

Fast sausage modes in transversely continuous coronal tubes

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Thanks to

S.-X. Chen, H. Yu, M.-Z. Guo @ Shandong U.
M. Xiong @ National Space Science Center

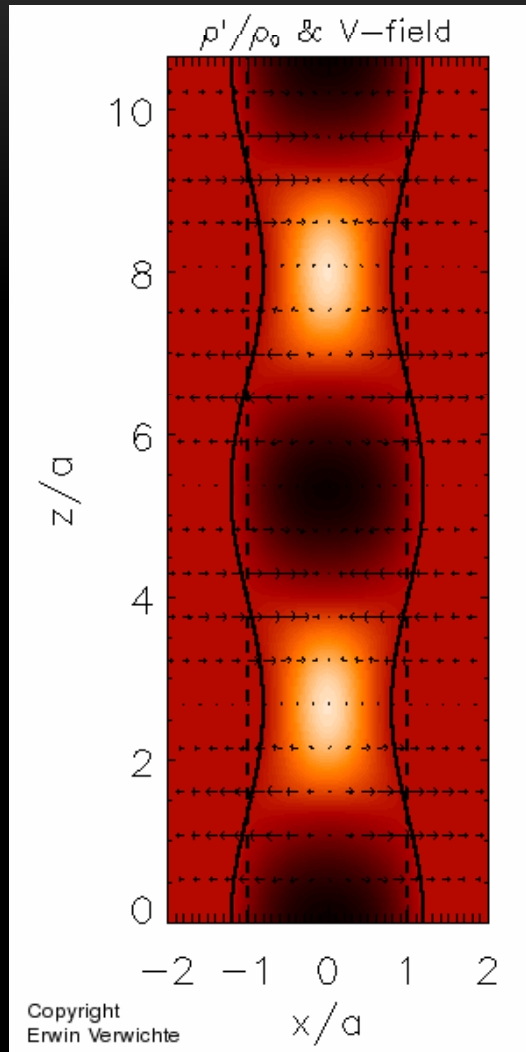
ISSI-BJ, 16-20 Jan 2016

Fast sausage modes in transversely continuous coronal tubes

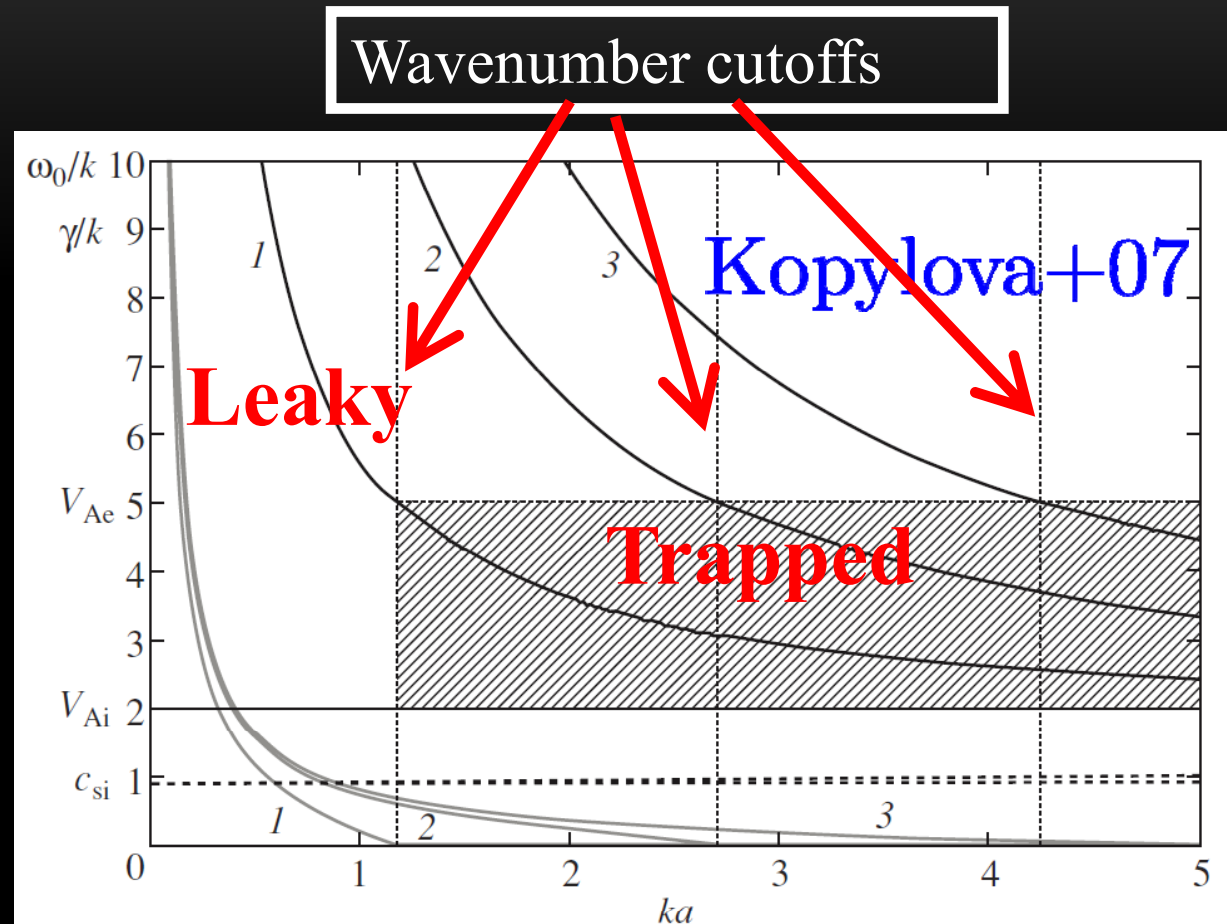
Contents

- Brief Intro to sausage modes
- Standing sausage modes in flare loops & their seismological applications
- Impulsively generated sausage wave trains in coronal tubes & implications for seismology

Fast sausage modes in tubes



Stationary Prop. Waves
 [Nakariakov & Verwichte 05
 LRSP]

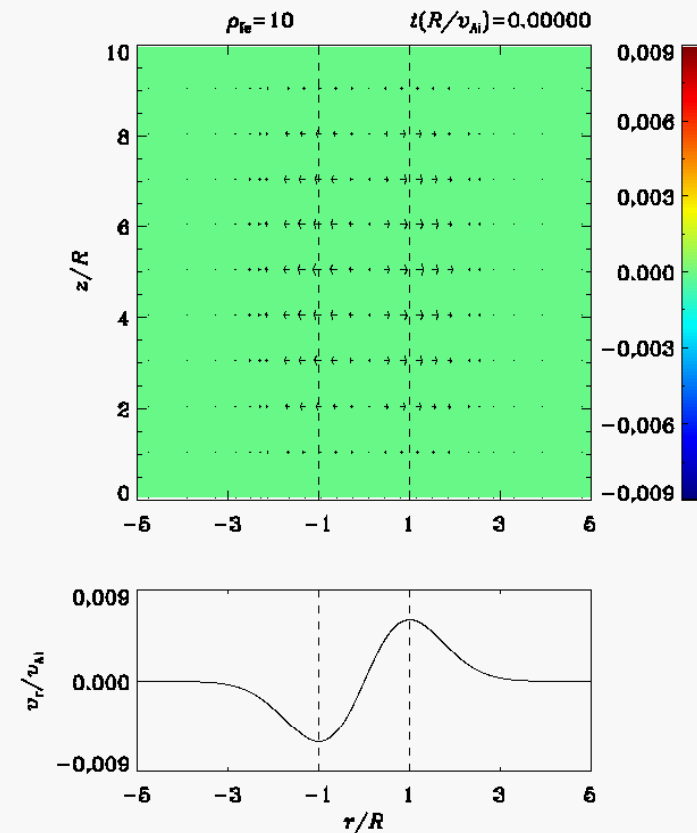
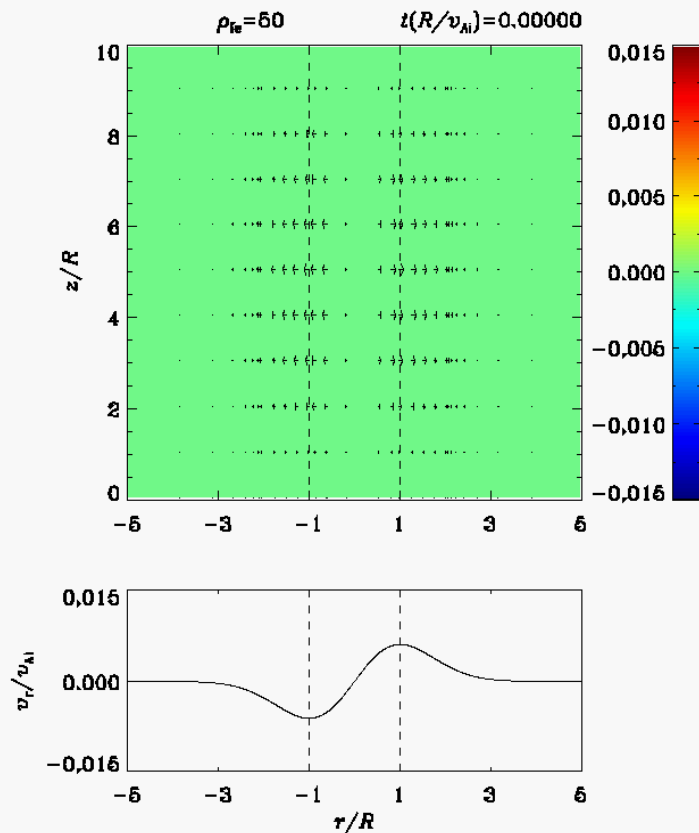


Phase speeds: real part (thick), imag. (thin)
 [Rosenberg 70; Zaitsev & Stepanov 75;
 Edwin & Roberts 83; Cally 86; ...]

