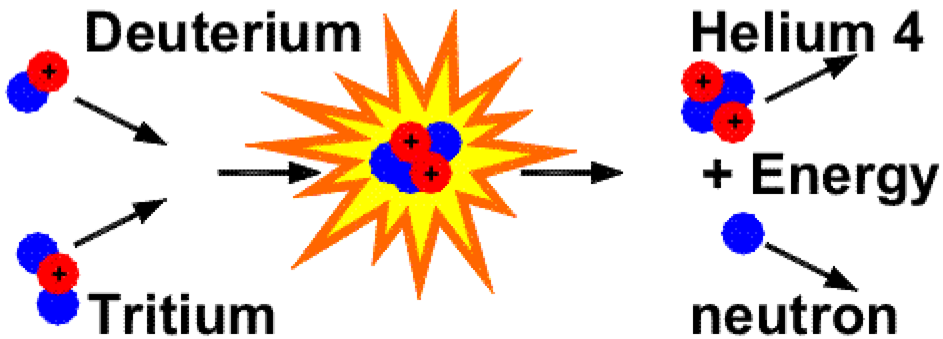
Magnetically Confined Fusion: Transport in the core and in the Scrape- off Layer

Bogdan Hnat

Joe Dewhurst, David Higgins, Steve Gallagher, James Robinson and Paula Copil

Fusion Reaction



$^{2}H + {}^{3}H \rightarrow {}^{4}He + n$

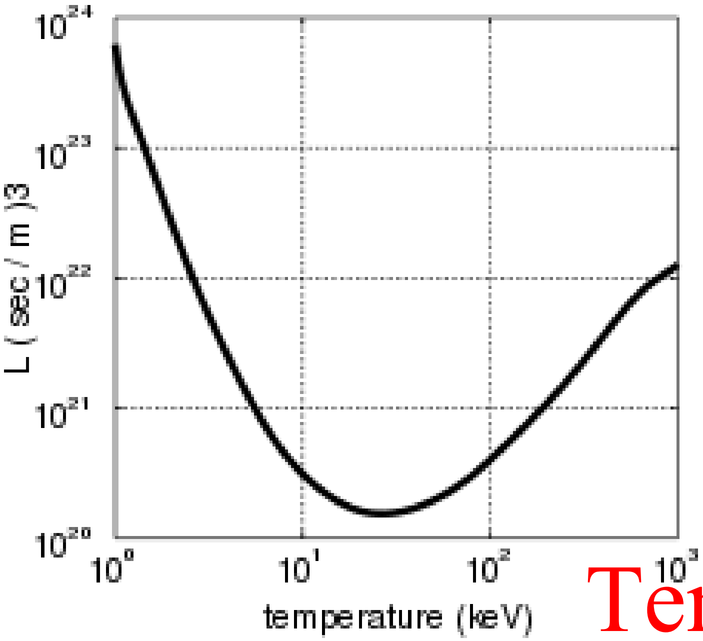
Lower binding energy for ^{4}He + energy of the neutron = ~ 17.6 MeV

- Extra neutron in each nuclei increases collision rate
- Electrostatic forces small – one proton per nucleus
- Result: cross section maximum at relatively low temperature of ~ 300 milion K.
- ^{2}H extracted from see water, ^{3}H can be produced

$^{6}Li + n \rightarrow {}^{4}He + T$ or $^{7}Li + n \rightarrow {}^{4}He + T + n$

Lawson's criterion

- Confinement time τ_E=Energy content / Rate of Loss
- Stationary state: equalise energy loss with input of thermal energy (keep T constant)
- Define $L=n_e\tau_E$, look for T at which fusion reaction produces enough energy to sustain itself



Take **50/50 D + T** fusion reaction
Assume cold ions (electron pressure only)

