularly at risk, however). Far out objects such as Pluto may be expected to have mixed success at survival. Pluto itself being expected to have a close encounter with Neptune after escaping from its protective resonance orbit somewhere between tens of millions of years and several tens of billions of years.

This combination of protracted semi-stability, with potential disruptions in the long term, creates a picture plausibly permitting the dynamic of occasional interactions resulting in an object falling in towards a white dwarf. The theoretical evidence points towards systems generally maintaining stability over the course of time required for a white dwarf to have formed and settled.

To date no direct evidence of a planet or planetary system has been found around a white dwarf. However, this does not necessarily imply anything about the likeliness of such systems, as white dwarfs have generally not been candidates for the surveys for exoplanets for a number of reasons. Radial velocity surveys tend to be reliant on high amplitudes of motion to provide small enough errors on the velocity measurements that planetary detections would be possible. In addition, many absorption lines are required in the stellar atmosphere in order to determine radial velocities to high enough precision.

Another plausible route to discovering planets around white dwarfs is direct imaging, as described by Hogan et al. [2009]. Additionally, through pulsation timing of IR photometry with Spitzer, a planet candidate has been determined around the white dwarf GD66 [Mullally et al., 2009].

The best observational evidence of the existence of planetary systems around post-Main Sequence stars is the observation of a super Jupiter around an 'extreme horizontal branch' star V391 Pegasi, a pulsating red giant [Silvotti et al., 2007]. This indicates it is possible that planetary systems can survive the late evolution of stars and, by extension, exist at white dwarfs.

#### 2.4 Tidal Disruption of Asteroids and Disc Formation

Another central aspect of the idea that asteroids or comets could create a debris disc are the conditions required for such objects to tidally disrupt.

Considering the tidal disruption of a fluid sphere around a mass, the equation for the Roche limit can be utilised to provide an initial estimate,

$$R_{\rm Roche} = A \left(\frac{\rho_{\rm Primary}}{\rho_{\rm Secondary}}\right)^{\frac{1}{3}} V_{\rm Primary}, \qquad (2.1)$$

where  $R_{\text{Roche}}$  is the closest radius to the primary without tidal disruption occurring, A is a factor relating to properties of the asteroid ( $A \approx 2.45$  for a fluid body, and  $\approx 1.26$  for brittle solid bodies),  $\rho_{\text{Primary}}$  and  $\rho_{\text{Primary}}$  are the mass and volume of the central object, a white dwarf in this case, and  $\rho_{\text{Secondary}}$  is the density of the asteroid [Boss et al., 1991].

For a white dwarf with mass M = 1.4 g  $(0.7M_{\odot})$  and an asteroid with a density of  $\rho = 2$  g cm<sup>-3</sup>, this gives a Roche limit in the region of one solar radius, or approximately 100 times the radius of a typical white dwarf. This is sufficiently far outside the atmosphere of the white dwarf that it is conceivable that an event resulting in an asteroid with a perapsis close enough to the white dwarf to be tidally disrupted could occur; rather than the asteroid simply impacting the white dwarf itself. This also appears to correspond to the radius range in which the disc emission is thought to originate from as described in Sections 1.3 and 1.4.

The exact scenario surrounding any disruption event should be complex and involves a great many parameters. The nature of the internal structure and composition of the object is important in determining how the object will break up within the tidal disruption radius of the white dwarf. Given the large variety in both structure and composition of asteroids observed in our own solar system, it is difficult to determine what an 'average' disruption event around a white dwarf might entail. The manner in which material spreads out after disruption, and the dynamics of it settling into a disc, is highly dependent on the initial disruption.

Saturn's rings provide a theoretical analogue of how such a process might occur at a different scale. Amongst the suggestions for the origin of the rings, is that a tidally disrupted comet may have been the source [Dones, 1991]. This illustrates the idea that a secondary body on a highly eccentric orbit (relative to Saturn), can become disrupted by the primary and contribute material to a disc around the primary within the Roche radius.

### **Chapter 3**

# **Description of Model**

#### 3.1 Concept of the Model

The intention of this thesis was to create a model of the gas disc systems observed, primarily to replicate the observed emission. The model was designed to be as self consistent as possible within the constraints of an MSc. The basis of the model is to start with the source of heating for the disc being a simulated white dwarf. The calculations described in Section (3.2) were done to determine the effective stellar flux projected onto the surface of a circumstellar disc. The disc is simulated through modeling as several radially sequential rings of gas/dust. This permits each ring to be simulated using the *Cloudy* photoionisation code. This would take the intended parameters of the ring (gas density, composition, location and size of the ring) to determine the properties of the gas. Using a simulated white dwarf spectrum, modified to represent the correct illumination, Cloudy can then provide the emitted spectrum, ionisation states, temperatures and a variety of other properties for the simulated ring of material. By simulating a range of concentric rings, the spectra from each can be summed up in order to replicate the emission

from an entire disc.

#### 3.2 Application of the Physical Situation to a Simulation

#### 3.2.1 Geometry



Figure 3.1: Geometry of white dwarf and disc

Figure 3.1 illustrates the geometry of the disc system. The axis  $\psi$  represents a rotational axis through the center of the WD and disc. This angle will be ignored as there will not be any  $\psi$  dependence, and will subsequently integrate to  $2\pi$ 

The physical system consists firstly of the central white dwarf of radius R and total luminosity  $L_{tot}$ . The second element of the system is the gas/dust ring, of radial extent t and height h, between inner radius  $r_{in}$  and out radius  $r_{out}$ .
#### 3.2.2 Incident Radiation per Unit Area on Disc

The angle  $\theta$  defines a point on the surface of an azimuthally symmetric white dwarf. Consider emission from this point to a point on the disc r along the line  $z_1$ . It can be seen that there will be an effective emission per unit area in that direction, which will be the perpendicular component (to  $z_1$ ) of the unit area on the WD surface, from the angle  $\phi$  between the tangent on the white dwarf surface and  $z_1$ , simply

$$I_e = I_0 \sin \phi_1. \tag{3.1}$$

Then, in addition to this, a proportion of this will be absorbed at that location on the disc, based in the same way upon the effective unit area, and the dispersion caused by the  $1/r^2$  relation,

$$I_a = I_e \sin \kappa_1 \left(\frac{R^2}{r^2}\right). \tag{3.2}$$

In these equations, (3.1) and (3.2),  $I_0$  is the flux at the surface of the white dwarf,  $\frac{L}{4\pi R^2}$ . The ratio  $\frac{R^2}{r^2}$ , resulting from the  $1/r^2$  relation, is modified from the actual ratio of  $\frac{R^2}{z_1^2}$  via the small angle approximation (assuming that  $a_2 \ll R$ ).

Therefore the green triangle in Figure 3.1(b) shows the actual relation, with  $z_1$  the path of the light between the point on the white dwarf defined by  $\theta$ and the point on the disc defined by r, with  $a_1$  and  $b_1$  its horizontal and vertical components. Obtaining an expression for  $z_1$  in terms of r and  $\theta$ , we can see that the components are

$$b_1 = R\sin\theta \tag{3.3}$$

and

$$a_1 = r - R\cos\theta. \tag{3.4}$$

This gives

$$z_1 = \sqrt{a_1^2 + b_1^2} = \sqrt{(R\sin\theta)^2 + (r - R\cos\theta)^2}.$$
 (3.5)

Multiplying out the squared terms in the final part of (3.5) would result in a very unwieldy equation to work with. To simplify the problem, instead of considering a triangle with both components changing with both  $\theta$  and r, we can define a system illustrated by the red triangle. In this system we are assuming that  $R \ll r$ , meaning that changes in the r direction because of R are negligible. This change basically results in 'a' being set as r.

This gives

$$z_2 = \sqrt{a_1^2 + b_1^2} = \sqrt{r^2 + R^2 \sin^2 \theta}.$$
 (3.6)

This will make the derivation easier to handle, especially for the  $\phi$  terms, with little difference in accuracy.

Returning to (3.1)  $\phi$  can now be seen to be  $\frac{\pi}{2}-\theta-\kappa$  or

$$\phi = \frac{\pi}{2} - \left(\theta + \tan^{-1}\frac{b_2}{a_2}\right) = \frac{\pi}{2} - \left(\theta + \tan^{-1}\frac{R\sin\theta}{r}\right). \tag{3.7}$$

Using the small angle approximation by assuming that  $R \ll r$  and a couple of trigonometric identities, Equation (3.1) for the new geometry system becomes

$$I_e = I_0 \sin \phi_2 = I_0 \cos \left(\theta + \frac{R \sin \theta}{r}\right).$$
(3.8)

The cosine term can then be expanded, and with the small angle approximation we obtain

$$I_e = I_0 \left( \cos \theta - \frac{R \sin^2 \theta}{2} \right). \tag{3.9}$$

Looking now to Equation (3.2) for  $I_a$ , we can again substitute the known variables,  $\kappa$  and  $I_e$ , as well as replacing  $I_0$  with  $L/4\pi R^2$ 

$$I_a = \frac{L}{4\pi r^2} \left( \cos \theta - \frac{R \sin^2 \theta}{2} \right) \left( \frac{R \sin \theta}{r} \right), \qquad (3.10)$$

or,

$$I_a = \frac{L}{4\pi r^3} \left( R\cos\theta\sin\theta - \frac{R^2\sin^3\theta}{2} \right).$$
(3.11)

To obtain a useful relation for the energy per unit area incident on the disc, this must be integrated over  $\theta$  over the surface of the white dwarf.

$$I = \int_0^{\frac{\pi}{2}} I_a \cdot d\theta = \frac{L}{4\pi r^2} \left(\frac{R}{2r} - \frac{R^2}{3r^2}\right).$$
 (3.12)

This is the solution for the incident radiation per unit area on any part of the surface of the disc. The assumptions involved include the disc inner and outer radii being significantly larger than the radius of the white dwarf,  $r \gg R$ , and that the white dwarf radiates homogeneously across its effective area (i.e. limb darkening has not been included). Another assumption is that the height of the disc is negligibly small compared to the other dimensions. It might be noted that the integral was taken between 0 and  $\pi/2$ , which assumes that the polar top or bottom of the white dwarf is visible from the point on the disc, which will not be the case. However the error resulting in this assumption is small.