Fast standing modes: an initial-value-problem perspective

trapped

leaky

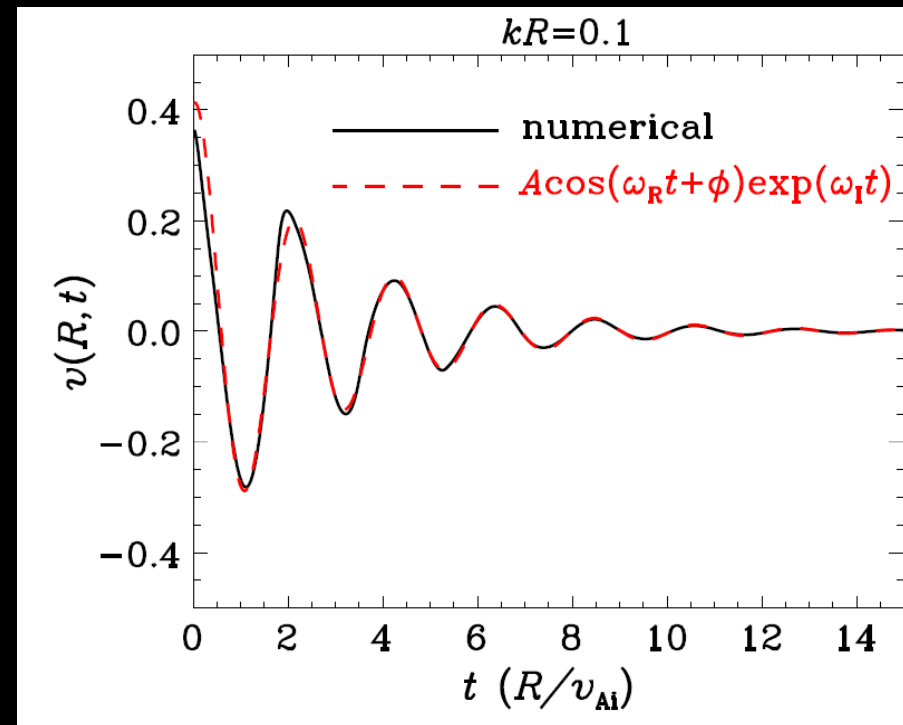
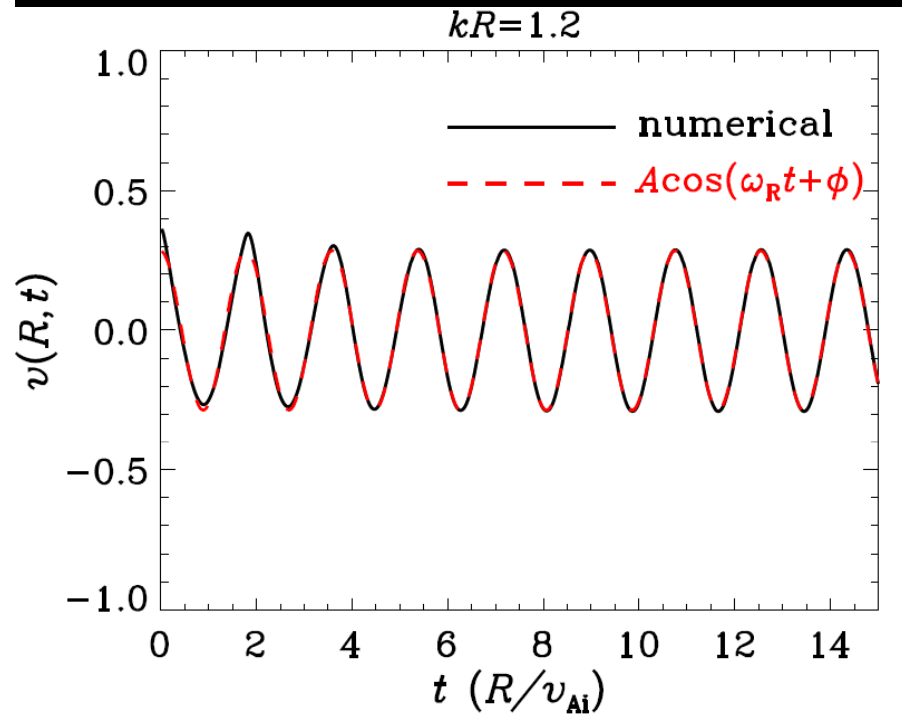


Upper row: Filled contours \rightarrow density pert.; vectors \rightarrow vel. Field
Lower row: radial distribution of radial velocity

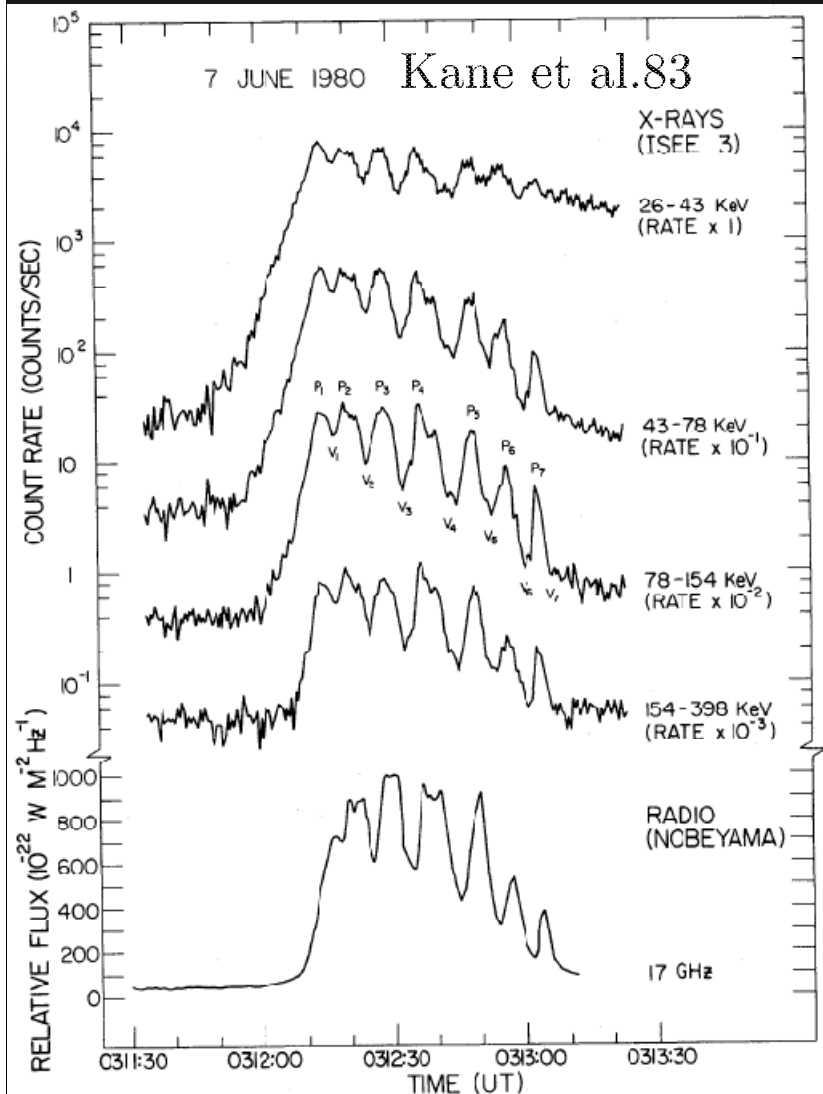
Fast standing modes: an IVP approach

trapped

leaky

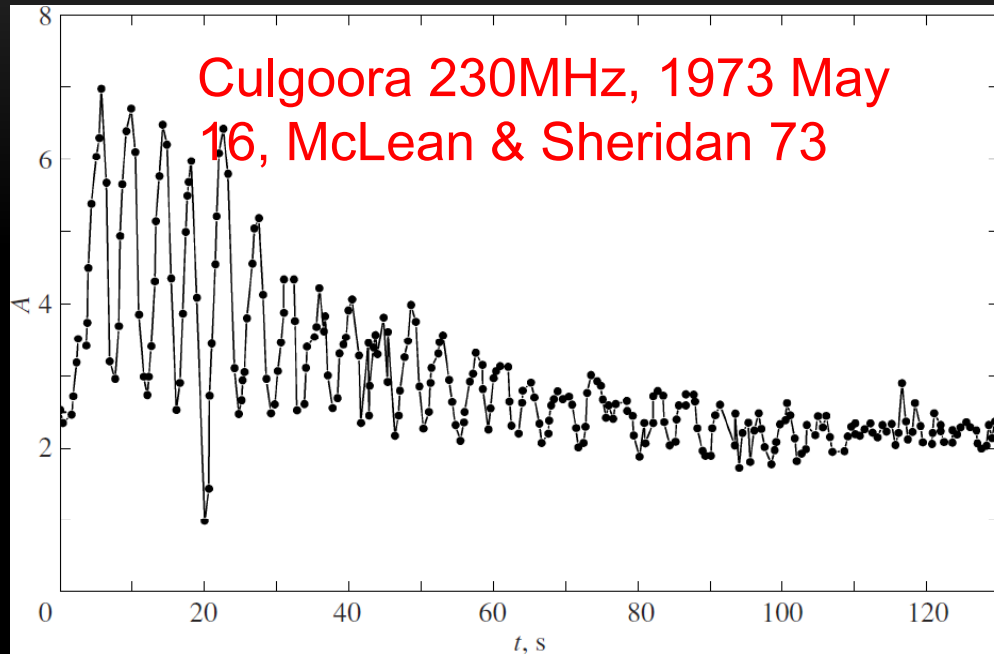


Quasi-periodic pulsations(QPPs) in solar flare lightcurves



- Discovered in late 1960's [Parks & Winckler 69, Frost 69, Rosenberg 70, ...]
- Seen in all phases, in nearly all (? Inglis+16) flares [Nakariakov+09, Van Doorselare+16]
- Imaging measurements possible
 - NoRH [e.g., Asai+01, Nakariakov+03, Kolotkov+15]
 - SDO/AIA [e.g., Su+12, Li, Ning+16]
 - IRIS [Tian+16]
- Standing sausage modes → QPPs with periods ~ secs [e.g., Aschwanden+04]