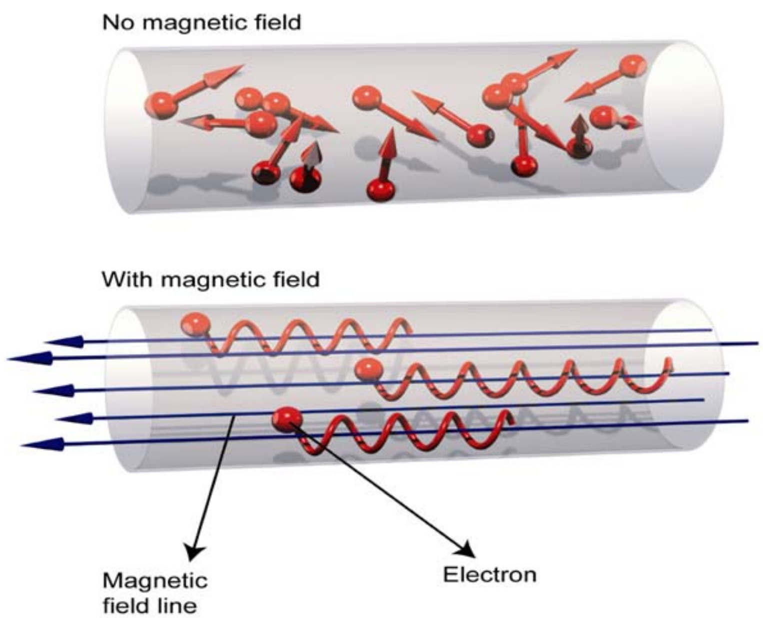
$$L = n_e \tau_E \geq 1.5\times10^{20}\,[\mathrm{s/m^3}]$$

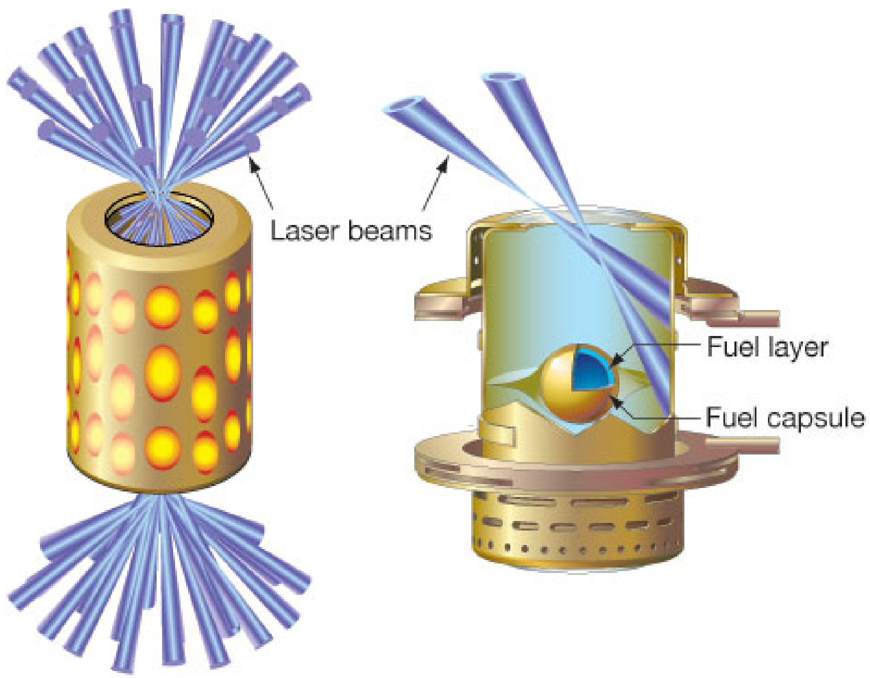
Temperature at min. ~ 300 milion K

Realising Lawson's criterion

Condition $L = n_e \tau_E \geq 1.5 \times 10^{20}$ can be achieved by:

- Large values of τ_E – magnetic confinement, or
- Large density n_e – inertial confinement