## 3.2.3 Radial Temperature Variation

The expression (3.12) can be evaluated to compare with radial temperature variation of discs to determine whether it is reasonable for the geometry in question. Several new assumptions must be made: first that the disc is optically thick while retaining a negligible height; second that the disc will radiate as a blackbody. Taking the expression for the Intensity of a blackbody as being  $I = \sigma T^4$ , where  $\sigma$  is the Stefan-Boltzmann constant, I is the incident radiation in Wm<sup>-2</sup> and T is the blackbody temperature in K. Using I as the same as found in (3.12), we obtain the following for the disc temperature

$$T_{\rm disc} = \left[\frac{L}{4\sigma\pi r^2} \left(\frac{R}{2r} - \frac{R^2}{3r^2}\right)\right]^{\frac{1}{4}}$$
(3.13)

For a more useful form, we can substitute  $4\pi R^2\sigma T_{\rm WD}{}^4$  for L, then solve for  $T_{\rm disc}/T_{\rm WD}$  to obtain

$$\frac{T_{\rm disc}}{T_{\rm WD}} = \left[\frac{1}{2}\rho^3 - \frac{1}{3}\rho^4\right]^{\frac{1}{4}},\tag{3.14}$$

with  $\rho$  substituted for R/r due to its recurrence. In King [1997] an expression is found for the same as

$$\frac{T_{\rm disc}}{T_{\rm WD}} \simeq \left[\frac{2}{3\pi}\rho^3 \left(1-\beta\right)\right]^{\frac{1}{4}},\tag{3.15}$$

where  $\beta$  is the disc albedo. This is a reduced expression for the assumption that  $r \gg R$ .



Figure 3.2: Comparison of the radial disc temperature model by King [1997] and the derivation presented here. The white dwarf temperature was taken as 20,000 K.

The discs are likely to have radii of at least  $r \approx 10R$  to  $\approx 100R$  (based on the models by Jura et al. [2007] with radii between 0.2 and 0.8  $R_{\odot}$ ), validating this approximation for the expected range of r. In the case of both of these estimates, the assumptions of the optically thick nature of the disc and re-emission as a pure blackbody become slightly tenuous. It can be shown that optically thin(ner) discs would likely reach a high(er) temperature than these estimates suggest. However, certainly in the case of Equation (3.15) and likely the case for Equation (3.14), this can act as an effective lower limit to the disc temperature [King, 1997].

## 3.2.4 Total Incident Disc Radiation

Since the simulation is limited to dealing with discrete 'rings' within the disc, it is necessary to integrate Equation (3.12) over r between an inner radius  $r_{\rm in}$  and outer radius  $r_{\rm out}$ .

$$I_{\rm disc} = \int_{r_{\rm in}}^{r_{\rm out}} 2\pi r I \cdot dr = \int_{r_{\rm in}}^{r_{\rm out}} \frac{L}{2} \left(\frac{R}{2r^2} - \frac{R^2}{3r^3}\right) \cdot dr,$$
 (3.16)

where the  $2\pi r$  term results from integration of the azimuthal angle. This solves to give

$$I_{\rm disc} = \frac{L}{2} \left[ \frac{R}{2} \left( \frac{1}{r_{\rm in}} - \frac{1}{r_{\rm out}} \right) - \frac{2R^2}{3} \left( \frac{1}{r_{\rm in}^2} - \frac{1}{r_{\rm out}^2} \right) \right].$$
 (3.17)

Substituting in some approximate values, we can examine the expected proportion of the white dwarf radiation incident on the disc. Using  $R = 6 \cdot 10^8 \text{cm}$ ,  $r_{\text{in}} \approx \frac{1}{3}R_{\odot} \approx 2.3 \times 10^{10}$  cm and  $r_{\text{out}} \approx \frac{3}{2}R_{\odot}$ , we get the ratio of radiation emitted from the white dwarf to incident on the disc as

$$\frac{I_{\rm disc}}{L} \approx 0.01. \tag{3.18}$$

[N.B. a factor of 2 was used to account for both hemispheres of the white dwarf. This has not been done in Equation (3.17) as the factor is accounted for separately within the simulation.]

## 3.3 Introduction to the 'Cloudy' Photoionisation Code

The photoionisation code for the simulation is called 'Cloudy' (Calculations were performed with version 08.00 of Cloudy, last described by Ferland et al. [1998], also see Ferland [2003] for a recent example of its application). This code is designed to take parameters defining a spherical shell of gas, with an optional dust component, and a central source of illumination. It then calculates successive iterations to determine the temperatures and distributions of the gas with radius. The focus of the simulation is photoionisation of the elements comprising the gas, as well as modelling the electron energy level transitions from absorption, and the resulting emission. Also taken into account are the various radiation transmission/absorption coefficients for the gas/dust, as well as numerous other gas properties and processes, all are described in Ferland et al. [1998]. The initial purpose of Cloudy was to model planetary nebulae and similar phenomena that could be approximated by spherical symmetry; would contain a high proportion of volatile elements (hydrogen and helium); would be in equilibrium and not dynamically changing on observational timescales. The code itself is one dimensional, to simulate radial pressures and distributions only, before applying the form to a spherical shell. This geometry is not suitable for mimicking a disc, so it was necessary to write a series of scripted input and output routines in order to utilise the relevant calculations done by Cloudy to then create the form of a disc geometry.

A key output feature of Cloudy is its capability to produce the simulated spectrum of the gas being modelled. This spectrum is composed of three components, the diffuse emission, which is the spherically emitted result of bound-bound transitions; the reflected spectrum, which is comprised of back-scattered radiation and diffuse emission back toward the source; and the transmitted radiation, being the component of the incident continuum which penetrates through the gas without interaction. The first two of these are all that are needed to replicate the disc, as, unless viewing at extreme inclinations (either edge on, or very nearly edge on), the transmitted continuum does not contribute to the observed spectrum.

Other key parameters are the gas temperature and the ionisation proportions of the elements in the gas. These properties are key in not only trying to replicate the observed spectrum, but also hopefully aid in inferring additional parameters able to constrain the physics of the system, and make predictions which may be testable via observations.

## 3.3.1 Physics

Cloudy deals with the radiation and equilibrium effects on and between atoms, molecules and dust. Most important being the atomic species, with several molecule types modelled, and two broad species of dust (silicates and graphites). The first 30 atomic elements (up to zinc) are treated with varying degrees of detail. Of these atoms several key properties are used to determine their behaviour: radiation cross sections, ionisation energies, electron energy levels and similar; mostly sourced from NIST data.

Working outwards from the inner face of the cloud is done along a one dimensional axis in adaptive size steps determined by rates of change of properties between one step and the next. This results in higher spatial resolution for areas with rapidly varying temperature, ionisation states, etc. At each step, effects of radiation are calculated in terms ionising rates on relevant atoms depending relevant photon energies in the SED supplied, the cross sections of the atoms, and ionisation fractions. In addition to ionising effects, electron transitions between energy levels for atoms are accounted for, as well as the density and hence cross section of free electrons for the step through the cloud being examined. Heating and cooling balances are dealt with at the same level, with radiation energy being transferred to free electrons or ionising atoms strongly contributing to heating, then radiation from the material, especially through recombination, contributing towards cooling. Balancing between these cells in terms of pressure and density equilibrium effects is performed based on the constraining effective boundary conditions chosen in the input (constant pressure, constant density or constant temperature).

This builds up the structure of material over the depth of the cloud, to result in estimates for temperature profiles, ionisation states, the proportion left of the initial incident radiation that has not interacted with the material, and varying types of emission from the cloud. This can be run through in several iterations to converge on the best fitting profiles for the material parameters.

Cloudy can be divided into two main types of calculations performed in order to obtain its results. The first is for determining the equilibrium of various properties of the gas as a function of radius. The second is the radiative transfer itself, computing the changes to each of the atomic species as subjected to the incident SED. While the code can broadly be separated into these categories, they are not separate systems and there is interaction between both processes.

Some of the drawbacks of Cloudy are in its treatment of the heavy elements, and dust which these elements can form. The processes for including dust in the calculations are limited to models of generic silicates and graphites (in terms of the grain types relevant to this project, it also has models for PAHs and 'grey' dust). Heavy elements have some limitations in terms of modelled energy levels. For each atomic species, maps of electron levels, absorption probabilities and other associated parameters are used. For the lighter elements and some of the more common heavier elements, a great many of these levels are included. However, for less common elements (broadly, those less frequently used as analytical tools in observed spectra of nebulae), these maps are more basic. While internal routines ensure energy conservation, not all energies of recombinations and similar emission effects are fully treated, and hence do not create lines to appear in output spectra.

This is by no means an exhaustive description of the processes used by Cloudy. The full code covers an extensive variety of parameters and physical processes, with equally varied background physics and as such, a complete understanding and description of the code as a whole was beyond the scope of this project.