Inferring flare loop parameters with QPPs



Kopylova+07

$$P \approx \frac{2.6a}{v_{Ai}} \quad \tau/P \approx \frac{\rho_0/\rho_e}{\pi^2}$$

$$P \approx 4.3\text{s}, \quad \tau/P \approx 10$$

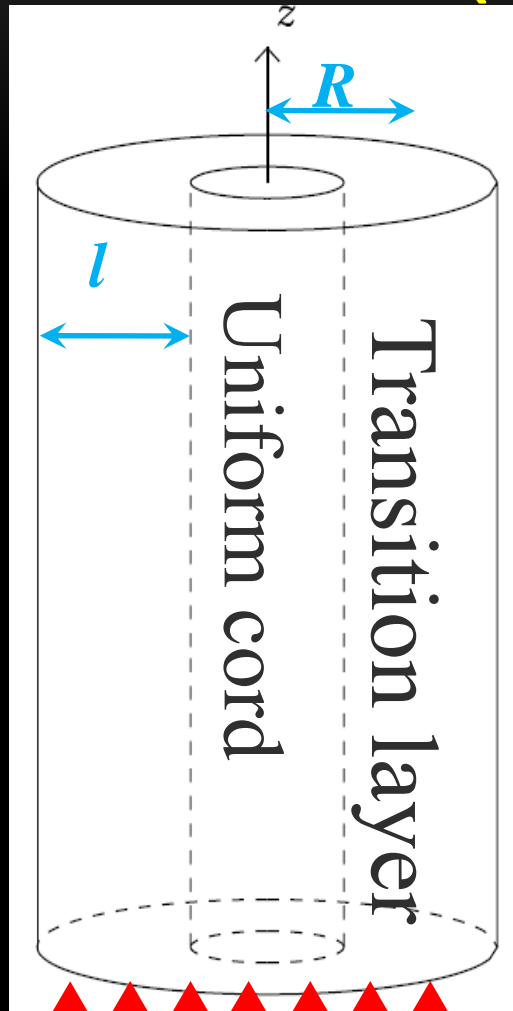
$$a/v_{Ai} \approx 1.6\text{s}, \quad \text{den ratio} \approx 10$$

- Transverse Alfvén time \rightarrow B strength
- Key assumptions
 - ~~Transverse distributions of parameters: Discontinuous~~
 - (very often) gas pressure neglected

Dispersive Properties of Sausage Modes

	Transversely discontinuous	Transversely continuous
beta = 0	<p>Eigenmode analysis: Rosenberg 70; Zaitsev & Stepanov 75; Meerson+78; Spruit 82; Cally 86; Vashegani Farahani+14; ...</p> <p>Initial-value-problem (IVP) Terradas+07</p>	<p>Eigenmode analysis: Pneuman 65; Lopin & Nagorny 14, 15; Chen+15a, Yu+16; summarized in Yu+17 ApJ</p> <p>IVP Nakariakov+12; Chen+15a,b; Guo+16</p>
beta ≠ 0	<p>Eigenmode analysis: Edwin & Roberts 83; Kopylova+07; ...</p>	<p>Eigenmode analysis: Chen+16</p> <p>Initial-value-problem (IVP) Chen+16</p>

Transversely continuous density profile (but still pressureless)

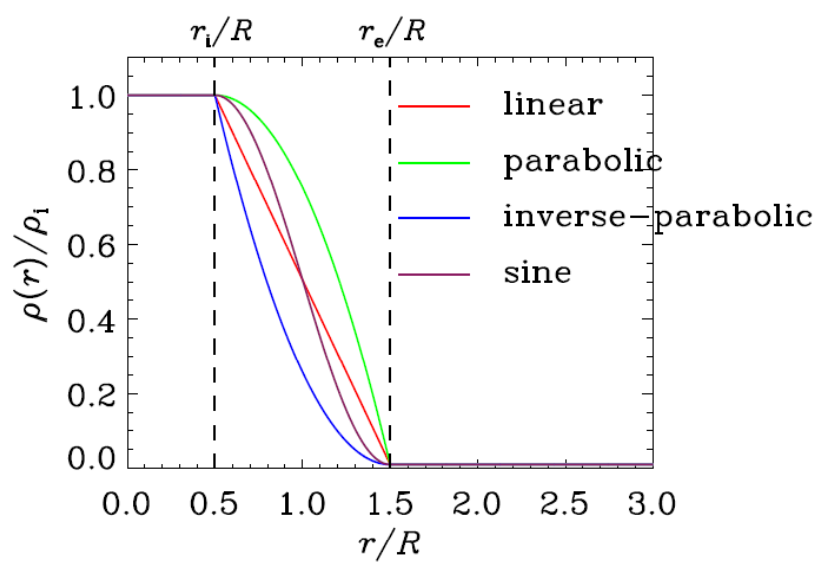


- Analytical DR [Chen+15], applicable to
- arbitrary transition layer thickness ($0, 2R$)
 - arbitrary profile prescription in the TL

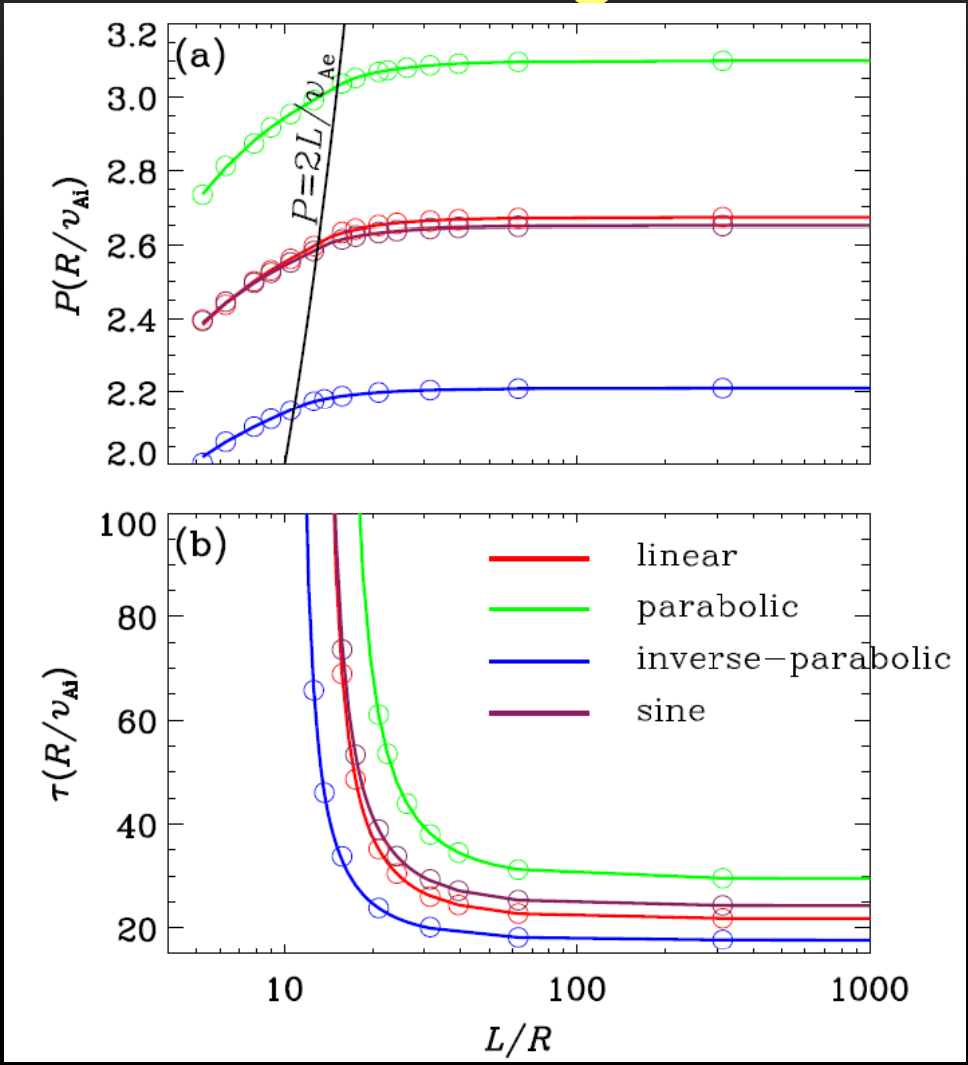
$$\rho_{\text{tr}}(r) = \begin{cases} \rho_i - \frac{\rho_i - \rho_e}{l} \left(r - R + \frac{l}{2} \right), & \text{linear,} \\ \rho_i - \frac{\rho_i - \rho_e}{l^2} \left(r - R + \frac{l}{2} \right)^2, & \text{parabolic,} \\ \rho_e - \frac{\rho_e - \rho_i}{l^2} \left(r - R - \frac{l}{2} \right)^2, & \text{inverse - parabolic,} \\ \frac{\rho_i}{2} \left[\left(1 + \frac{\rho_e}{\rho_i} \right) - \left(1 - \frac{\rho_e}{\rho_i} \right) \sin \frac{\pi(r - R)}{l} \right], & \text{sine.} \end{cases}$$

\vec{B} — Uniform external medium

Dispersive properties of sausage modes



$$k = \pi / L$$



Chen, Li+2015 ApJ, 812, 22

- Overall, similar to top-hat case

Re-analysis of the Mclean & Sheridan event

$$P_{\text{saus}} = \frac{R}{v_{\text{Ai}}} F_{\text{saus}} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_i}{\rho_e} \right),$$

$$\frac{\tau_{\text{saus}}}{P_{\text{saus}}} = G_{\text{saus}} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_i}{\rho_e} \right).$$