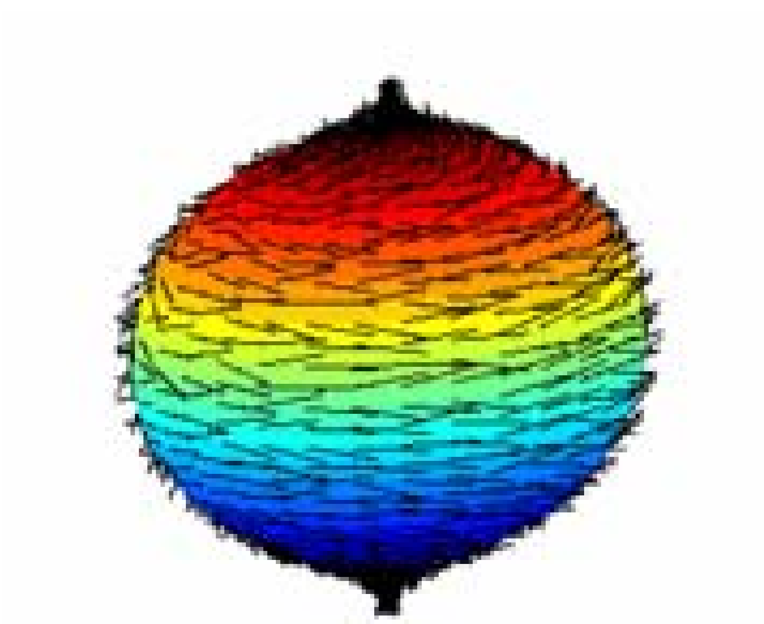


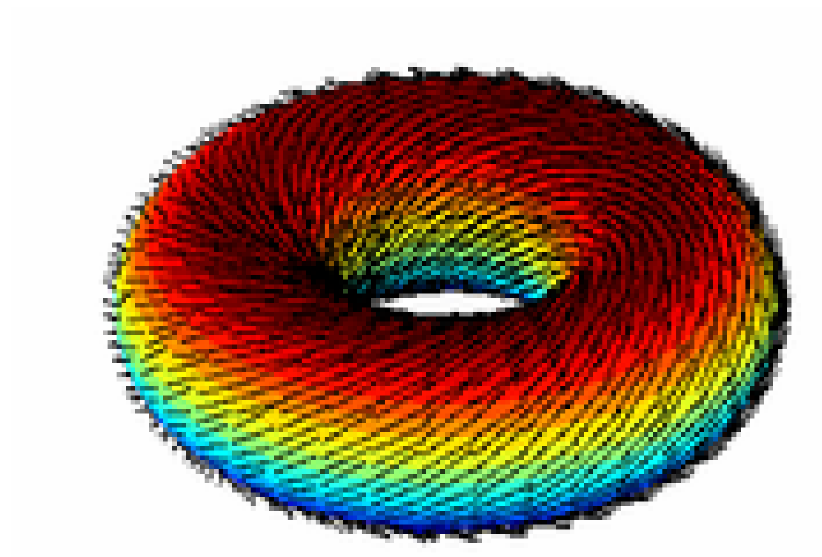
Confinement and topology

Poincare's theorem (in my own words…)

Let ***S*** be a smooth, closed surface and ***C(x)*** be well behaved vector field such that the component of ***C*** tangent to ***S*** never vanishes.

The surface ***S*** must then be a torus.





