Thermal Physics II – Problem Sheet 8

Part I: Questions

1. What are Fermions and what are Bosons? Find examples!

2. What are the approximations that gives the classical limit of the Fermi-Dirac and Bose-Einstein distributions?

3. If we would consider interacting particles instead of noninteracting, what would change: the density of states or the energy distribution or both?

4. What is named Bose-Einstein condensation? State the conditions for it to occur at fixed density and at fixed temperature.

5. What are the limitations on the chemical potential for Fermions/Bosons?

Part II: Problems

1. Number of States for ideal Bosons and Fermions
Consider a system that contains 12 noninteracting particle that can occupy 29 states that are energetically available. What is the number of states if
a) the particles are distinguishable (classical particle),
b) the particles are Fermions (indistinguishable + Pauli principle) and
c) we have identical Bosons?
What could you say about a system with 29 particles and 12 states available?

2. Conduction Electrons as ideal Fermi Gas
In solid aluminium, every atom gives three electrons into the conduction band. The density of the atoms is $n_{\text{atom}} = 2.7 \times 10^{23} \, \text{cm}^{-3}$.
a) Show that the Fermi temperature, $T_F = E_F/k_B$, is much higher than the room temperature (E_F is the Fermi energy).
b) If we treat the electrons as a Fermi gas at $T = 0$, what is the highest momentum p_F that is occupied?
c) How large is the Fermi pressure p_F (pressure of the electron gas at $T = 0$)?

3. Critical Values for the Bose-Einstein-Condensation
Bose-Einstein condensation of an ideal gas was first achieved by cooling a gas of rubidium atoms (atomic weight 85.47) in a laser trap. At a temperature of roughly $T = 10^{-7} \, \text{K}$, the transition to the new state was reached. Which rubidium isotope can be used? What was the minimum average density in the trap at the transition to a BEC? If the trap roughly contains 10.000 atoms, how large is the volume occupied by the BEC?

4. Comic Microwave Background as Blackbody Radiation
Find the temperature that fits the Planck spectrum of the cosmic microwave background. Derive at which frequency this radiation field has its maximum intensity. Why is this result changing over (very!) long times?