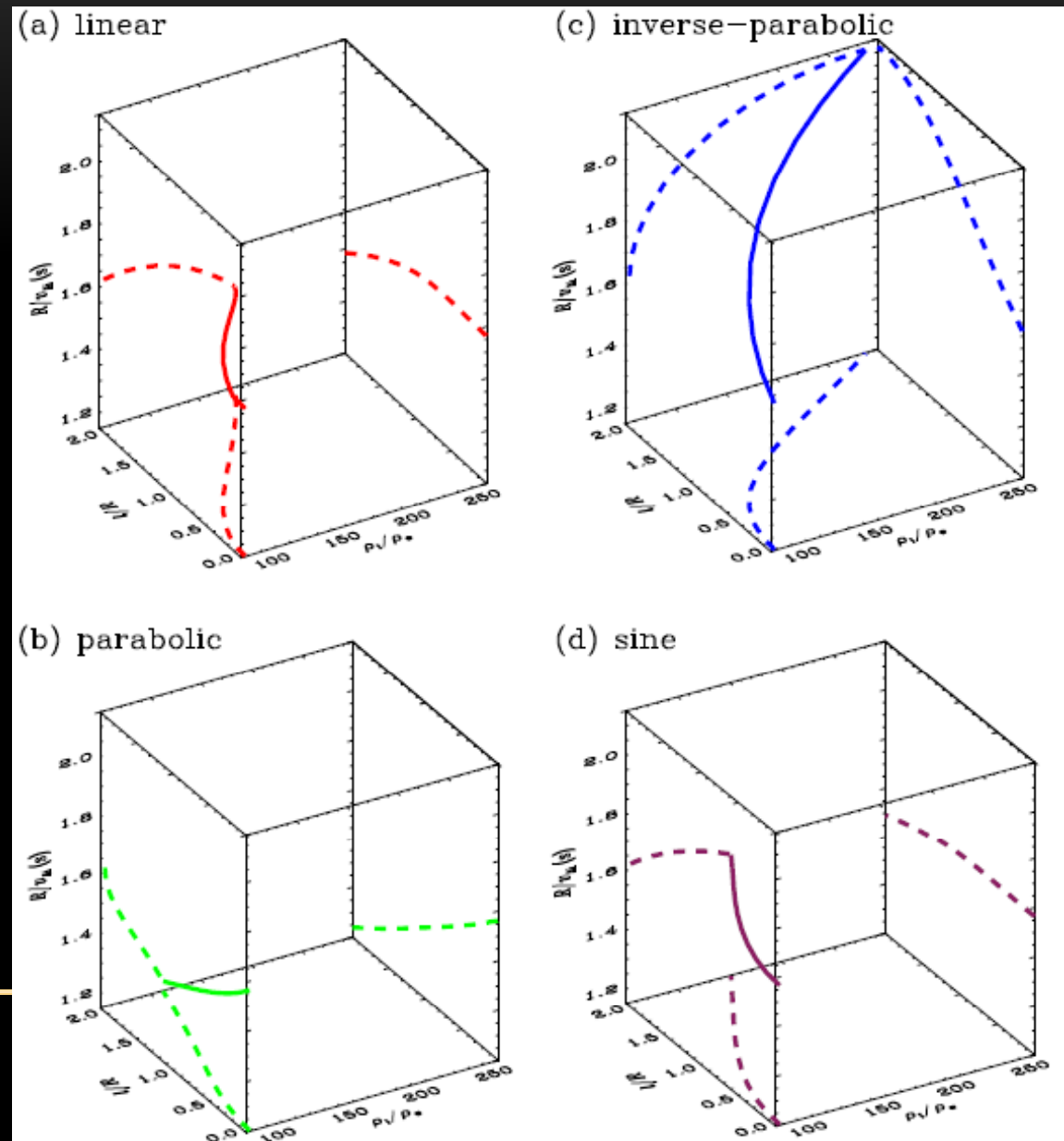
$$k = \pi / L$$

Chen+15 ApJ, 812, 22

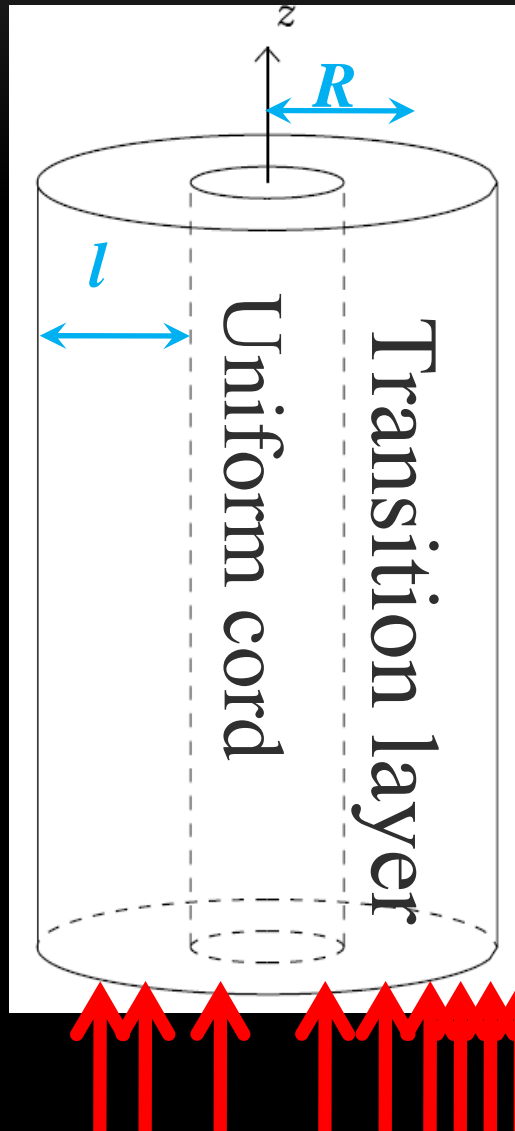
- Deduced transverse Alfvén time: Max/Min = 1.8
- Not possible to constrain TL width

Guo+16, 2016 SoPh, 291, 877

- Possible to improve, if QPPs spatially resolved & involve multiple modes



Finite gas pressure & continuous distribution



Gas/magnetic pressure may reach unity

- Hot Active Region Loops [SUMER, Wang+07,...]
- Hot & Dense flare loops [Nakariakov+03, Melnikov+05]

$$\rho(r) = \begin{cases} \rho_i, & 0 \leq r \leq r_i = R - l/2, \\ \rho_{tr}(r) = \mathcal{F}(\rho_i, \rho_e; r), & r_i \leq r \leq r_e = R + l/2, \\ \rho_e, & r \geq r_e, \end{cases}$$

$$T(r) = \begin{cases} T_i, & 0 \leq r \leq r_i, \\ T_{tr}(r) = \mathcal{F}(T_i, T_e; r), & r_i \leq r \leq r_e, \\ T_e, & r \geq r_e. \end{cases}$$

[Chen, Li, et al. 2016 ApJ 833, 114]

Uniform external
 B
medium

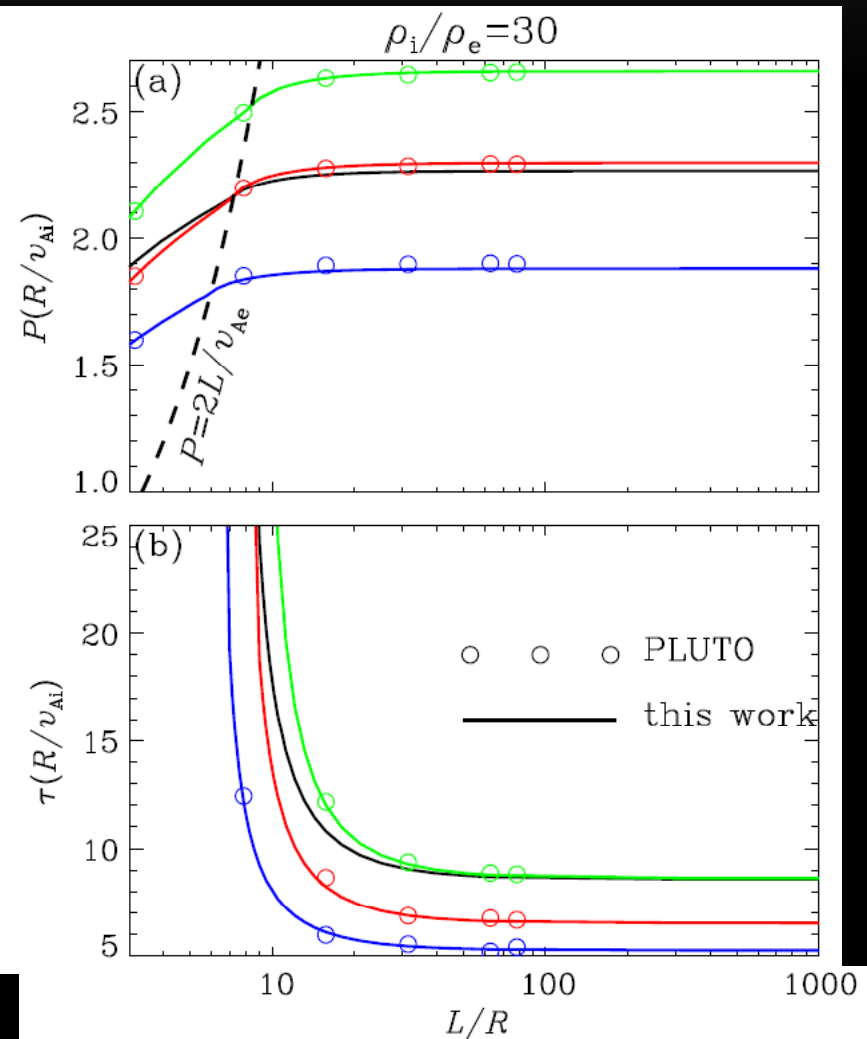
$$\frac{\frac{\rho_i J_0(\mu_i r_i)(\omega^2 - k^2 v_{Ai}^2)}{\mu_i r_i J_1(\mu_i r_i)} y_1(x_i) + \Lambda_i y_1'(x_i)}{\frac{\rho_i J_0(\mu_i r_i)(\omega^2 - k^2 v_{Ai}^2)}{\mu_i r_i J_1(\mu_i r_i)} y_2(x_i) + \Lambda_i y_2'(x_i)} - \frac{\frac{\rho_e H_0^{(1)}(\mu_e r_e)(\omega^2 - k^2 v_{Ae}^2)}{\mu_e r_e H_1^{(1)}(\mu_e r_e)} y_1(x_e) + \Lambda_e y_1'(x_e)}{\frac{\rho_e H_0^{(1)}(\mu_e r_e)(\omega^2 - k^2 v_{Ae}^2)}{\mu_e r_e H_1^{(1)}(\mu_e r_e)} y_2(x_e) + \Lambda_e y_2'(x_e)} = 0 ,$$