Confinement and topology

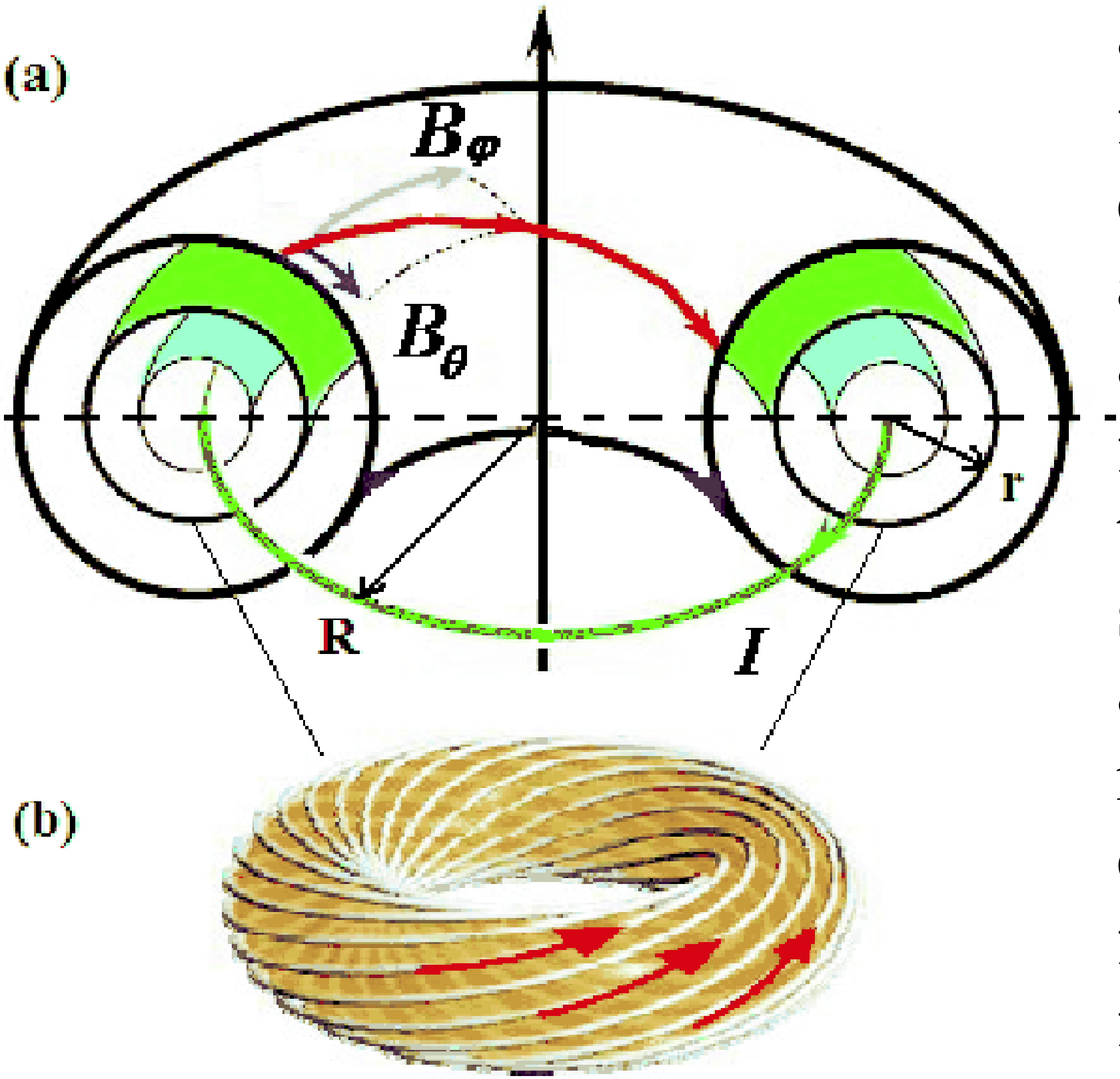
Consider the outmost bounding surface of some confined plasma.

- Particles can stream free along the magnetic field lines
- An ideal confining magnetic field should have no component normal to the bounding surface
- Magnetic field B must cover the entire surface and the tangent component can not vanish anywhere

Conclusion:

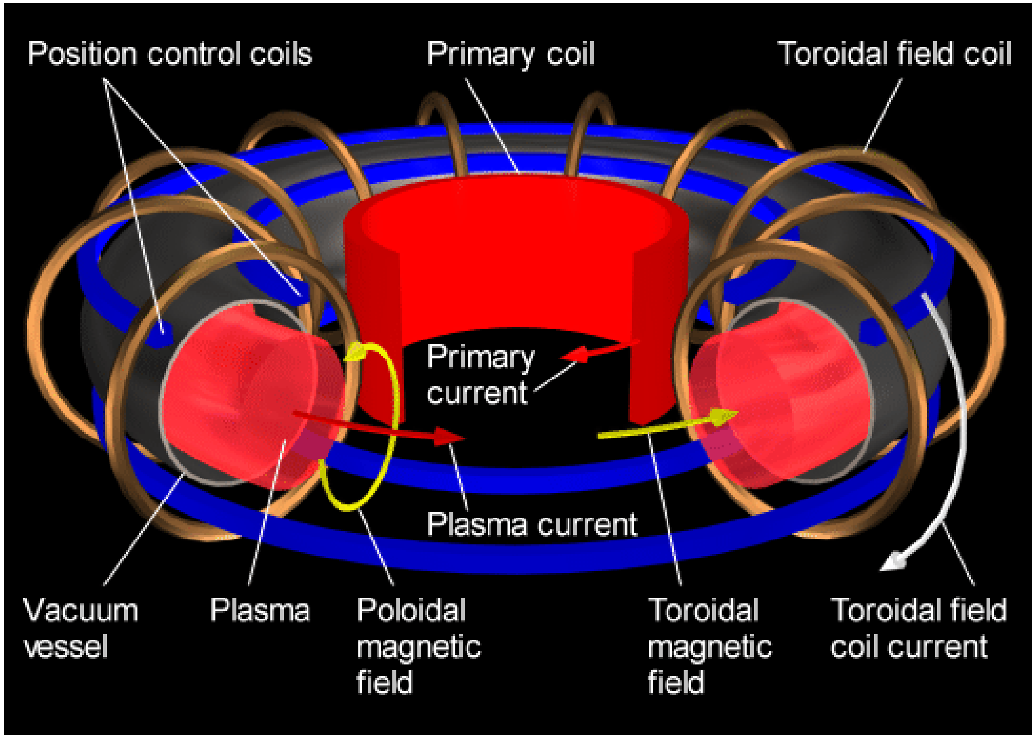
The bounding surface must be a torus.