## 3.3.2 Capabilities

The vast majority of the parameters and the code pathways used behind physical processes can be output from a run of Cloudy. Of these, the key parameters for this project would be temperatures, densities of ionisation states, and the distribution of these properties through the radial depth of the cloud.

Critical for comparison with observational data regarding the discs, are the model spectra. In combination with the simulated white dwarf spectra, these simulated emission spectra from the Cloudy disc-analogue should provide a spectrum which is directly comparable to the observational data, as well as being able to make predictions for regions of the spectrum for which observations have not yet been made.

## 3.3.3 Operation

Cloudy is operated by input files with lines specifying desired parameters through the means of keywords and values. These commands encompass all of the desired initial conditions, limiting boundary conditions and what output should be created at the end of the run. Default output is in the form of a file specifying the overall end conditions of the test run. This includes basic line strength data, ionisation state proportions for elements, densities as well as a number of other summaries of parameters from the test. This also includes data relating to the success of the test, any problems encountered, to what extent various parameters had converged and any areas which might push boundaries of what constitutes reliable results.

All other specified output are in files of tabulated ASCII data with commented headers permitting easy graphing or post-processing. In the case of this project, some post-processing is necessary to translate the adapted Cloudy spherical simulation, into the desired version representing disc properties.

The spectra given by Cloudy are of an arbitrary resolution, emission lines are points at a single wavelength. In order to represent both the broadening of the lines, and the effect of the double peaked Doppler shifting resultant from the emission being from a rotating disc of material. It was necessary to fold the spectra through a Doppler broadening code provided by B. Gänsicke. This was done separately for each simulated ring, with the average Keplerian velocity for that ring used for the Doppler broadening velocity.

## 3.4 Simulation Setup

## 3.4.1 Overall Model Configuration

Cloudy works with a 1-D slice representing a spherically symmetric system, assuming that all illumination must radiate from the center outwards through the material. Since a disc is illuminated primarily on it's upper and lower surfaces, with a very small proportion radiating out from the center star through the disc, this is an incompatible setup.



Figure 3.3: Layout of the model

In order to accommodate this difference in mechanics, instead of having Cloudy represent the entire disc, outwards from the center, it can represent ringlike segments of the disc in separate runs of the simulation. This permits us to manually feed the correct luminosity for the disc system to each ring segment.

Figure 3.3 illustrates the two projections of the system. Separate simulations are run for each segment, where the parameters for a segment are annotated with a square bracketed index. An ideal number of segments can be chosen for a balance between sufficient accuracy and sufficient speed. For N segments, a segment n will have an inner radius  $r[n]_{in} = r[n-1]_{out}$  and outer radius  $r[n]_{out} = r[n+1]_{in}$ , with the innermost and outermost segments having  $r[0]_{in} = r_{in}$  and  $r[N-1]_{out} = r_{out}$ .

The system for the segment radial width can be set up in one of two ways. Either the radial extents can be made uniform, such that t[n] = t/N. However, an alternative set up is to have non-uniform radial thicknesses. This would be for cases such as accounting for Doppler shifting, for which higher segment density is needed closer to the inner disc edge, due to greater difference in orbital velocities between segments. Were small variations due to the Doppler shifting being modelled, for instance in a full 3D simulation, this would need to be taken into account. However, since the bulk properties is being replicated rather than small details, as well as the ring segments being larger than the resolution at which the velocity distributions could be satisfactorily modeled, it was deemed suitable to use uniform radial extents.

## 3.4.2 Disc Height

There is nothing in the observations that provides a constraint on the disc height. Inner and outer radii can be determined from elements such as Doppler broadening and the line profiles, even disc temperatures can be estimated based on line strengths. Without observational evidence of the vertical disc profile, it is still possible to use theoretical models. For this project the scale height of an accretion disc was used.

Consider a slab of material at a height z above the midplane with a thickness dz. The surfaces on the top and bottom of this slab are of area A. Density of the medium of which the slab is a part of, and the gravity acting on the medium, both vary with height, the gravity and density at z are notated as g and  $\rho$  respectively. The weight of the slab is therefore,

$$W = A \rho g dz. \tag{3.19}$$

This will be subject to pressure from the surrounding medium acting on the slab's top and bottom surfaces. For the system to be in equilibrium, the combination

of the forces due to surrounding pressure and the weight of the slab must sum to zero,

$$F_{\rm tot} = 0 = F_{\rm top} + F_{\rm bottom} + W, \tag{3.20}$$

where  $F_{\rm tot}$  is the resultant force,  $F_{\rm top}$  is the force due to the pressure from above the slab and  $F_{\rm bottom}$  due to the pressure from below. This can be rewritten as,

$$A P_z - A P_{z+dz} = W = A \rho g dz,$$
 (3.21)

where  $P_z$  is the 'upward' pressure at the base of the slab z, and  $P_{z+dz}$  the 'downward' pressure from the top of the slab z + dz (hence opposite in sign to  $P_z$ ). This can be rearranged to obtain

$$\frac{P_{z+dz} - P_z}{dz} = \frac{dP}{dz} = -\rho g.$$
(3.22)

The ideal gas equation for an isothermal fluid of temperature T gives

$$P = \frac{\rho kT}{\mu},\tag{3.23}$$

where  $k_b$  is Boltzmann's constant and  $\mu_{m_H}$  the average particle mass multiplied by the atomic mass constant. Substituting this in to Equation (3.22) gives only one parameter in the differential which is dependent on z, the density  $\rho$ , this results in the expression

$$\frac{d\rho}{dz} = -\frac{\mu g}{kT} \ \rho. \tag{3.24}$$

Should g be independent from z, this would solve to

$$\rho = \rho_0 \exp\left(\frac{-z}{h}\right). \tag{3.25}$$

With the scale height h as  $\frac{kT}{\mu g}$ . However, g varies with height for an accretion disc, such as being considered, by the following relation:

$$g = \frac{GM}{r^3}z\tag{3.26}$$

Substituting this expression into Equation (3.24) gives

$$\frac{d\rho}{dz} = -\frac{G M \mu}{r^3 k T} \rho z, \qquad (3.27)$$

which has the solution

$$\rho = \rho_0 \, \exp\left(-\frac{G M \mu z^2}{2 r^3 k T}\right). \tag{3.28}$$

To obtain a form for a scale height h such that  $\rho = \rho_0 \exp(-(z/h)^2/2)$ , from Equation (3.28) we obtain an expression for the scale height of

$$h = \left(\frac{k T r^3}{G M \mu}\right)^{1/2}.$$
(3.29)

The drawbacks of this derivation is that it assumes a constant vertical temperature profile, which would almost certainly not be the case. However, since the internal structure of such discs can only be speculated on, both temperature and density are highly uncertain, even within the Cloudy simulation. Since the radial dependence is a simple  $r^{\frac{3}{2}}$  form, having the scale height determined primarily by this relation, rather than relying on a more complex combination of parameters whose variance within the discs would be hypothetical, provides a more consistent, if perhaps slightly less physically accurate, basis.

## 3.4.3 Application of the Physical Situation to the Model

Variables must be adjusted to 'convert' between the 2-D basis disc geometry, with axial symmetry, to the 1-D basis Cloudy model, with spherical symmetry. In this case  $r_{\rm in}$  and  $r_{\rm out}$  define the inner and outer edges of a ring segment.

Figure 3.4(a) illustrates the physical situation at the disc for an average line of radiation for the white dwarf to a point on the disc (defined by the angle  $\kappa_{avg}$ ). Using the assumption that the situation can be modified assuming an average



Figure 3.4: Transforming the physical geometry to Cloudy equivalent

angle of incidence for the radiation, we can also define an average effective area for a disc ring element,  $A_{\rm eff}$ , and an average penetration depth for the radiation,  $d_{\rm eff}$ . The difference between the actual surface area of the disc and the effective area has already been accounted for and so is already contained in the expression for incident radiation, Equation (3.17).

Therefore we are left with the difference between the radial thickness t and the Cloudy radial thickness, which is related to the penetration depth  $d_{\text{eff}}$ . Also the difference between the actual surface area of the disc A, and the interior surface area of the Cloudy sphere. We can set up the system such that we consider slicing the disc in two, a top half and bottom half, and applying them such that their surface areas are spread, as much as they can, across the inner surface of the Cloudy sphere. This would then mean that the incident radiation in the Cloudy model, having already been modified by Equation (3.17), must be further modified by the ratio  $A_{\text{cloudy}}/(A_{\text{up}}+A_{\text{down}})$ , where  $A_{\text{cloudy}}$  is the area of the interior surface of the Cloudy sphere,  $A_{\text{up}}$  is the top surface area of the disc, and  $A_{\text{down}}$  is the bottom surface area of the disc.

This setup is shown as part of Figure 3.4(b). It may be considered to then make the Cloudy shell thickness equal to half the disc height, h. However, this would only hold if the incident radiation was perpendicular to the disc surface, which is not the case. Since the average distance that the radiation must travel through is  $d_{\rm eff}$ , then it follows that the Cloudy shell thickness can be set as  $d_{\rm eff}$  to most closely mimic the path the radiation must take through the disc material in the physical situation. However, this will only be representative of the situation averaged over the ring segment being calculated. Were the disc of a constant height,  $d_{\rm eff}$  would be given by

$$d_{\rm eff} = \frac{h}{\sin \kappa_{\rm avg}}.$$
 (3.30)

However, since the disc height increases with radius use of this relation overestimates  $d_{\rm eff}$ . Instead, for a given radius being calculated,  $r_{\rm calc}$ , it is required to find the point at which the line representing the average path of the radiation (at an angle  $\kappa_{\rm avg}$  to the mid plane) and the height function cross over. This provides the best representation of the penetration depth,  $d_{\rm eff}$ .

