Particle Acceleration in the Presence of Weak Turbulence at an X-Type Neutral Point

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Abstract

It is well known that particles in space plasmas can be energised by interaction with reconnection regions, which form at magnetic topological features such as nulls and separatrices. Such energisation has been studied in simple, large-scale fields. Here we extend these studies to include noisy, turbulent electric and magnetic fields. The magnetic field is perturbed by a superposition of cold plasma eigenmodes,including self-consistent electric field oscillations,constructed as in the work of Craig and McClymont.

Weak turbulence is modelled by adopting random phases for these eigenmodes. Using a numerical method we numerically integrate particle orbits in realisations of this field and describe the resulting particle distributions.

Form of Perturbations

We consider a situation in which an X-type neutral point is perturbed by a reconnective disturbance. Reconnection can only occur in azimuthally symmetric modes, so they are the only ones which will be considered here. Petkaki and Mackinnon (1997) consider only a disturbance at the the fundamental, n=0, mode of oscillation. Here, we consider a superposition of eigenmodes as our disturbance.

A standard X type magnetic field of the form B=y**x**+x**y** is perturbed by oscillations of the form (Craig and McClymont 1991, Petkaki and Mackinnon 1997):

$$\bar{B}_y = \bar{x}\Big[1 + A_0\frac{1}{2\eta}\exp(-\kappa t)[\kappa(\cos(\omega t)f'_{\Re} - \sin(\omega t)f'_{\Im}) + \omega(\sin(\omega t)f'_{\Re} + \cos(\omega t)f'_{\Im})]\Big], \qquad (1)$$

$$\bar{B}_x = \bar{y}\Big[1 - A_0\frac{1}{2\eta}\exp(-\kappa t)[\kappa(\cos(\omega t)f'_{\Re} - \sin(\omega t)f'_{\Im}) + \omega(\sin(\omega t)f'_{\Re} + \cos(\omega t)f'_{\Im})]\Big], \qquad (2)$$

Where η is the dimensionless resistivity, A_0 is the amplitude of the fluctuations, $f'_{\Re}$ and $f'_{\Im}$ are the derivatives of the real and imaginary parts of the hypergeometric function, κ is the eigenvalue decay time, and ω is the eigenvalue of the oscillation frequency. The electric field is perturbed by oscillations of the form:

$$\bar{E} = A_0[\exp(-\kappa t)[\kappa(\cos(\omega t) f_{\Re}\sin(\omega t) f_{\Im}) + \omega(\cos(\omega t) f_{\Im} + \sin(\omega t) f_{\Re})]] \qquad (3)$$

Where $f_{\Re}$ and $f_{\Im}$ are the real and imaginary parts of the hypergeometric function. The values of κ and ω were determined by finding the roots of the hypergeometric function using Broyden's method. Craig and McClymont (1991) give analytical estimates for the eigenvalues:

$$\frac{1}{2}\omega\ln\eta \approx -(n+\frac{1}{2})\pi \qquad (4)$$

$$\kappa = \frac{\omega^2}{2} \qquad (5)$$

Where n is the eigenmode number. These approximate values were supplied as initial guesses for the numerical recipes routine `broydn`, which returned accurate values for κ and ω.

These values were then inserted into (1), (2), and (3), and also used to calculate appropriate values for $f_{\Re}$, $f_{\Im}$, $f'_{\Re}$, and $f'_{\Im}$. A random phase difference was also introduced for each eigenmode. The value of the electric and magnetic field disturbances were then summed over all eigenmodes, to produce a simple approximation of plasma turbulence.

Particle Orbits

The trajectories of 100 electrons in the above electric and magnetic fields were followed. Particles were chosen randomly from a Gaussian distribution centred on 5 x 10^6 K. Distances were normalised to d, where $d=(c^2m_e/eB_0)^{1/2}$, Where c is the speed of light, m_e is electron mass, e is electron charge and B_0 is the magnetic field strength at the boundary. For $B_0=10^{-7}$, d=1.3x10^5cm. Speeds were normalised to the speed of light, and times were normalised to the electron gyroperiod at d.

The particle orbits were integrated using a fourth order Runge Kutta method taken from Numerical Recipes. Since we did not wish to plot orbits exactly, a large stepsize of 10^{-2} was used. This is allowable as long as the numerically integrated orbits stay within a 'shadow' of a real orbit (Hayes, 2003). Such an approach yields correct distribution functions, even although individual particles are followed only crudely. This technique is adequate for determining the bulk motion of particles. For a more accurate picture of the behaviour of individual particles, an adaptive stepsize method should be used.