Flux surfaces



- **Rational surfaces**: magnetic field line on such surface closes on itself after n toroidal and m poloidal turns
- **Ergodic surface**: magnetic field line never closes on itself, thus covering densly the surface
- **Stochastic regions (volums):** magnetic field has radial component which allows for fast transport between different flux surfaces (can not support pressure gradient)

Tokamak

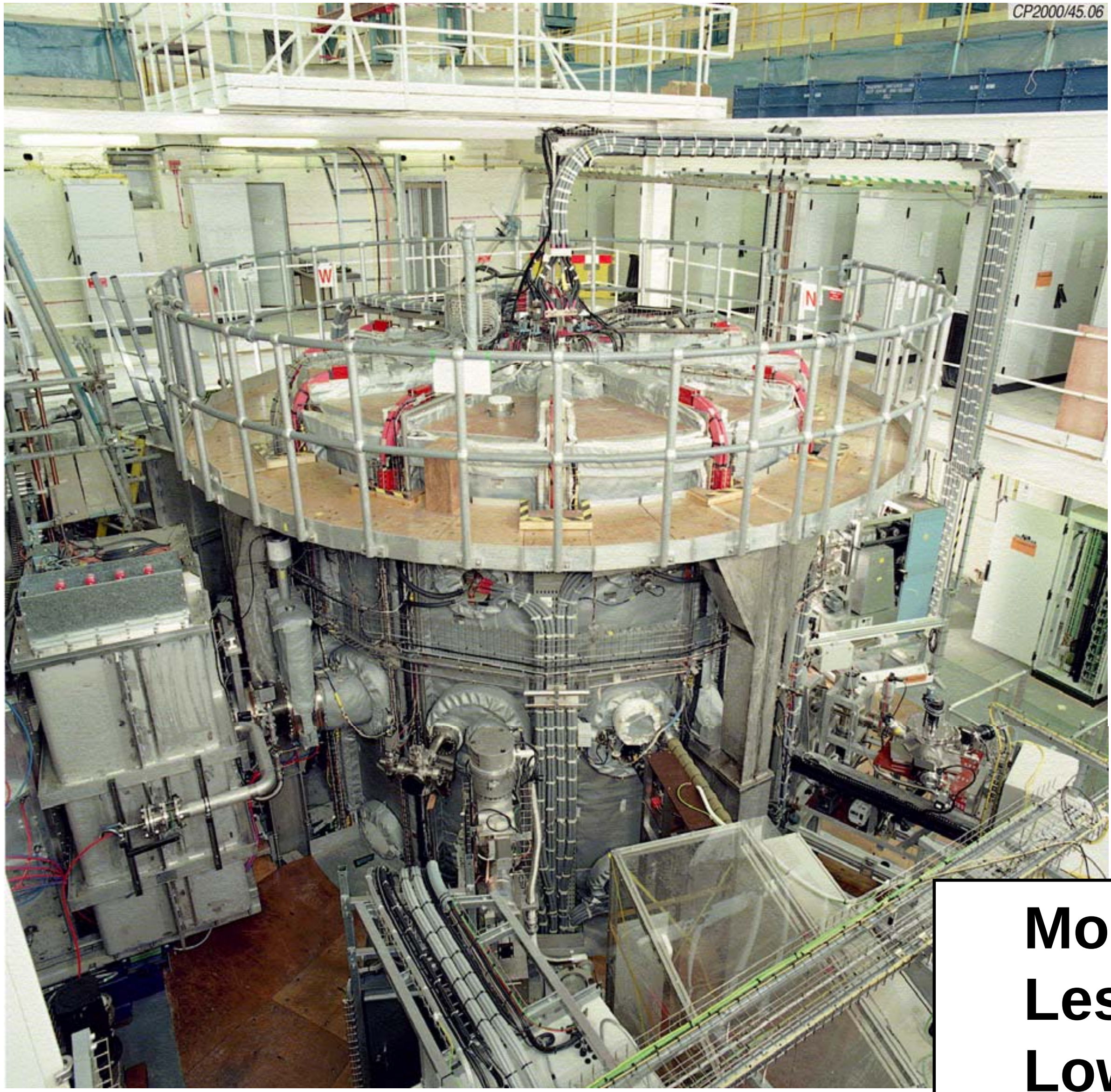


- Primary coil indices toroidal magnetic field
- Pure toroidal field configuration is unstable
- Plasma current is driven in toroidal direction, inducing poloidal field