## Chapter 4

# **Results and Analysis**

## 4.1 Single Ring Models

Before considering a full disc model, the single ring model can be used alone to examine directly the effects that changing single input parameters have on line strengths. Even if all but one input parameter are kept constant, many interdependant parameters would be expected to vary and interact across the radius of the disc. By restricting a test to a single ring, the amount of unintended variation of parameters that may have an effect on the output can be limited.

## 4.1.1 Density

#### **Example Spectra**

Being the first set of tests, it is worth looking initially at one example simulated spectrum in order to observe and analyse the general features. It would be unfeasible to conduct analysis by direct comparison of every single one of the spectra generated. In addition, with tests comprising a single ring, the disc spectrum cannot be combined with the white dwarf spectrum, as the flux from a single ring would not be sufficient to overcome the white dwarf flux, nor would it be possible to directly compare with the observational data.

The spectrum in Figure 4.1 is that of a ring with an inner radius of  $4.0 \times 10^{10}$  cm (0.58  $R_{\odot}$ ) and outer radius of  $4.5 \times 10^{10}$  cm (0.65  $R_{\odot}$ ), which was run as a single ring test. The white dwarf parameters for this test and all subsequent density tests were 20,000 K,  $6.0 \times 10^8$  cm ( $\approx 0.95R_{\oplus}$ ) radius and luminosity of  $4.07 \times 10^{31}$  erg s<sup>-1</sup>. Henceforth an illumination source with these properties will be referred to as the 'standard'. The abundances used for the gas were based on the calculated white dwarf atmosphere abundances by Zuckerman et al. [2007] and the gas density was  $1.4 \times 10^{-12}$  g cm<sup>-3</sup>, being approximately the value previously estimated by Werner et al. [2009].

Several of the stronger lines in Figure 4.1 have been labelled. With Call clearly visible around 8600 Å. The pair of lines labeled 'Call Forb.' are from two Call forbidden transitions, which, in reasonable densities, should be collisionally depopulated before being able to emit. Their presence in the simulated spectra is a consistent problem, but in any real situation should not be expected to be present. Additionally, unless stated otherwise, the intensities of all spectra of emission are in units of erg s<sup>-1</sup> Å<sup>-1</sup>.

#### Line Strengths and Ratios

For the ring geometry, line strengths have limited direct use. The flux for a given line from a ring does not necessarily say much about how visible that line would be for the entire disc (though it should give an indication). However, line strengths for ring geometry tests can be analysed based on their relative strengths and the ratios between the lines. Figures 4.2 and 4.3 show the line fluxes and ratios of various prominent lines from the set of simulations. Marked by vertical lines in



Figure 4.1: Simulated spectrum for a single ring at radius  $4.25\times10^{10}$  cm and density  $1.4\times10^{-12}$  g cm<sup>-3</sup> around a standard white dwarf.

these Figures are densities chosen as representing interesting points in the line flux profile. There appears to be significant variation in line strength up to densities of approximately  $5 \times 10^{-13}$  g cm<sup>-3</sup>, at which point the emission seems to saturate.

## Opacity

Displayed in Figure 4.4 are the opacity profiles for several of the densities. Opacity is in units of  $cm^{-1}$ , indicating the ratio of absorbed to transmitted per cm through the disc from the point of emission.

The sharp set of edges between 1500 and 2500 Å form a division between opaque at shorter wavelengths and transparent at longer wavelengths. This is an important result as it indicates that, for a wide range of densities, the absorption will be high as a large percentage of the white dwarf flux is in the UV. This would mean that, for absorption, the disc is optically thick. Emission in the



Figure 4.2: Line fluxes for various density disc rings at a radius of  $4.25\times10^{10}$  cm, illuminated by a standard white dwarf. Vertical lines indicate the densities of interest.



Figure 4.3: Line fluxes and lines ratios for various density disc rings at a radius of  $4.25 \times 10^{10}$  cm, illuminated by a standard white dwarf. Vertical lines indicate the densities of interest. Horizontal lines of a given colour indicate the observed ratio for SDSS 1228 for the line represented by the same colour to Call y.



Figure 4.4: Opacity profiles as a function of wavelength for selected densities with the disc rings at radius  $4.25\times10^{10}$  cm around a standard white dwarf.

optical wavelength range in which lines have been observed is not subject to much attenuation, indicating that emission from throughout the vertical structure of the disc may contribute towards the observable line emission from the disc as a whole. This is contrary to the general view of an optically thick disc, as that model requires a higher emission per unit volume, or for the disc to have a larger surface area, as all of the observed flux would have to be from the surface layers of the disc.

In addition to Figure 4.4 (which displays only continuum opacity), we can look at the transmitted continuum. In this case the 'transmitted' continuum is the attenuated incident continuum upon reaching the mid plane of the disc. This shows the strength of absorption associated with lines. The transmitted continua for a density of  $3.4 \times 10^{-12}$  g cm<sup>-3</sup> is displayed in Figure 4.5, and shows the spectrum from UV to IR in order to give an indication of effects in the UV region of the spectrum.



Figure 4.5: Transmitted spectrum for a density of  $3.4 \times 10^{-12}$  g cm<sup>-3</sup> with the disc rings at radius  $4.25 \times 10^{10}$  cm around a standard white dwarf.

One problem with the model as set up is that it works based on emission and contributions to emission up to the end of the Cloudy run, or in other words the mid plane of the disc. This is fine for the optical region, which, as shown in Figure 4.4, has an extremely low opacity, any contribution from throughout the disc will end up in the resulting spectrum in this region. However, extremely optically thick material, as is the case in the UV and higher energy part of the spectrum, the treatment for emission and escape of photons is less accurate. This means that the emission spectra for the UV lines will be less accurate and less representative of actual physics. In particular, it means that prediction of line fluxes in the UV is less reliable.

By using the transmitted continuum, it is possible to see just how strongly these UV lines are absorbed, with strongly absorbed lines likely being less reliable. By looking at these unreliable lines, this may give an indication as to how UV emission from the surface layer of the disc may appear. At the surface of the disc, it would be expected that there will be strong bound-free absorption, corresponding to these high energy lines with very high opacities. However, it is also reasonable to expect that there will be free-bound emission for the same lines. Within the disc it would be expected that such emission would be rapidly re-absorbed due to the high opacities. However, at the surface it would be likely that some of this emission would escape, which may not be represented in the simulated emission spectra. While it is difficult to estimate how strong these lines may appear, from such a potentially thin layer of the disc, it should give an indication as to what such lines might be expected to be detected, and in what ratios.

Figure 4.5 potentially suggests a dense forest of lines. Some of the main absorbers in this simulated disc include magnesium, aluminium and sodium as the strongest. As such, it would not be unreasonable to expect to see some UV lines from these elements, albeit potentially quite weak due to only a thin effective layer contributing to the emission.

Also noticeable in this spectrum is the stark contrast between the absorption of the Call H & K lines and the Call x, y & z lines. The H & K absorption is many orders of magnitude higher than the x, y and z absorption. The absorption is sufficiently high that one can conclude Call H & K emission is only coming from a thin layer at the surface of the disc, as with some of the UV lines. The difference in optical thickness between the H & K lines and x, y & z lines provides an explanation of the key inaccuracy in the LTE and NLTE models, which predict very strong H & K lines which are not observed.

The integral of the transmitted spectra will show what percentages of the radiation is absorbed over the depth of the disc. This ratio ranges from 3.4% at the lowest density, up to 79.8% at the highest. This indicates that it is likely that, even for fairly thin and sparse discs, a large amount of energy will still be

deposited. Note, however, that these percentages are the amount of absorbed radiation *incident to this thin ring of the disc*, as opposed to the percentage of the total flux from the white dwarf. In the case of this particular ring, only approximately 0.5% of the flux from the white dwarf is incident to the ring. Also noteworthy is that the lowest density included in these tests is lower by  $10^{-3}$  compared to the hypothesised density. Additionally, the format of the tests means that only the upper half of the disc is modeled as the situation is expected to be mirrored about the mid plane, with low radiation going between the two halves. In cases where a high percentage of the radiation is transmitted, this assumption is less accurate and it is likely that at least some transmitted radiation from one half would subsequently be absorbed by the opposite half of the disc. This is also for a gas-only disc, a disc with dust content should be expected substantially increase the opacity up to 20  $\mu$ m.

## **Temperature Profiles**

Figure 4.6 displays the vertical temperature profiles for different densities relating to key values of line strengths and line ratios (Figures 4.2 and 4.6).

Low densities have a fairly smooth profile, up to densities of around  $10^{-13}$  g cm<sup>-3</sup>, at which kinks in temperature begin to appear. The profile of temperature better understood in context with the ionisation profiles in the following Section.

#### **Ionisation Profiles**

Shown in Figure 4.7 are vertical ionisation profiles for calcium at different densities. These display the population of each calcium ionisation state. Low densities show a consistent gradual change for the states generally with a single state dominating. CallI is the dominant state between  $\rho$  greater than  $\approx 10^{-17}$  and less than  $\approx 10^{-14}$ 



Figure 4.6: Vertical temperature profiles for selected several densities with the disc rings at radius  $4.25 \times 10^{10}$  cm around a standard white dwarf.