The particles were started at randomly chosen positions close to the neutral point, within the central diffusion region of the system, where they can become energised. Their orbits were then allowed to evolve temporally and spatially.

Results

A superposition of 30 different eigenmodes was used to produce weakly turbulent electric and magnetic fields. The variation of the electric field with distance from the neutral point is shown in fig. 1. The field is strongest in the central dissipation region, so particles gain most of their energy here. The magnitude of the electric field decreases with r, and falls to zero at the system boundary.

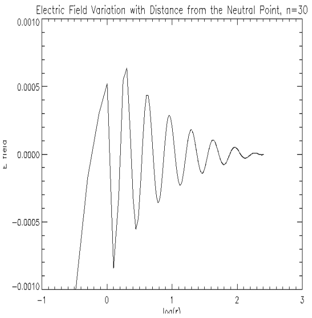

Fig 1: Variation of electric field with distance from the neutral point for a superposition of 30 eigenmodes.

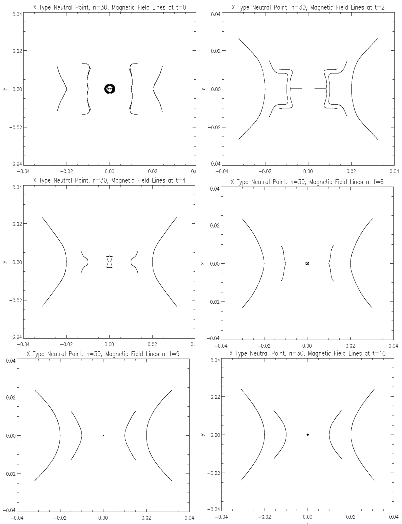

Fig 2: Variation of magnetic field near the neutral point with time. The amplitude of the fluctuations has been increased for the purposes of this figure so that they can be seen more clearly.

Figure 2 illustrates the temporal fluctuation of the magnetic field very close to the neutral point, for a superposition of 30 eigenmodes, each with a random phase. The initial disturbance relaxes into a stable X-point field as time increases. The field lines were integrated numerically.

Such fluctuations could be a driver of magnetic reconnection in plasmas.

The magnetic field strength is zero at the centre of the region, and is 100G at r=1, when r is normalised to d.

Figure 3 shows the energies of 100 electrons at the start of the simulation, and at t=100 (where t is normalised to the electron gyroperiod at the system boundary). Energies are normalised to the electron rest mass. Over this time scale, the majority of electrons are accelerated, although some do lose energy. Further work will extend this simulation to run to greater times, and to include more electrons.

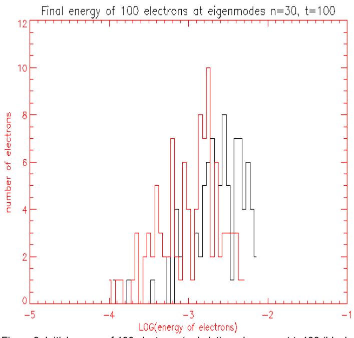

Figure 3: Initial energy of 100 electrons (red plot), and energy at t=100 (black plot). Energies are normalised to the electron rest mass, and times are normalised to the electron gyroperiod at the system boundary.

Conclusions

A superposition of 30 different eigenmodes of electric and magnetic field oscillations produced a weakly turbulent, inhomogeneous plasma capable of accelerating electrons to high energies. Future work will investigate the effect of increasing the number of eigenmodes present, and also allowing the phase difference of the oscillations to vary randomly. It is hoped that this will produce a more turbulent field.

In these simulations, particles were chosen from a Gaussian distribution, however it could be of interest to select particles from other distributions, and also to allow for pitch angle scattering, which was not taken into consideration here.

References: Craig, I.J.D. & McClymont, A.,1991,ApJ...371L..41L ; Craig, I.J.D. & McClymont, A.,1993,ApJ...405,207; Craig, I.J.D. &Watson, P.G..,1992,ApJ...392,385; Hannah, I.G,2005,PhD Thesis, Univeristy of Glasgow; Hassam, A.B., 1992,ApJ...405,207 ; Hayes, W.B., 2003, Phys.Rev.Lett. 90, 054104; Petkaki, P., 1996, PhDThesis, University of Glasgow; Petkaki, P., & MacKinnon, A. L. 1997, Sol. Phys., 172, 279;Petkaki, P. & MacKinnon, A.L.,2007, A&A 472, 623