Magnetic field: 0.6 T
Density: $2 \mathrm{x} 10^{19}$ [1/m^3]
Plasma current: 1-2 MA

$$\nabla_r p = (\vec{j} \times \vec{B})_r$$

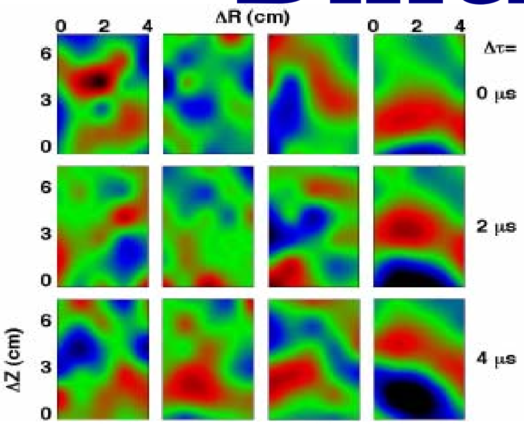
MAST – Spherical tokamak

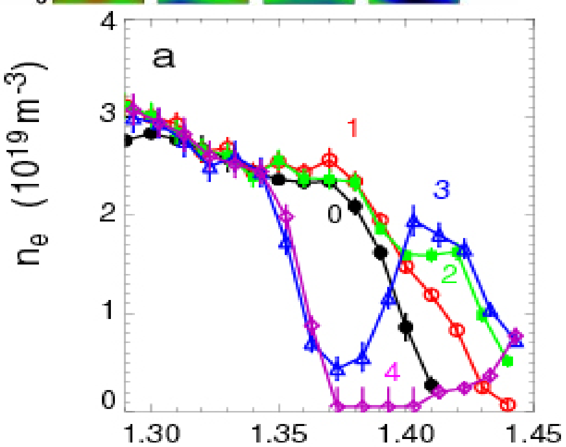


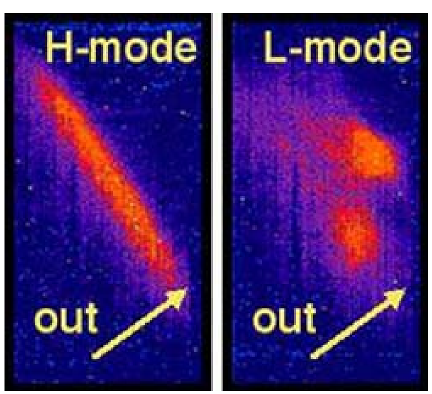
Electricity cost $\sim \beta^{-0.4}$

More compact in size
Less prone to MHD instabilities
Lower B – cheaper electricity