$$\frac{\omega R}{v_{Ai}} = \mathcal{G} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_i}{\rho_e}, \beta_i, \beta_e \right)$$

$$\beta_i = 0.5, \beta_e = 0.01, l/R = 1.0$$

- step
- linear
- parabolic
- inverse-parabolic

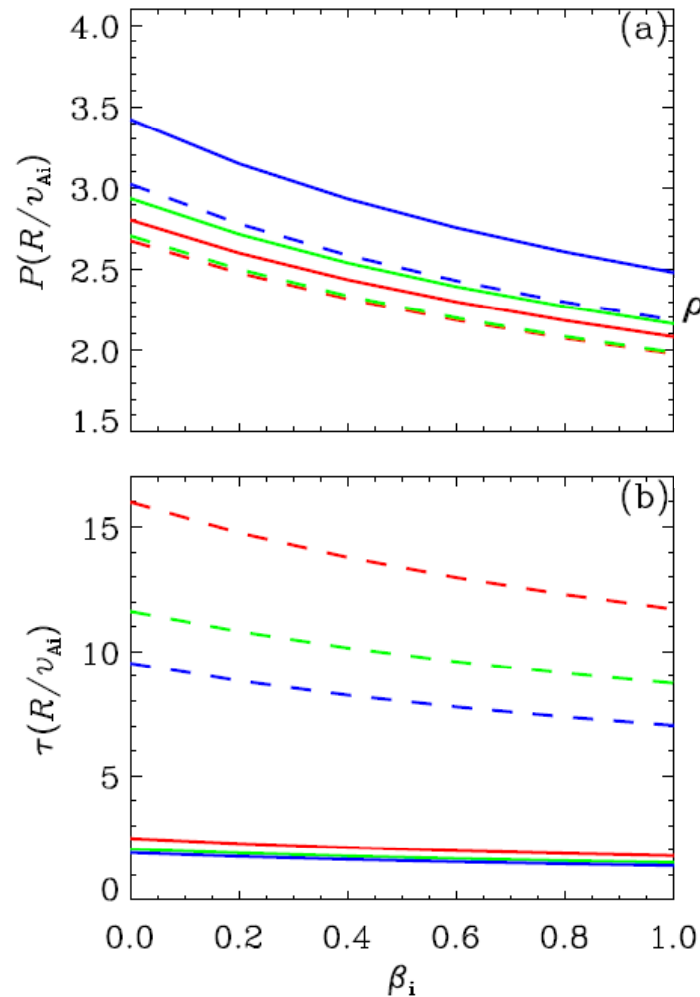


Importance of units of P & tau

$$\frac{\omega R}{v_{Ai}} = \mathcal{G} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_i}{\rho_e}, \beta_i, \beta_e \right)$$

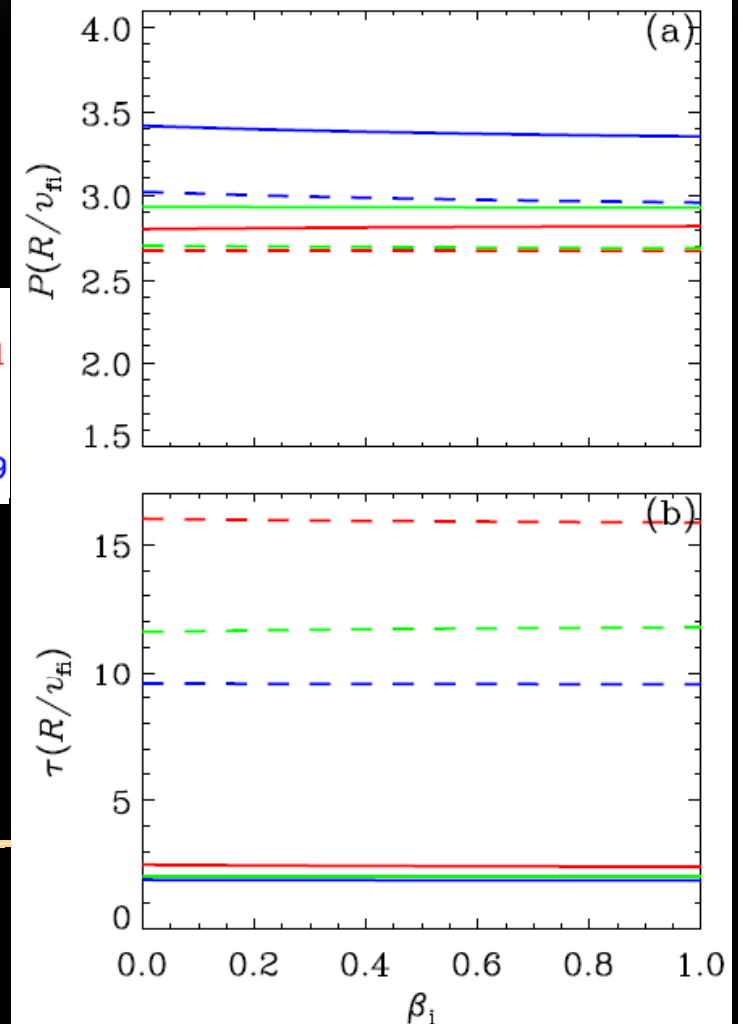
$$\frac{\omega R}{v_{fi}} = \mathcal{H} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_i}{\rho_e}, \beta_i, \beta_e \right)$$

linear



$$v_f^2 = c_s^2 + v_A^2$$

linear



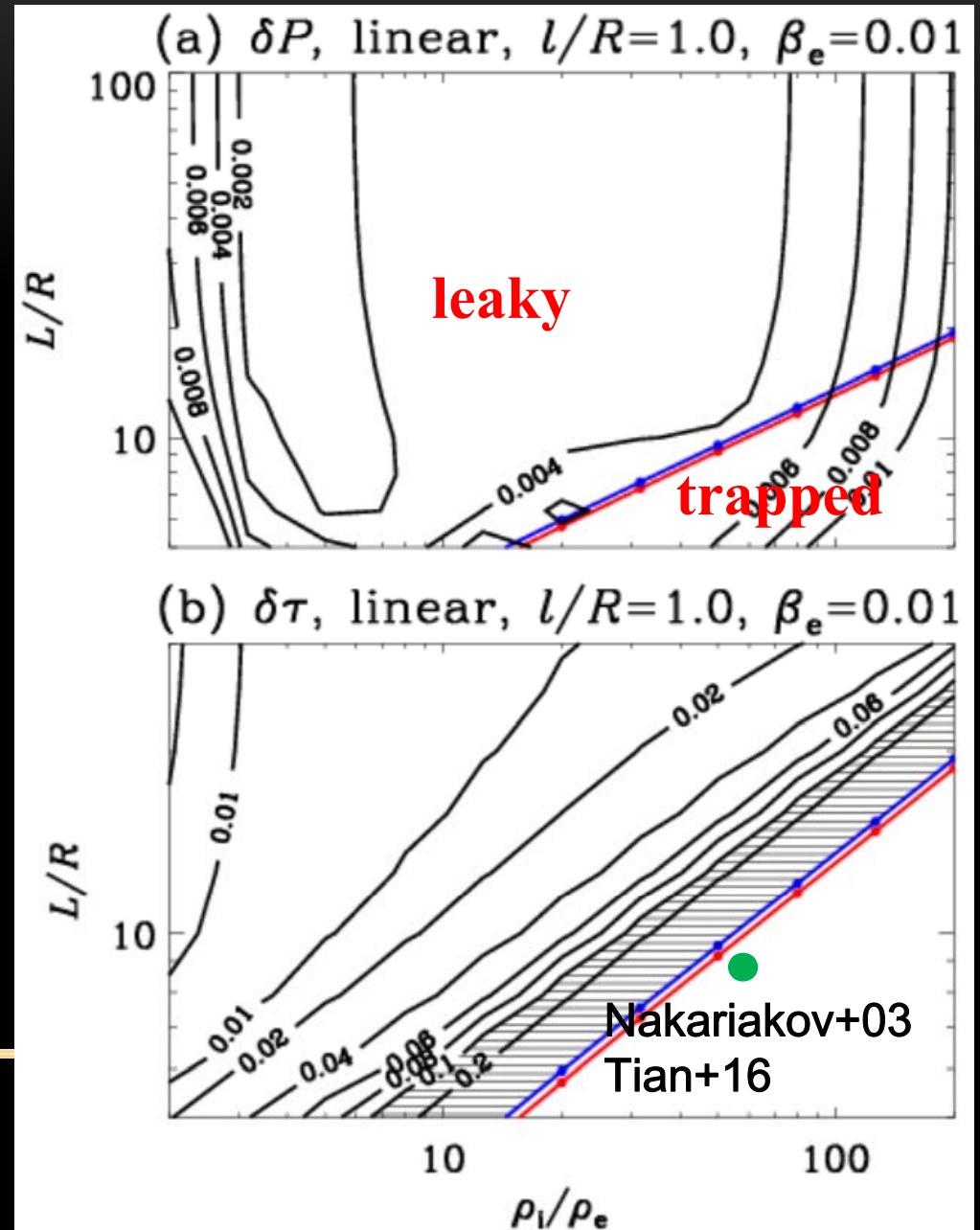
Finite vs. zero gas pressure

$$\frac{\omega R}{v_{fi}} = \mathcal{H} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_i}{\rho_e}, \beta_i, \beta_e \right)$$

$$\delta P = \max \left| \frac{P^{\beta \neq 0}(\beta_i \in [0, 1])}{P^{\text{cold}}} - 1 \right|$$

$$\delta \tau = \max \left| \frac{\tau^{\beta \neq 0}(\beta_i \in [0, 1])}{\tau^{\text{cold}}} - 1 \right|$$

- negligible changes to cutoff wavenumbers & periods
- Changes to damping times may be substantial