 $10^{-14}$  g cm<sup>-3</sup> (as shown in Figures 4.7(a) and 4.7(b). As density rises, the Calll population decreases, and above a density of approximately  $10^{-14}$  g cm<sup>-3</sup>, the Call becomes comparable to Calll. This also corresponds to densities at which a more complex vertical structure begins to emerge. A noticeable feature seen in Figure 4.7(d) is a 'bump' in the profile of neutral calcium, corresponding to a sharp cut off of any Calll. A similar pair of features can be seen in 4.7(e), first with CallI falling rapidly near the surface of the disc, and then Call does the same at a greater depth.

These may be the result of high densities resulting in attenuation of ionising radiation to the point at which energies associated with particular ionisation states cease to be able to propagate through the whole disc. This would match the concept of distinct, well defined ionisation edges as found in Strömgren Spheres. At a certain depth for a given density of gas, the photons associated with a



Figure 4.7: Vertical ionisation profiles of Calcium for selected density disc rings at a radius of  $4.25\times10^{10}$  cm, illuminated by a standard white dwarf.

specific ionisation jump are essentially expended. At higher densities the distance over which photons of that energy are available decreases. The noted discontinuity in the temperatures and ionisation at about  $\rho = 5.4 \times 10^{-13}$  g cm<sup>-3</sup> (Figures 4.6 and 4.7(d)) as well as the leveling off of the line fluxes (Figure 4.2) could likely be the point at which this depth becomes less than the depth of simulated disc. Up to that density, photons of the ionisation energy are still making it past the midplane. After this point, although the depth at which it occurs changes, all of the photons are still being absorbed on the same actual number of atoms/ions, hence the line strengths stay similar from that point.

The general switch from hot, highly ionised material to colder, more neutral material with increasing density is to be expected. The results of looking at the opacity has shown that, even geometrically thinner discs will absorb a very high percentage of incident radiation. As such, it can be seen that, since the total amount of absorbed radiation will be about the same regardless of density, higher densities will result in less energy available per unit volume of the disc. The result of this would be lower temperatures and lower ionisation states.

Figures 4.8 and 4.9 show profiles comparing the gas temperature to the ionisation states of calcium, sodium, magnesium and iron, all for a density of  $3.4 \times 10^{-12}$  g cm<sup>-3</sup>. The Mg and Fe profiles both show a shallow jump, similar to the temperature profile, however both Ca and Na feature a switch between two ionisation states. It would seem that the end of the Call ionisation region triggers a temperature jump, subsequently inducing similar jumps and changes in the other elements. Na can be ruled out as the cause, as its jump is from neutral Nal up to Nall.

50



Figure 4.8: Comparison of several gas parameters in a ring of density  $3.4 \times 10^{-12}$  g cm<sup>-3</sup> at a radius of  $4.25 \times 10^{10}$  cm, illuminated by a standard white dwarf. Depth scales are the same in each graph, showing the link between the temperature profile (top) and the ionisation levels of Calcium and Sodium (middle and bottom)



Figure 4.9: Comparison of several gas parameters in a ring of density  $3.4 \times 10^{-12}$  g cm<sup>-3</sup> at a radius of  $4.25 \times 10^{10}$  cm, illuminated by a standard white dwarf. Depth scales are the same in each graph, showing the link between the temperature profile (top) and the ionisation levels of Magnesium and Iron (middle and bottom)

## 4.1.2 Luminosity

It would be helpful to understand the effect that radius has on the properties of the disc. However, changing the radius has many concurrent changes due to the radially varying geometry. The disc height is expected to vary, along with the length of the radiation path through the disc. While these could be set to a constant value, it may bias the results. One parameter which can be changed without such a bias is luminosity. A basic relation would be that with increasing radius, the main nongeometry effect on the properties of the gas will be a decreasing amount of incident luminosity. It is important to note that this is clearly a non-physical situation, since changing the white dwarfs luminosity without changing other parameters is not possible. This test is purely as a useful method to investigate the effects of altered incident flux without geometric changes to the disc itself.

Figure 4.10 shows the line fluxes for the range of luminosities examined. The tests used a standard disc with a density of  $1.7 \times 10^{-12}$  g cm<sup>-3</sup>. The white dwarf spectral energy distribution shape was kept constant, and the luminosity of the white dwarf was varied between  $10^{30}$  and  $10^{32}$  erg s<sup>-1</sup> in multiples of  $10^{0.1}$ .

The profile for this test appears fairly straight forward, with generally increasing emission for larger incident fluxes. This test is of more interest compared with varying of the actual radius and with varying white dwarf temperature.

## 4.1.3 Radius

It is also possible to look directly at radius with its associated changes in geometry, in order to compare the two. Using ring segments of  $\Delta r = 2 \times 10^9$  cm, from a minimum inner radius of  $6 \times 10^9$  cm ( $10R_{\rm wd}$ ) out to a maximum outer radius of  $1.2 \times 10^{11}$  cm ( $200R_{\rm wd}$ ) for a total of 57 separate rings.



Figure 4.10: Line fluxes for disc rings illuminated by a white dwarf of varying luminosity only, using the standard white dwarf SED. The disc rings were at a radius of  $4.25 \times 10^{10}$  cm and had a density of  $1.7 \times 10^{-12}$  g cm<sup>-3</sup>.

Since the ring segments for this test are contiguous (the outer radius of segment n is the inner radius of segment n + 1) it is effectively a full disc test. However, it will be analysed in terms of it's individual segments, rather than as a whole, as the total range for this disc would be larger than any estimates for the disc size. Using the same results however, it is possible to use the rings to look at overall disc size properties, by combining different ranges of tested radii. This analysis is presented in Section 4.2.2.

Figures 4.11 and 4.12 show the fluxes and ratios for several important emission lines as a function of radius. From the Figures it is clear that the general trend as the radius becomes larger is for emission to gradually increase, with less energy spread over more material.

In comparison with the previous test of varying luminosity illustrated in Figure 4.10 this would appear to show the same relation over the course of equivalent



Figure 4.11: Line fluxes for disc rings at a range of radii from  $\approx 6.3 \times 10^9$  cm to  $\approx 1.2 \times 10^{11}$  cm  $(0.09 - 1.7 R_{\odot})$ . The rings had a density of  $1.7 \times 10^{-12}$  g cm<sup>-3</sup> and were illuminated by a standard white dwarf.

luminosities.

Additionally, looking at the line ratios, it is clear that the relative strengths of SDSS 1228 for the Call triplet are not reproduced at any radius fro this parameter set, with the Call x line being stronger than the y line at the majority of radii, and always stronger than the z line. This graph also highlights the low simulated emission of the Fell line present in the observed spectra, around an order of magnitude lower than the other lines.

## 4.1.4 White Dwarf Temperature

The surface temperatures of the gas disc host white dwarfs range from 22,000 K down to 13,500 K. As such, it should be expected that the generation of Call lines which have been observed should be replicated by the simulation over this range of temperatures. Preferably the results would also display effects in a wider range of



Figure 4.12: Line ratios to the Call y line for disc rings at a range of radii from  $\approx 6.3 \times 10^9 \, {\rm cm}$  to  $\approx 1.2 \times 10^{11} \, {\rm cm} \, (0.09 - 1.7 R_\odot)$ . The rings had a density of  $1.7 \times 10^{-12} \, {\rm g} \, {\rm cm}^{-3}$  and were illuminated by a standard white dwarf. Horizontal lines of a given colour indicate the approximate observed ratio for SDSS 1228 for the line represented by the same colour to Call y.

temperatures than already observed, in order to make predictions about candidate host white dwarfs for such discs.

In this set of tests, simulated white dwarf spectra between 8,000 K and 26,000 K in 1,000 K increments were selected, using relevant luminosities for the temperatures chosen and the standard white dwarf radius simply using the relation

$$L_{\rm wd} = 4\pi\sigma R_{\rm wd}^2 T_{\rm wd}^4. \tag{4.1}$$

This results in luminosities ranging from  $1.05 \times 10^{30}$  to  $1.17 \times 10^{32}$  erg s<sup>-1</sup> for 8,000 K to 26,000 K respectively. The disc ring used had the inner radius and outer radius of the standard test ring, and a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup>.

#### Line Fluxes

Figure 4.13 shows the line fluxes for the discs illuminated by white dwarfs. The relationship between the temperature of the source of illumination and the line strengths appears to be fairly straightforward, with few drastic changes. The line strengths increase slightly toward higher temperatures, but appear to do so in the same relation as was found for simply varying the source luminosity.

For the higher temperatures the variance in line flux is fairly small, beginning to fall off more quickly when reaching the lower temperatures. At 14,000 K the flux has dropped almost an order of magnitude compared to the value at 20,000 K. While the drop of an order of magnitude between 20,000 K and 14,000 K is significant, it is not outside the realms of detectability, though it does suggest that line fluxes for white dwarfs any cooler than 14,000 K would become increasing difficult to detect.

This suggests that apart from the changes in luminosity of the white dwarf associated with different temperatures, the effect of the changing temperature



Figure 4.13: Line fluxes for disc rings illuminated by white dwarfs of varying temperature (and associated luminosity). The disc rings were at a radius of  $4.25 \times 10^{10}$  cm and had a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup>.

itself is minimal. This would appear to be in agreement with the observed data, supporting the idea that it is possible to obtain the calcium IR triplet line emission even from fairly cool white dwarfs, with emission from as low as 10,000 K being within an order of magnitude as that at 18,000 K.

## **Average Triplet Strength**

Figure 4.14 shows the average of the simulated Call triplet fluxes against temperature normalised to 22,000 K. It also displays observed EW data, normalised to the SDSS data of SDSS 1228. Whilst comparing the observed effective widths to the fluxes of the simulated emission is technically incorrect, when combined with the normalisation of both to 22,000 K it provides a reasonable measure of the effective strength of each.