Different confinement regions



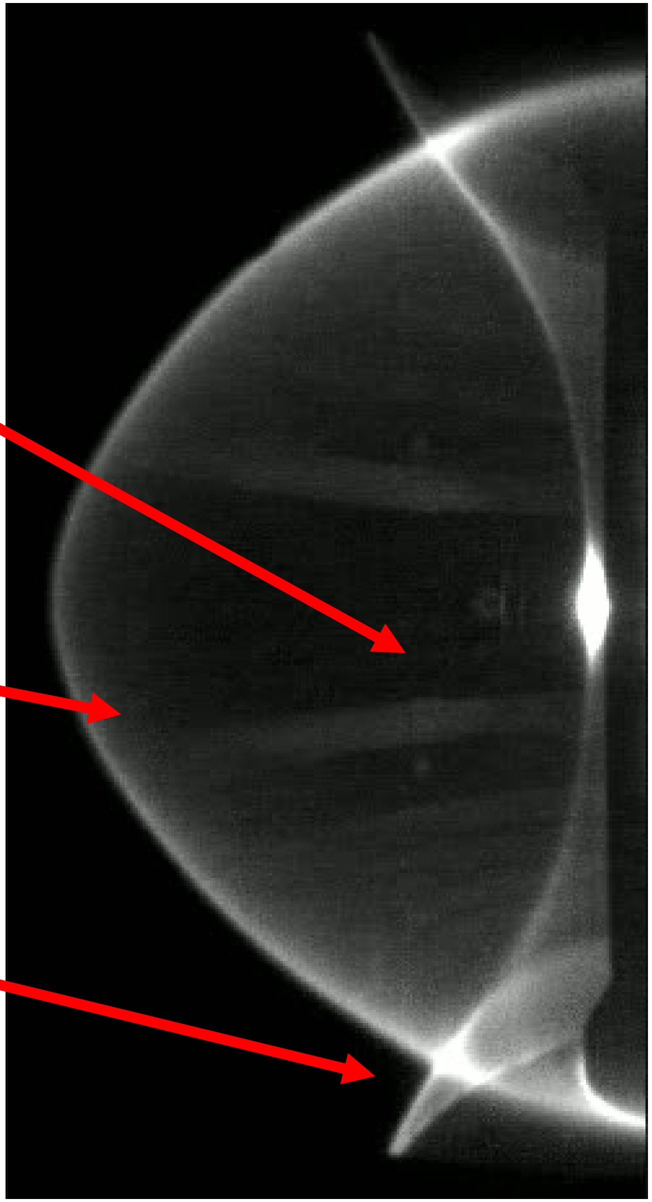




Core: Hot collisionless plasma, $\beta \sim 1$, $\delta x / \langle x \rangle \ll 1$

Edge: strong flow shear, steep gradients

Scrape-Off Layer: Cooler, low **B**, atomic physics effects



Particle Transport: classical estimates

$$\partial_t n + \nabla \bullet \Gamma = S_{part} \qquad \partial_t (3/2nT) + \nabla \bullet Q = S_{heat}$$

Normally we assume that: $\Gamma = -D\nabla_\perp n \quad Q = -\kappa\nabla_\perp T$

$$\Gamma_r \approx (n_r - n_{r+\Delta r})v_r = \left[n_r - \left(n_r + \partial_r n \Delta r + \ldots\right)\right]v_r = -(v_r \Delta r)\nabla_r n$$

$v_r \approx \dfrac{\Delta r}{\tau_{ei}}$ and we assume that $\Delta r \approx \rho_e$ thus $D_r^e = \nu_{ei}\rho_e^2$

$\rho_i = \left(m_i / m_e\right)^{1/2} \rho_e$ and $\nu_{ie} = \left(m_e / m_i\right)\nu_{ei}$ thus $\Gamma_r^e = \Gamma_r^i$

- Ions and electrons contribute equally to particle transport
- Same specie collisions do not contribute (momentum conservation)

Heat Transport: classical estimates

$$\partial_t n + \nabla \bullet \Gamma = S_{part} \qquad \partial_t (3/2nT) + \nabla \bullet Q = S_{heat}$$

Assume temperature gradient in radial direction

$$Q_r \approx \frac{1}{2} mn\left[(v_{th})_r - (v_{th})_{r+\Delta r})\right] v_r = -\kappa_r \nabla_r T; \; \kappa_r^i = \frac{n_i \rho_i^2}{2\tau_{ii}} = n_i \chi_i$$

In parallel direction the step size in the mean free path

$$\kappa_{\|} \approx nT\tau_{ee} / m \propto T^{5/2}$$

- Same specie collisions important for heat flux
- Electrons dominate parallel heat transport
- Collision increase $\chi_{\perp}$ and decrease $\chi_{\|}$

Transport: neoclassical estimates

Assume particle is moving on a flux surface with magnetic field magnitude between B_{min} and B_{max}. Define:

$$\lambda \equiv \frac{2\mu B_0}{mv^2} = \frac{v_\perp^2 B_0}{v^2 B}$$

A particle can never enter the region where $\mu B > E - Ze\Phi$

All particles must then satisfy $0 \le \lambda \le (B_0/B_{min})$.

$$\frac{B_0}{B_{\max}} \le \lambda \le \frac{B_0}{B_{\min}}$$

Particles trapped by the mirror force, move on the outboard side of the flux surface.

This trapped particle orbits are called banana orbits.