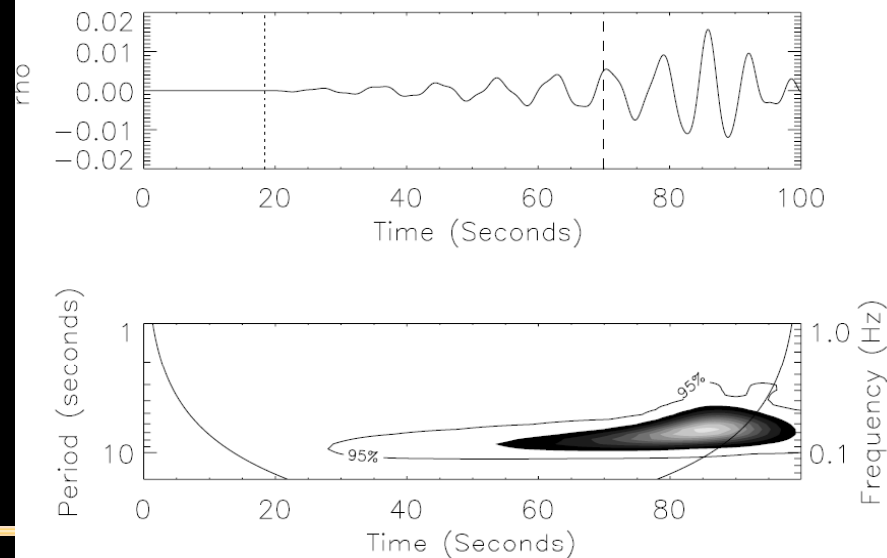
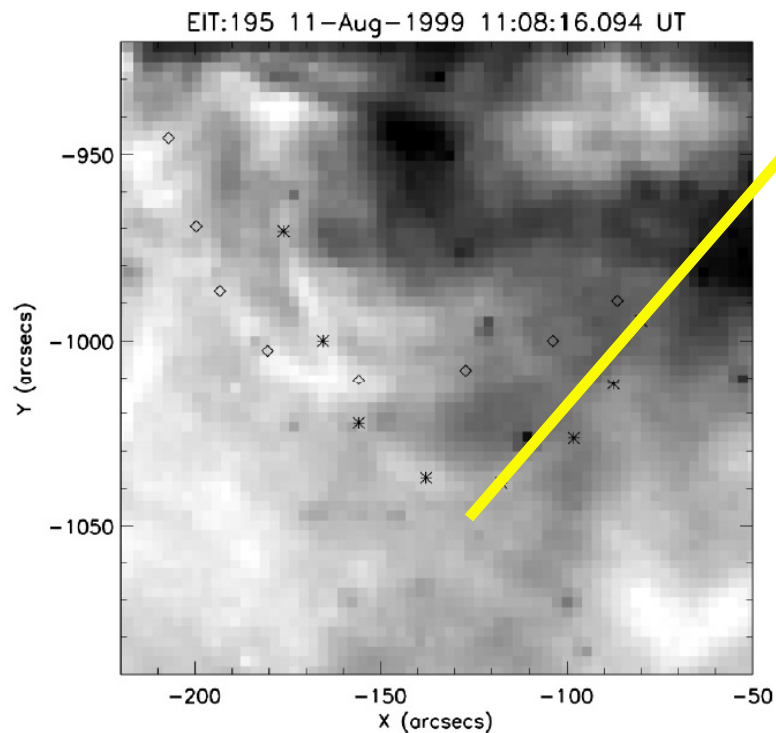
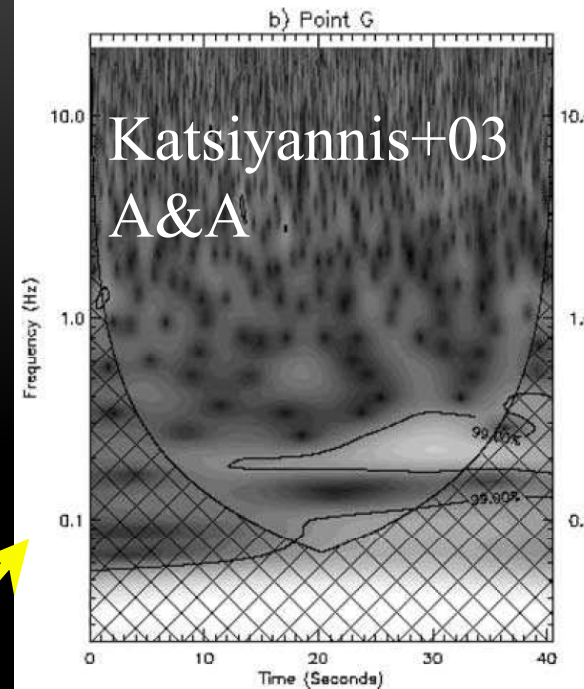


Sausage wave trains in coronal tubes

Oscillatory behavior in optical measurements of the corona

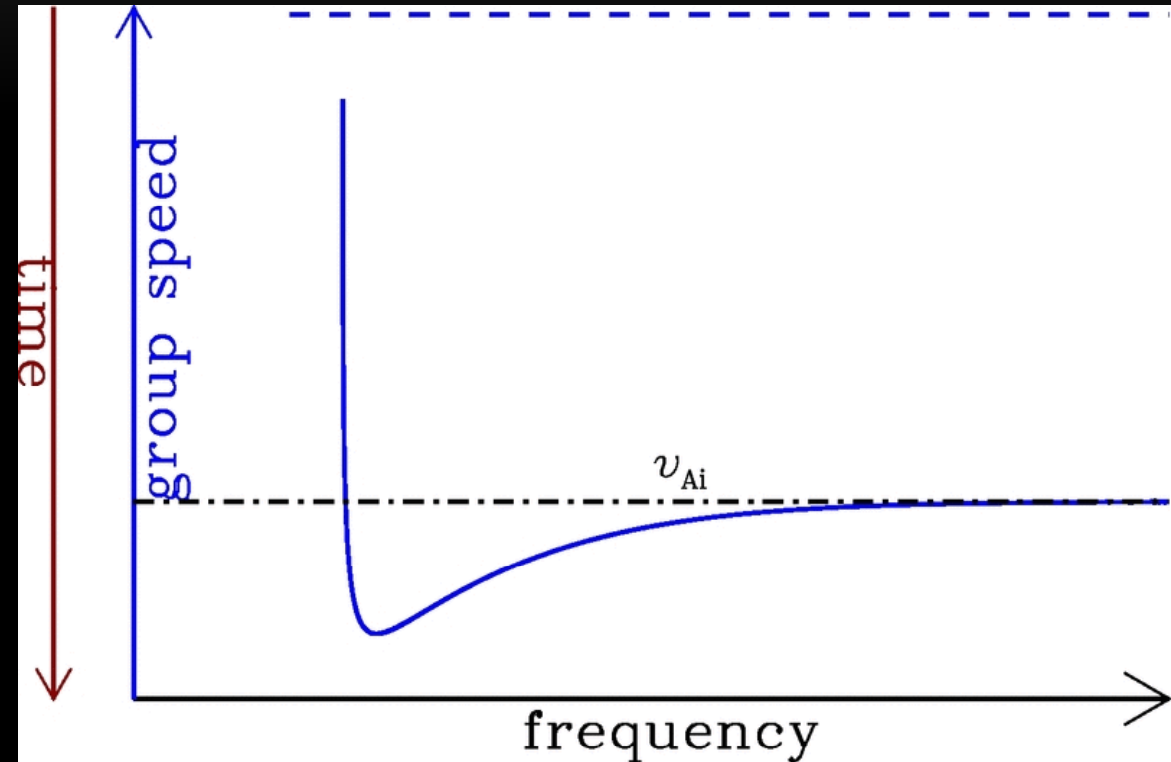
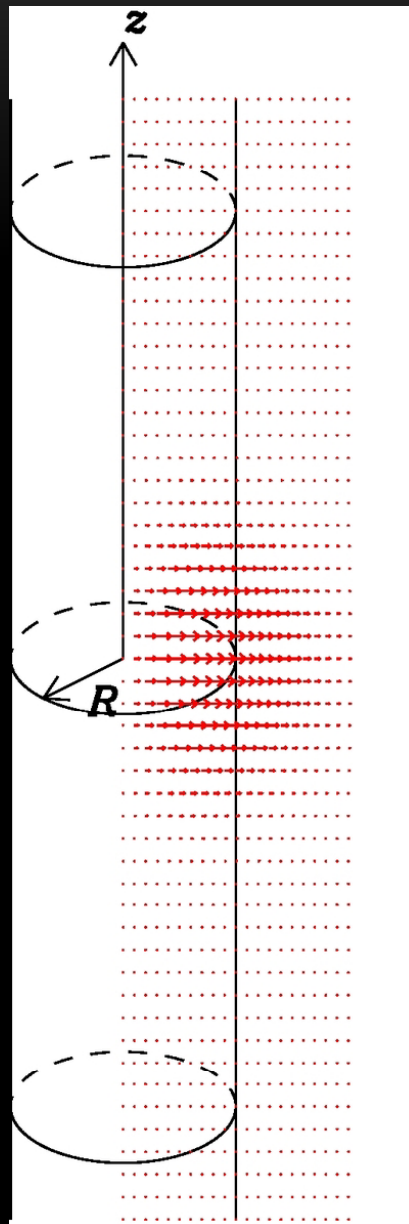
- Originated in Billings 59; Tsubaki 77... (5303; periods > minutes)
- Rapid oscillatory behavior at total eclipses
 - 5303; 0.5-2 secs (?) [Pasachoff & Landman 84; 1980 Hyderabad]
 - 5303; 0.5-4 secs (?) [Pasachoff & Ladd 87; 1983 Indonesian]
 -
 - 5303; 6 secs [Williams+01, 02; Katsiyannis+03; 1999 Bulgaria]
 - 5303 & 6374; 6-25 secs [Samanta+16, 2010 Chile]