Going on face value, this would appear to fit with the observational data



Figure 4.14: Average Calcium triplet line strength, normalised to the observed value for SDSS 1228+1040 at 22,000 K. Simulated points are in black, points from SDSS data are in red and points from WHT data are in green. The disc rings were at a radius of  $4.25 \times 10^{10}$  cm and had a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup>.

on the relative Call IR triplet line strengths. Both of the observed white dwarfs at 18,000 K had line strengths of approximately one third of SDSS1228 at 22,000 K. The simulated data suggests a ratio closer to about one half, but, with only three effective datapoints, it is difficult to determine to what extent the model follows the observation. The two cooler white dwarfs also appear to sit near the simulated fluxes, again with some error between the observed and simulated values. When variations in density, size of disc and other parameters are included, it is not expected that the observed objects will fit the exact relation for white dwarf temperature alone.


Figure 4.15: Line fluxes for a range of disc heights. Disc height is given in terms of the multiplier used to modify it from the theoretical disc height as described in 3.4.2. The disc rings were at a radius of  $4.25 \times 10^{10}$  cm, had a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup> and were illuminated by a standard white dwarf.

## 4.1.5 Disc Height

The hydrostatic equilibrium model used to determine disc height suffers from several deficiencies (described in Section 3.4.2). As such, it is useful to investigate the effects that the height of the disc has on the emission from a ring. Figure 4.15 shows a plot of the line fluxes, and Figure 4.16 shows the line ratios to the Call 'y' line, against the multiplier used to modify the calculated disc height.

The main effect is the change on the relative strength of the 'x' line. A problem with the tests so far has been the lack of agreement with the observed spectrum with regards to the triplet line ratios, the 'x' line being persistently stronger than observed. In SDSS 1228 the x line is the weakest, similarly in other observations the x line is either weaker or, at its strongest, approximately equal to the y and z lines. The results of this test suggest that the disc heights may



Figure 4.16: Line ratios against the Call 'y' line for a range of disc heights. Disc height is given in terms of the multiplier used to modify it from the theoretical disc height as described in 3.4.2. The disc rings were at a radius of  $4.25 \times 10^{10}$  cm, had a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup> and were illuminated by a standard white dwarf.

be larger than the hydrostatic equilibrium calculations by up to a factor 5. Due to the low opacity of the disc in the near IR to near UV region of the spectrum, another effect of increasing disc height is increasing line fluxes.

This effect seems at least partly to do with the temperature distribution along the length of the radiation path. More rapidly increasing disc height with radius results in a steeper slope to the disc material relative to the central plane. This would result in interception of a larger amount of the flux, and more hot surface material responsible for the observed emission. However, it is difficult to directly compare the temperature profiles for different geometries in the way that this test affects them, as they represent physically different paths through the material, rather than the same path but for different lengths. As such, it is difficult to say the exact effect causing the changes in relative fluxes without some degree of remodeling of the tests to ensure that the same paths are used for each test.

One implication of these tests, is a possible explanation for both the triplet ratios of SDSS 1228 and additionally the comparative strength of the lines of SDSS 1228 against several of the other similar systems found. The other systems do not have quite the same distinctive ratios between Call x, y and z as SDSS 1228. This may imply that, although it is likely that the all of the discs are taller than the hydrostatic equilibrium predictions, SDSS 1228 is especially tall. This would have the dual result of modified line ratios in the manner observed, as well as generally stronger emission.

## 4.1.6 Inclusion of Grains/Dust

Until now the tests have been exclusively a composition of gases. However, it is known that there is dust present in these discs [Brinkworth et al., 2009], meaning that attempts to integrate its existence into the model should be made. There are several disadvantages to the current simulation setup. First, comparatively little is known of the properties of dust for an average disc. The models by Reach et al. [2009] provide some basis, but are not definitive. In addition the evolution of these discs is not clearly known. The manner in which the discs form could have very strong effects on the distribution of grain sizes and types, as such it is difficult to use a theoretical model to base these distributions on. The second point is that Cloudy, while proficient at modeling gases and diffuse ISM dust, is not designed for the varied grain sizes, as well as difficulties in recreating reasonable dust densities with the simulation as set up for this project. In addition, Cloudy does not self-consistently handle sublimation of grains in regions too hot for them to exist. While they are removed from the simulation they do not cause an increase

in population of the gas phase in that region. This means that although the dust grains will not contribute absorption and IR emission in regions where they should not exist, as if they had sublimated, they won't effect the gas abundances.

However, it is possible to discuss at least a basic model of dust which should hopefully perform well enough in order to draw generalised conclusions.

From the temperature profiles of the previous tests, it is clear that there are a large range of conditions under which the temperature nearer the midplane of the disc drops below 2,000 K, which can be thought of as an absolute cut-off point for the presence of dust due to sublimation. In addition, due to the mass and density of any dust, coupled with its effectively zero viscosity, it could be expected that the dust would settle in the midplane.

This means that a skeletal theory can be drawn up regarding the dynamics of a dust + gas disc. Over the course of a disc's evolution from the initial tidal disruption to the stable configuration, some manner of size distribution of particles would arise, from pieces (or aggregates) possibly as large as several cm (as seen in actual discs such as Saturns rings), down to the microscopic, as well as a gas component. Larger elements of dust will settle towards the midplane over this course of evolution, sitting beneath smaller elements, which are in turn beneath a layer consisting of evaporated, gaseous material. In addition to this there will be some level of cross over between the layers, in particular with gas likely throughout the entire vertical structure.

The gas near the surface receives the majority of the incident radiation from the white dwarf, efficiently absorbing the high energy component of the spectrum. This protects the lower layers of dust from the incident radiation and provides the main, ionised region from which line emission may originate. Meanwhile the dust will be kept at a more controlled temperature, contributing an IR continuum up through the gas (most of which would appear from previous tests to be fairly optically thin from near IR to mid IR).

## 4.2 Disc Models

Although tests utilising a single ring are useful for determining relations as to what effects certain variables have on aspects of the disc, they cannot be directly compared to the actual observations. For this purpose, a model of an entire disc, consisting of multiple ring elements can be used. Whilst these full disc tests provide data that is directly comparable to known properties and observed data of discs, they have several disadvantages. Especially for discs of higher densities, the processing time for a single ring can become a limiting factor. Times of upwards of 4 hours for a single ring can be reached. While with the single ring tests, with only one ring for each modification to a parameter this would not be too large a problem, as tests were run on the Warwick University Centre for Scientific Computing 'Cluster of Workstations' permitting parallel processing of several tests at once. With many rings per test, as with modelling a full disc, time starts to become more of a factor.

## 4.2.1 Density

In order to examine the effects of density on a whole disc several densities were selected around  $1.0 \times 10^{-12}$  g cm<sup>-3</sup>, shown to have peak Call emission as well as having been the previously estimated density from Werner et al. [2009]. Step sizes between densities were larger than in previous Sections, being steps in power of 0.5. The discs have an inner radius of  $1.4 \times 10^{10}$  cm (0.2  $R_{\odot}$ ) and outer radius of  $6.4 \times 10^{10}$  cm (0.92  $R_{\odot}$ ), which are subdivided into 25 ring segments

with  $\Delta r \ 2.0 \times 10^9$  cm.

Figure 4.2.1 shows the total spectra (white dwarf plus disc) for four of these discs. In addition, the red line in the plots are the observed spectrum for SDSS 1228 to provide a reference. In 4.17(a) there are also labels indicating the element/ion source of several of the lines. Since the standard simulated white dwarf parameters have been used, these are not expected to provide an exact fit. SDSS 1228 was used simply due to it having the most prominent and well defined emission lines.

Focusing on a density of  $5.42 \times 10^{-12}$  g cm<sup>-3</sup> as the best fit, there are several notable features. First, the Call IR triplet line strengths fail to match the observed, with the 'x' line being much stronger. The line strengths also fall slightly short, but it is important to keep in mind that the parameters of these simulations were not tailored to match SDSS 1228 specifically. Additional lines are those of Nal 5892 Å, Fell 5169 Å and the two forbidden calcium lines, as well as a neutral calcium line at 4233 Å. The Fell 5169 Å line appears to be much weaker than would be ideal to even proportionally reflect the observational data, in the same manner as the Call IR triplet. In addition, the clear divergence from the observed is the presence of the Nal line at 5892 Å.

Whilst the presence of this sodium line is not in agreement with the observed spectra, it is not necessarily a major problem. Unlike the Call H & K lines which, in the LTE and NLTE models had been extremely strong compared to the observations and closely linked to the desired IR triplet, this Nal line could plausibly be removed with just modification of abundances. The scope for modification of the abundances, without theoretical backing would represent an unwieldy number of free parameters, with little idea of what alterations would be reasonable. This is the reason for having stuck to the single set of abundances specified in Zuckerman



Figure 4.17: Total spectra (white dwarf plus disc) for discs of several different densities, plotted with the observed spectrum for SDSS1228. The simulated discs span radii of  $1.4 \times 10^{10}$  to  $6.3 \times 10^{10}$  cm (0.2 to 0.9  $R_{\odot}$ ) and were illuminated by a standard white dwarf.

et al. [2007] derived from observational measurements of white dwarf absorption lines.

Another positive outcome of these simulations are the Call H & K line fluxes. For simulated discs, the lines strengths of H & K need not be disproportionately stronger than the Call triplet in order for the latter to be of a detectable strength. Previous LTE and NLTE models have suggested that very strong H & K lines would be expected in a standard gas emission spectra. However, this more detailed modelling of the gas appears to suggest that the H & K lines rapidly become very optically thick, more so than the IR triplet lines, resulting in a much better agreement with the observed spectra.