Transport: neoclassical estimates

The width of the banana orbit is given by: $\Delta r \simeq \rho_p \sqrt{\varepsilon}$

$$\rho_p \equiv \frac{v_{th}}{\Omega_p}, \text{ where } \Omega_p = \frac{eB_p}{m} \text{ and } \varepsilon \equiv \frac{a}{R}$$

Taking f_t as fraction of trapped particles heat diffusivity χ is

$$\chi_i^{ban} = f_t (\Delta r)^2 \nu_{eff} = (2\varepsilon)^{1/2} (\rho_{pi} \sqrt{\varepsilon})^2 \frac{\nu_{ii}}{\varepsilon} = \sqrt{2\varepsilon}\, \rho_{pi}^2 \nu_{ii}$$

Comparing classical and neoclassical heat diffusivities

$$\frac{\chi_i^{neoc}}{\chi_i^{c}} = \sqrt{2\varepsilon} \left(\frac{B}{B_p} \right)^2 \sim 10 - 50$$

Turbulent transport estimate

- Turbulence driven by micro scale instabilities (drift wave instability, Ion Temperature Gradient-ITG, etc…)
- Gradients are sources of free energy – large fluctuations at plasma edge are expected
- Often modelled as purely electrostatic

$$m d_t \vec{v} = -\nabla p + q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{multiply } \vec{B}\times$$

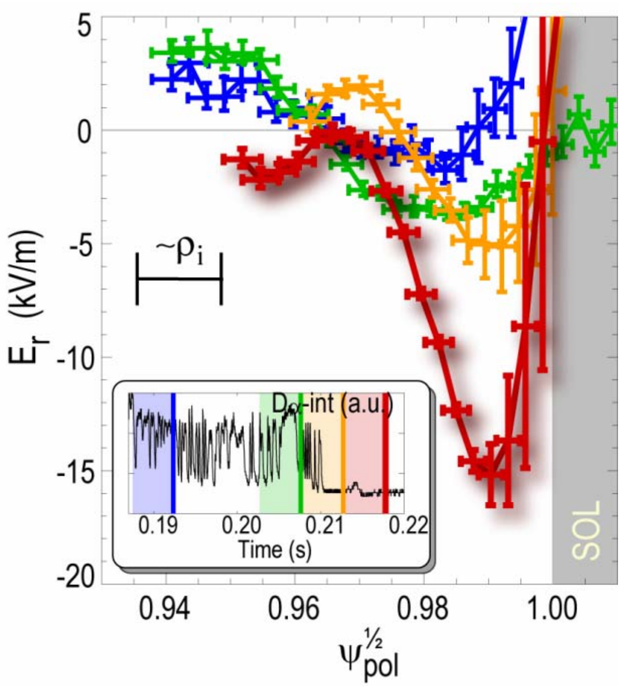
- Neglect inertial terms, take velocity perpendicular to B

$$\vec{v}_\perp = \vec{v}_E + \vec{v}_{diam} = \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\vec{B} \times \nabla(nT)}{B^2}$$

Turbulent transport estimate

- Often modelled as purely electrostatic
- Neglect diamagnetic part

$$\vec{v}_\perp \approx \vec{v}_E = \frac{\vec{E}\times\vec{B}}{B^2} = -\frac{\nabla\phi\times\vec{B}}{B^2} \Rightarrow v_\perp \approx \frac{\nabla\phi}{B}$$



$$D = \frac{(\Delta r)^2}{\tau} \approx v_E \Delta r = \frac{\phi}{B}$$

$$\frac{D_i^{turb}}{D_i^{neoc}} \sim 10 \qquad \frac{D_e^{turb}}{D_e^{neoc}} \sim 1000$$

Sources of turbulence – drift waves

- **Ions dominate perp dynamics, electron parallel dir.**
- **Quasineutrality: $n_e = n_i$, cold ions: *grad(p_i)=0***
- **No e-i collisions: $\delta\varphi$ is in phase with δn**

$$v_{E\times B} = \frac{\vec{E}\times\vec{B}}{B^2}, \quad v_P = -\frac{1}{\omega_g B}\frac{d\vec{E}_\perp}{dt}$$

$$v_d = \frac{\omega}{k_y} = \frac{T_e}{eB}\frac{\nabla_x n_o}{n}$$

$$\delta n = n_0(1 - e^{e\phi/kT})$$