“crazy tadpole”-like Morlet spectra

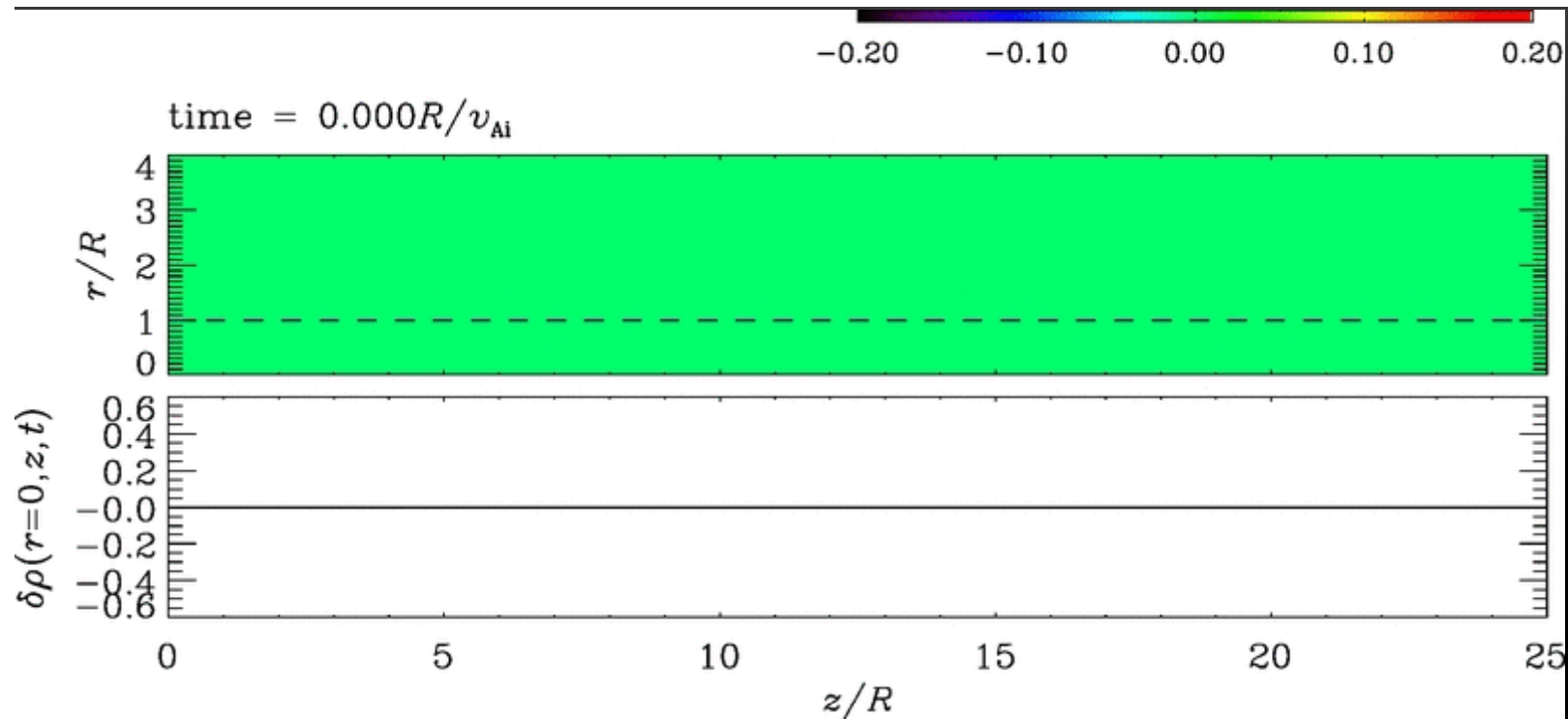


Slabs: Nakarikov+04
Tubes: Shestov+15

Interpretation in terms of sausage wave trains

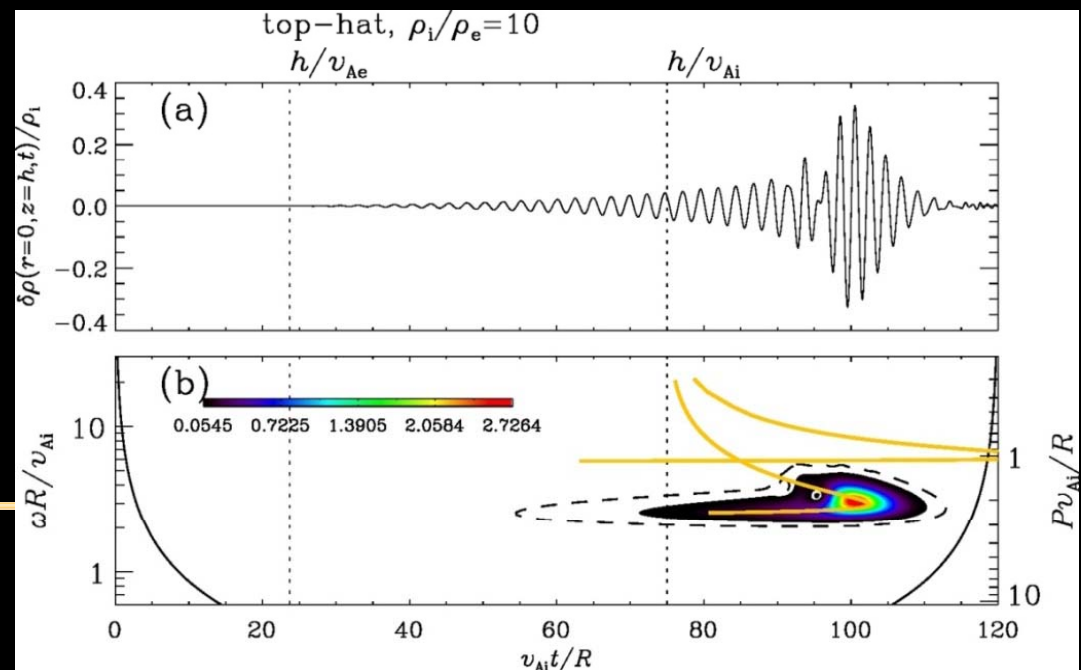


[Roberts+83, 84, Edwin & Roberts 86, 88; Nakariakov & Roberts 95, Oliver+15]

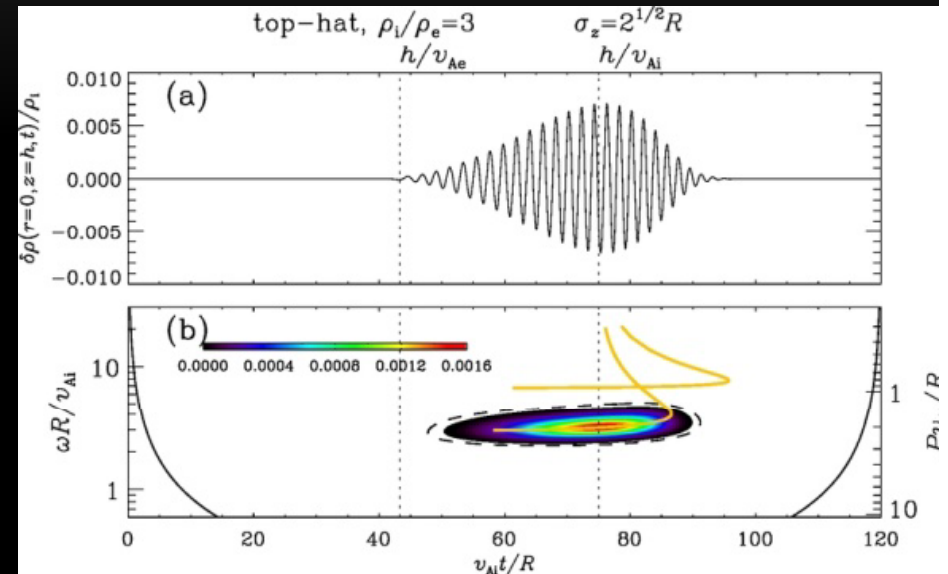
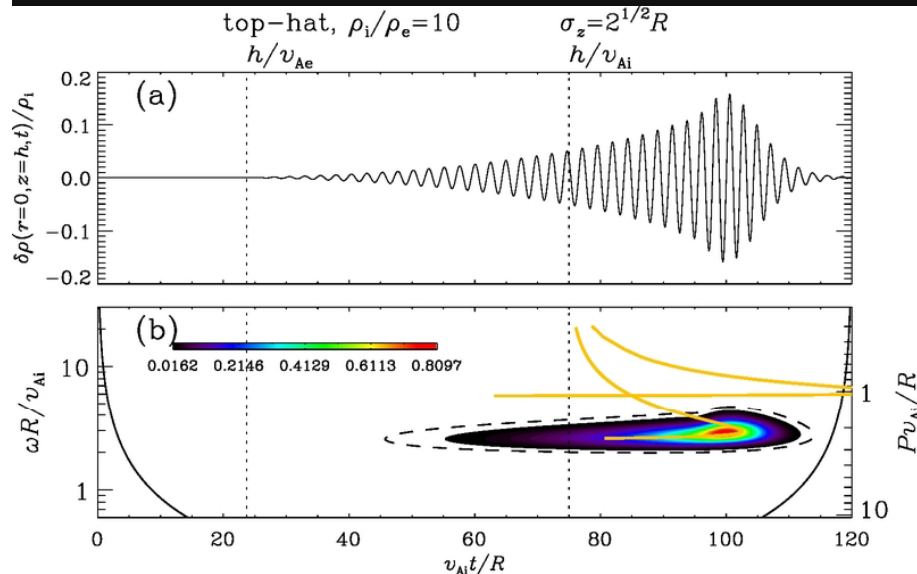


Yu, Li, et al. 2016ApJ 833, 51
 Yu, Li, et al. 2017ApJ, in press
 arXiv:1612.09479

yellow curves: $\omega - h/v_{gr}$



Spatial extent of initial perturbations



yellow curves: $\omega - h/v_{gr}$

- Spatial extent of init. pert. important
- The reasoning by Roberts & co-workers?
- Group speed curves helpful for providing the context
 - Cutoff wavenumbers
 - Whether curves are monotonical