## 4.2.2 Size

Because simulations of rings are independant from the overall structure of the discs, in order to investigate the effect that the overall radial extent of the disc has on the emission it is possible to reuse the previous ring calculations.

The observations can only determine the extent of the disc that is detectably emitting. The disc could conceivably extend closer in to or further out from the white dwarf than the inner and outer radii of the region of detectable emission may suggest. However, in addition to constraints on the emitting region made possible by the observations, it is possible to estimate limits based on other likely parameters of the disc. Since it is assumed that the source of the atmospheric contamination of these white dwarfs is accretion from the discs, at least some material from the disc must extend all the way to the white dwarfs surface. However, at such small radii it may be that the actual emitting area, coupled with a very thin disc height, would mean that the actual contribution from this region to the strength and shape of emission lines may be minimal.



Figure 4.18: Total line flux for each of 57 rings at radii between  $\approx 6.3 \times 10^9$  cm to  $\approx 1.2 \times 10^{11}$  cm  $(0.09 - 1.7R_{\odot})$ . Illumination was from a standard white dwarf.

Figure 4.18 shows the contribution to the total flux from each ring, as opposed to the flux of the ring per unit area as in similar previous plots. The effect of increasing emitting area for the outer rings to some extent counteracts the lower emission per unit area, however it also provides line ratios less like those observed. The Call x line becomes even stronger relative to y at fairly large radii, having already been a problem in earlier tests. In addition the Call z line gradually becomes stronger than the Call y line, also disagreeing with the observations. The Fell line also becomes rapidly weaker at greater radii. Figure 4.19 shows the spectrum from a disc composed of the entire range of ring radii.

## 4.2.3 White Dwarf Temperature

To complement the initial single ring models for different temperatures of illuminating white dwarf, several full disc tests were performed. As with the other disc



Figure 4.19: Total spectrum (white dwarf + disc) for a disc with inner radius with inner radius  $\approx 6.3 \times 10^9$  cm and outer radius  $\approx 1.2 \times 10^{11}$  cm  $(0.09 - 1.7R_{\odot})$  with a density of  $1.71 \times 10^{-13}$  g cm<sup>-3</sup> illuminated by a standard white dwarf.

models, the parameter being varied was chosen at larger intervals than the equivalent single ring tests. In this case five white dwarf temperatures were chosen, from 8,000 K to 24,000 K in steps of 4,000 K. White dwarf luminosities were varied based on their temperature in the same manner as the single ring tests, as given by the equation 4.1. Figure 4.21 displays the output spectra of these tests, with the total spectra in black (white dwarf + disc emission) and the disc emission alone in red.

Appreciable emission can be seen right down to the 12,000 K test, with the test for 8,000 K resulting in substantially lower line fluxes. As with the single ring tests, this appears to agree with the observational results of line emission being detected from white dwarfs as cool as 13,500 K. Figure 4.20 shows normalised versions of the calcium triplet line fluxes along with the equivalent widths of the same lines from the observational data. The ratios of normalisation are the same



Figure 4.20: Graphs of average Calcium triplet line strength, normalised to the same proportions as in Figure 4.14. Simulated points are in black, points from SDSS data are in red and points from WHT data are in green. The discs spanned radii between  $1.4 \times 10^{10}$  cm and  $6.4 \times 10^{10}$  cm and had a density of  $5.4 \times 10^{-12}$  g cm<sup>-3</sup>.

as in Figure 4.14, updated with simulation data from the entire discs, rather than single rings.

## 4.2.4 SDSS 1228+1040

By combining the parameters which have produced preferred effects so far, it is possible to create a model designed specifically to approximate the conditions of SDSS 1228. For this simulation the white dwarf properties were set as close to those of the observed as possible. A white dwarf effective temperature of 22,000 K, radius of 6  $\times 10^8$  cm, luminosity of 6.03  $\times 10^{31}$  erg s<sup>-1</sup> and mass of  $1.53 \times 10^{33}$  g (0.77  $M_{\odot}$ ) was selected. The inner and outer radii of the disc were  $1.4 \times 10^{10}$  cm (0.2  $R_{\odot}$ ) and  $6.4 \times 10^{10}$  cm (0.92  $R_{\odot}$ ) as in several previous disc tests. The disc density was  $8.6 \times 10^{-12}$  g cm<sup>-3</sup> and radial disc height was modified by a factor



Figure 4.21: Total spectra (black) and disc spectra only (red) for disc systems illuminated by white dwarfs of varying temperatures and associated luminosity. The simulated discs span radii of  $1.4 \times 10^{10}$  to  $6.3 \times 10^{10}$  cm (0.2 to 0.9  $R_{\odot}$ ) and had a density of  $1.71 \times 10^{-12}$  g cm<sup>-3</sup>.

based on the results in Section 4.1.5 and reduced based on early versions of the test. Dust was included in a low density permitted for the test to run (a dust to gas ratio by mass of  $8.7 \times 10^{-5}$ ), in an effort to mimic some of the likely effects that the dust would have despite not being able to model it fully. Figure 4.22 shows the spectrum from this test.

The simulated Call x, y and z lines appear to conform well to the observational data. The problem with the NaI 5890 Å line persists, though this could stem from an issue with the selected abundances. Additionally with this simulation, further neutral lines of various elements appear to become visible, in particular the Cal line at 4233 Å as well as MgI at 4561 Å and to a lesser extent FeI at 3884 Å. Slight emission from FeIII, FeII and several trace elements of the gas such as Mn, V and Li are also present as weak lines. This may suggest that the abundances used in the simulation do not fit the abundances in the disc of SDSS 1228. However, given the wide variety of elements which are the source of the neutral lines, it may be more likely that the simulation is lacking in another area, that some other property of the disc is the cause of the neutral lines. This may relate to the condition of constant density across the radial and vertical structure disc. A more complex vertical structure could result in vertical layering of emission. A density profile with a dense midplane out to a low density surface would redistribute the populations of different ionisation states. Neutral material may be confined to the higher densities nearer the midplane, potentially resulting in less neutral emission escaping the disc.



Figure 4.22: Spectrum of the simulated version of SDSS 1228+1040. Simulated disc spectrum is in blue, total spectrum (disc + white dwarf) in black, and the observational data in red. Inset is a zoomed in view of the Call x, y and z lines.

## **Chapter 5**

# Discussion

## 5.1 Summation of Findings

## 5.1.1 Call H & K Lines

We had hoped to address the previously over-predicted strengths of the near-UV Call H & K lines. Both simulation and theory had suggested that a calcium gas, at a temperature required to produce the observed Call near-IR triplet, Call x, y & z, should also produce the H & K lines disproportionally strong than seen in the observations. The results of testing carried out in this project appear to show that, while a large percentage of calcium in the disc will indeed be strongly emitting in the H & K lines, the optical depths for these in comparison to x, y & z differ drastically. As such very little of the H & K emission would be likely to leave the disc without being reabsorbed, whereas a far greater percentage of the x, y & z lines would be permitted to exit the disc and contribute towards detectable emission.

## 5.1.2 Call x, y & z Relative Strengths

Using disc heights based on the hydrostatic equilibrium calculations, the observed Call x, y & z ratios could not be reproduced. For those disc heights, most of the tests exhibited Call x lines which were significantly stronger than the y and z lines, contrary to the observations. The relative strengths seem to be strongly dependant on gas temperature, with high temperatures required for ratios which more closely match the observations. With most of the test geometries, the regions with sufficiently high gas temperature are fairly small. The disc height tests seem to provide the most favourable alteration to the relative line strengths. In addition to this, these tests also appear to provide some degree of an explanation for both the relative strengths of SDSS 1228 emission compared to other observed systems. With increasing disc height, due to the low opacity in the observed emitting region of the spectrum, more material is available to contribute to the line strengths. Also, more material can reach the higher temperatures which favour the line ratios observed in SDSS 1228. Furthermore, the handling of a dust component of the disc was unfortunately limited in this project. Should the dust and gas not be in LTE, dust in the midplane of a disc may help increase the gas temperatures further due to the wide IR emission the dust provides.

## 5.1.3 White Dwarf Temperature Range

With observations covering white dwarfs with 13,500 K to 22,000 K, it was hoped that this simulation would produce Call emission over the same temperature range. As illustrated in previous sections, emission appears to be present across the same range of white dwarf temperatures. Additionally, when looking at the emission strengths from these tests in comparison to the observed strengths, there appears

to be good agreement, when considering that other parameters are also expected to be affecting the observed strengths.

## 5.1.4 UV Spectrum

We had hoped to determin the presence (or otherwise) of emission lines in the UV region of the spectrum. Results on this area are mixed, the opacities of the gas in the UV for these simulations are higher than Cloudy can reliably calculate radiative transfer for. High opacities result in a high degree of scattering. Cloudy treats the transfer of radiation radially outwards from a source in a manner which does not permit full treatment of back-scattered radiation. It would appear from looking at the absorption lines in Section 4.1.1 that there are a great number of active lines in the UV. No emission is predicted by the spectra output from Cloudy. However, evidence from the absorption lines, and missing energy from the emission spectra due to incomplete treatment of the escape of lines in such an optically thick environment, suggests that this may not indicate that such discs do not emit in the UV. In fact, when considering both the strong absorption lines and missing energy, it seems likely that at very least some emission would be detectable from the hot surface layer of the discs, though not from throughout the entire depth of the discs as is the case for the Call x, y & z lines. Whilst excessively strong Call H & K emission relative to the x, y & z emission is not produced as in previous models, emission of these lines for this model appears to fall slightly short of the observations. Contributions to these line strengths from the hot surface layer, not included in the output from Cloudy due to their high optical depth, may make up this missing flux and bring the H & K lines to a more equal footing with the x, y & z lines.

## 5.1.5 Neutral Lines

The simulations conducted appear to consistently predict the presence of neutral emission lines, primarily the Nal line at 5890 Å. The Nal line had a tendency to be very strong, of the order or above that of the Call lines. One potential reason for this may simply be in the relative abundances, it may be that there is less sodium in the observed discs than has been used in these simulations. However, other neutral lines from elements other than Na present in the full disc simulation based on SDSS 1228 shown in Section 4.2.4 may suggest that the Nal 5890 Å line present in the other Sections may merely be symptomatic of a wider fault with the model. Potentially due to the structural assumptions of the disc such as the constant density criteria, which could conceivably modify the proportions of ionised material significantly. This is discussed further in Section 5.2.2.

## 5.2 Critique of Method

## 5.2.1 Advantages

## Illumination

An improvement of this model over previous LTE and non-LTE models is the use both of a full simulated white dwarf spectrum as the source of illumination, and an approximation of the actual radiation incident to the surface of a geometrically thin disc.

## Self-Consistency

The simulations in this project using Cloudy are self consistent, dealing with more parameters interacting and their effect than previous gas models.

### **Optical Properties of the Discs**

By using Cloudy with its comprehensive treatment of optical depths and the extents to which radiation may be produced but reabsorbed before leaving the disc entirely, a number of curiosities regarding the discs can be dealt with. In particular, the mechanics observed with the Call H & K lines, which previously had presented somewhat of a problem. Previously anticipated to be strongly emitted, but with only minimal fractions of the predicted present in the observations, these simulations using Cloudy would appear to show that the H & K lines become optically thick more quickly than the Call x, y & z lines. This results in a set of predictions closer to the observed in that regard, cutting down H & K emission from strengths as much as several orders of magnitude larger than x, y & z in some previous models.

## **Reduced artificial criteria**

One disadvantage specific to the non-LTE model as in Werner et al. [2009] is that it had a heating source related to the assumption of an actively accreting alpha disc. While in some respects this relates to the illumination element mentioned above, the point here is in the artificial criteria of an unphysical accretion rate in that model in order to provide the required Call emission. While it was intended only as a proxy for heating by other methods (meaning that the unphysical accretion was essentially just a tool to provide heat), it does result in a limit as to which other aspects of the discs such a simulation can provide insights.

## 5.2.2 Disadvantages

### 1D Model

An unavoidable and major inadequacy of this model is that the 1D geometry of the Cloudy format is not suitable for the type of discs being modelled. From the results obtained, it is likely that the discs will have a non-linear, varied, vertical and radial structure. Distribution of heat and radiative transfer are likely to be more complex than that for which a 1D model allows.

A 2D approach to the radiative transfer and the equilibrium conditions of the gas discs would ideally be performed in order to properly model the structure of the discs, rather than the 1D models of this project. The 1D models used here appear to give a rather good approximation of averaged disc properties, as well as apparently simulating the emission spectra to a degree which appears to agree with observations without major alterations. In addition it has provided an approximation and qualitative description of conditions by which the gas and dust properties of the disc can simultaneously exist. However, in order to obtain more detailed specifics of properties of the discs, or perform fitting to determine characteristics of specific observed discs, a more complex system, such as the 2D one described, would be required.

## Illumination

While the fundamental treatment of the illumination as a self-consistent simulated white dwarf source is an advantage of this system over previous models, it has disadvantages as well. The incident flux to the disc was not treated to its full extent. The assumptions used were that each ring of the disc was flat, that the disc height was very much smaller than the radius of the white dwarf, as well as incomplete treatment of the illumination source itself with respect to such aspects as limb darkening. While the extent to which they represent the system were sufficient for this model, ideally a more comprehensive treatment of these assumptions would have been performed. While the simulated disc is only gradually sloped, it is not flat, which is not coincident with the assumption that each component ring is flat. The height of the disc, while always smaller than the white dwarf radius, does not genuinely fulfil the criteria of being very much smaller (heights ranged up to almost half of the radius of the white dwarf).

These were deemed acceptable inaccuracies to the level of detail to which the illumination was being modelled, however.

## Disc Structure and Hydrostatic Equilibrium Assumptions

Another area in which assumptions had to be made were the conditions determining the geometric structure of the disc. The disc height was treated as a function of radius only, in reality the variations described in section 5.2.2 above would all contribute to the shape and form of the disc. For a given ring segment at a given radius, the height would be affected by the variation of density and temperature throughout the height of the disc, as well as the radius itself, due to the restoring effect of gravity at any displacement from the orbital plane of the disc as a whole. In practice, to model these effects correctly, modification to a full 2D model would not necessarily be required, but at least a highly iterative process would be needed. Once general vertical structure of at least one parameter had been determined by the 1D model, it would be necessary to repeat the simulation with modifications based on this parameter, first altering the hydrostatic equilibrium and then fixing the chosen initial parameter and varying another, gradually converging on a solution. For example, initially the temperature could be varied, this would determine a new disc height and the same temperature profile could then be used in a second test varying density. This is a similar process to that used by Cloudy itself to determine just the temperature profiles for a fixed model such as was used in this project. As such, the number of iterations required for more and more varying parameters could easily become unworkable. So, while modifications could potentially be made to the current system in order to better determine vertical structure and more accurately model the discs, it would likely be more beneficial to go to the full 2D simulation already mentioned, appearing as it does to provide a better balance between effort and accuracy.

### Line Asymmetry

The asymmetric nature of the Doppler profile of the Call x, y & z triplet for SDSS 1228 is an illustration of how dynamic these discs could be expected to be. They show that, for at least one of the objects, in order to genuinely replicate the observed, a fully 3D model would be required. The asymmetry of these lines removes the ability to rely on axial symmetry as a means to simplify the model used. If it is assumed that the material originated from an asteroid on a highly eccentric orbit, it is not unreasonable to assume that it would take some time for the disrupted material to circularise into a disc, during this time it would likely display a high degree of asymmetry in its emission.

## Chapter 6

# Conclusions

The findings of this project support the concept that the observed emission lines are created in a disc which is effectively optically thick to the incident white dwarf UV flux, geometrically thin and made of material sourced from a tidally disrupted asteroid (or asteroids). It also suggests that both dust and gas could exist in a layered geometry. This would involve upper and lower surfaces of the disc being composed primarily of gas at fairly high temperatures (3,000 to 7,000 K), absorbing the majority of incident UV and shielding the dust from reaching high temperatures at which it would sublimate. Observed Call emission, as well as potentially FeII and others, would come from these outer gas layers, which are optically thin in the visible and near IR continuum. This transparency at near IR would also permit IR emission from the mid-plane dust to be emitted from the disc.

The results also appear to explain the previously undetermined reason for the lack of the Call H & K, in the observed spectra. A cloud of gas of the composition examined for the temperatures reached in the modeled disc rapidly become optically thick in the H & K lines, many orders of magnitude more so than it does for the x, y & z Call IR triplet lines. Emission of the Call IR triplet has been shown to be simulated at detectable strengths for the full range of white dwarf temperatures that the observed objects possess. Gas densities between  $\approx 10^{-12}$  g cm<sup>-3</sup> and  $\approx 10^{-11}$  g cm<sup>-3</sup> seem to provide the most considerable contribution to the Call IR triplet line strengths, corroborating previous gas density estimates. An entire disc of this gas density would possess approximately  $10^{19}$  g of material, excluding the mass of the dust component. Given prior estimates for the mass of dust being somewhere between  $10^{19}$  and  $10^{24}$  g of material, it seems plausible for this quantity of gas to be sourced from evaporated dust forming the rest of the disc.

The relative line strengths for the components of the Call IR triplet can also be recreated using this model. The parameter which appears to have the strongest overall effect on the line ratios is disc height. While the actual reason is likely the change in the overall distribution of temperatures that this causes, the effect of increasing emitting material and hence increasing the line strengths in general is also favourable, when looked at in relation to the comparatively strong emission lines of SDSS 1228+1040.

The prominance of the neutral sodium line in the simulated spectra may be indication that the discs with observed emission lines are sodium deficient with respect to GD362, upon which the simulation abundances were based.

While results are not conclusive for predictions of the UV emission from the disc, the data suggests that the hot ( $\approx$  3,000 K +) top layer of the surface could potentially source a large variety of reasonable strength UV lines for elements such as sodium, magnesium, calcium, aluminium and iron. Eventual UV observations of the objects possessing Call IR triplet emission should provide more information about the plausibility and existence of these possible lines (HST/COS observations of SDSS 1228 are scheduled for 2010).

Further uncertainties in the results arise from the imperfect nature of the model as an analogue for a disc with varying parameters in multiple dimensions. Density and temperature gradients through the disc can only be dealt with to a limited capacity in this model. In particular, the temperature profiles determined do not affect the hydrostatic equilibrium determining the disc height as a function of radius. Also, the density is forced to be constant throughout the vertical disc structure. While density could potentially have been varied as a function of radius, with little data as to in what manner such variance would occur, it also was kept constant across any given discs. The extent to which the vertical temperature profile varied suggests that the vertical structure of these discs would be expected to be complex merely for the gas. When including a layer of dust in the midplane of the disc, vertical structure would take on further additional complications.

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