Continuous Transverse Structuring

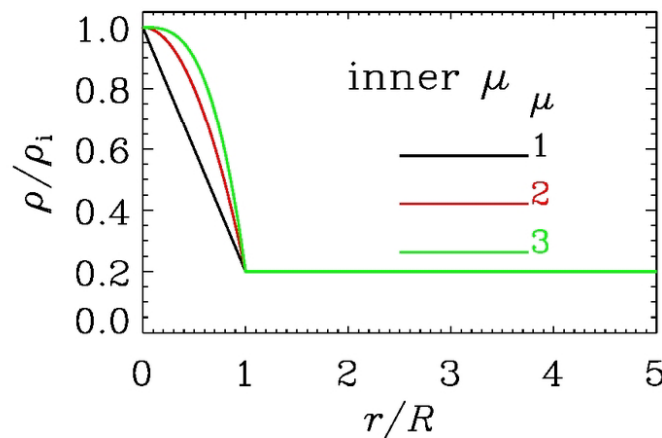
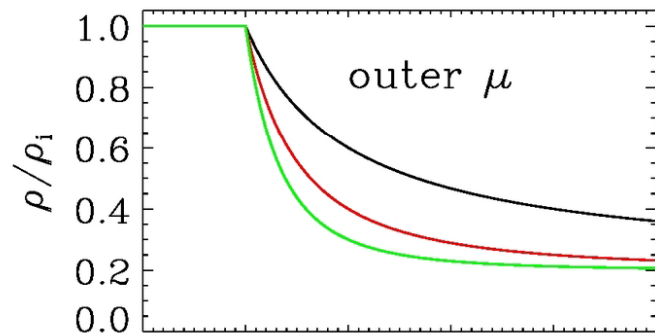
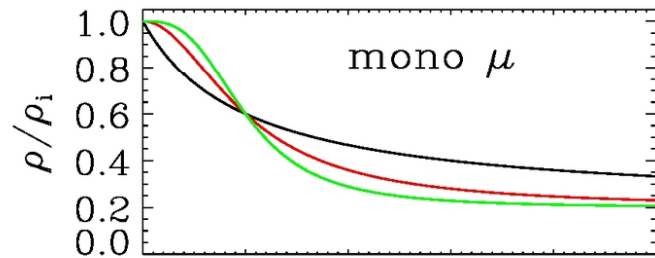
$$\rho(r) = \rho_e + (\rho_i - \rho_e)f(r)$$

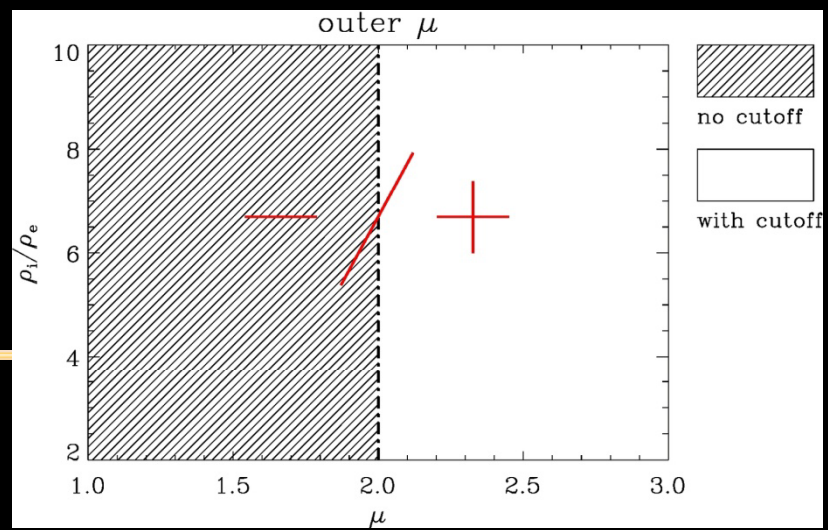
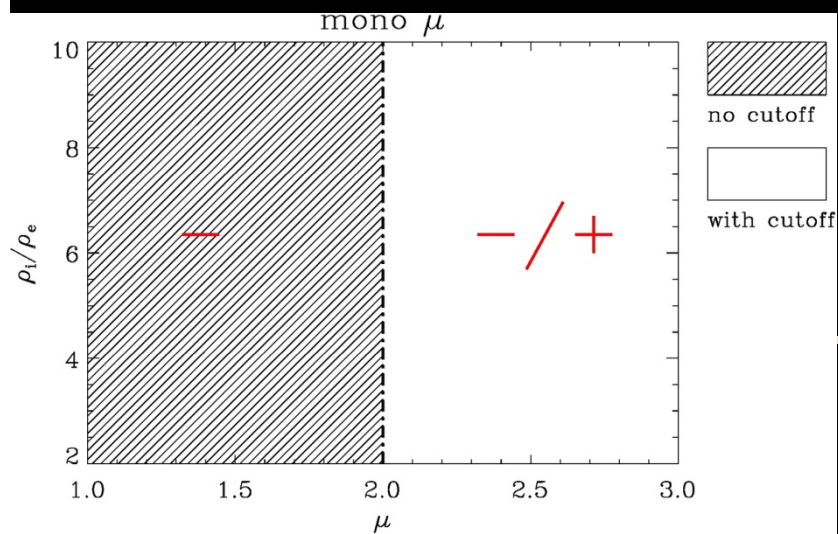
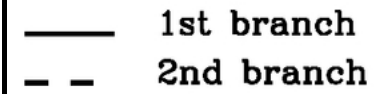
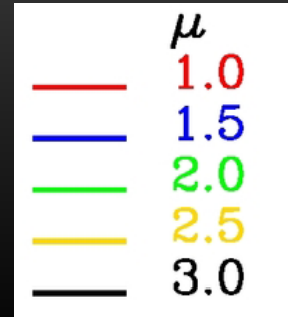
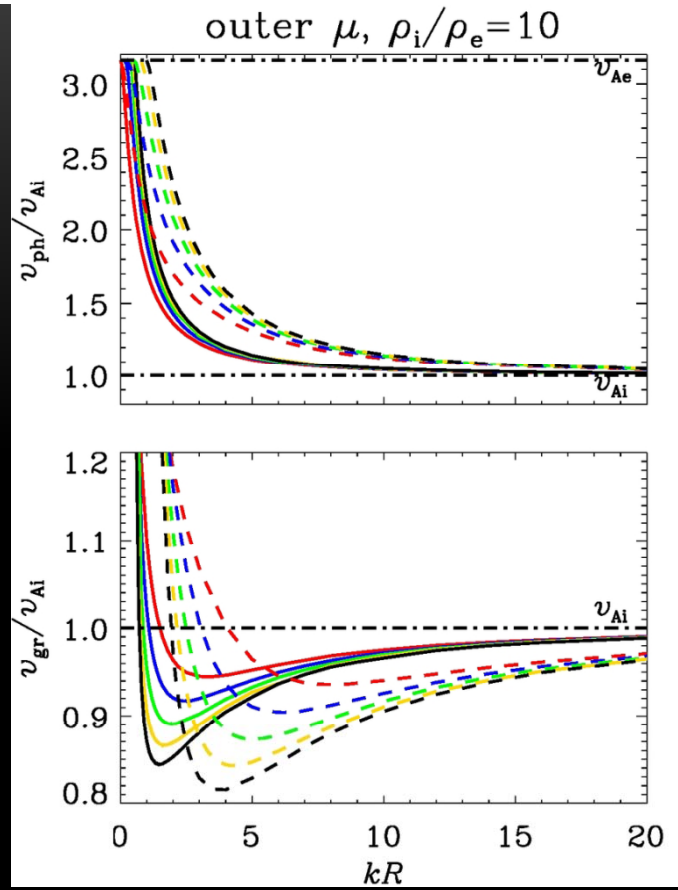
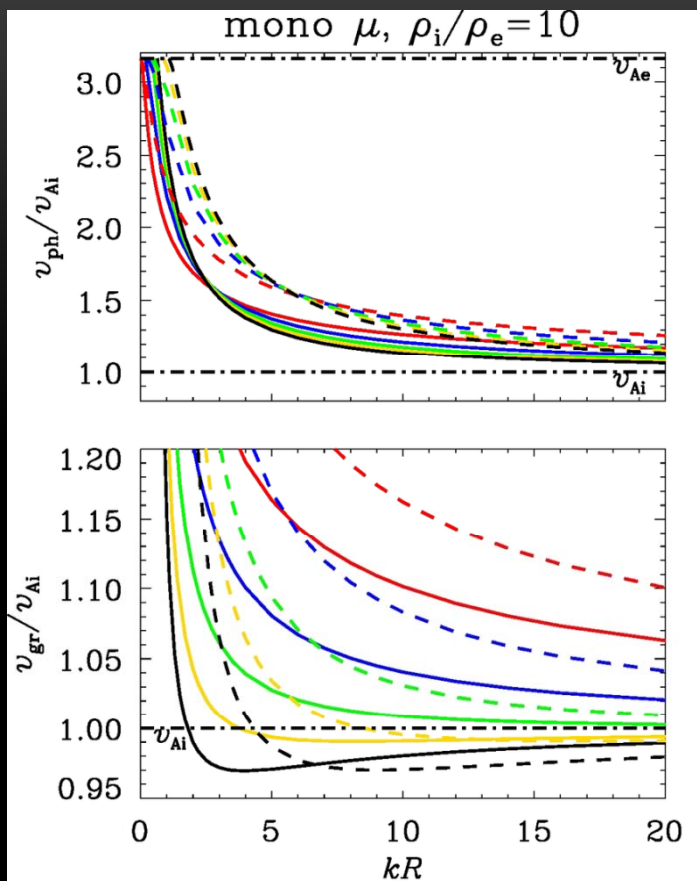
$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\tilde{\xi}}{dr} \right) + \left(\frac{\omega^2}{v_A^2} - k^2 - \frac{1}{r^2} \right) \tilde{\xi} = 0$$

$$f(r) = \frac{1}{1 + (r/R)^\mu}$$

$$f(r) = \begin{cases} 1, & 0 \leq r \leq R \\ (r/R)^{-\mu}, & r \geq R. \end{cases}$$

$$f(r) = \begin{cases} 1 - \left(\frac{r}{R} \right)^\mu, & 0 \leq r \leq R, \\ 0, & r \geq R. \end{cases}$$

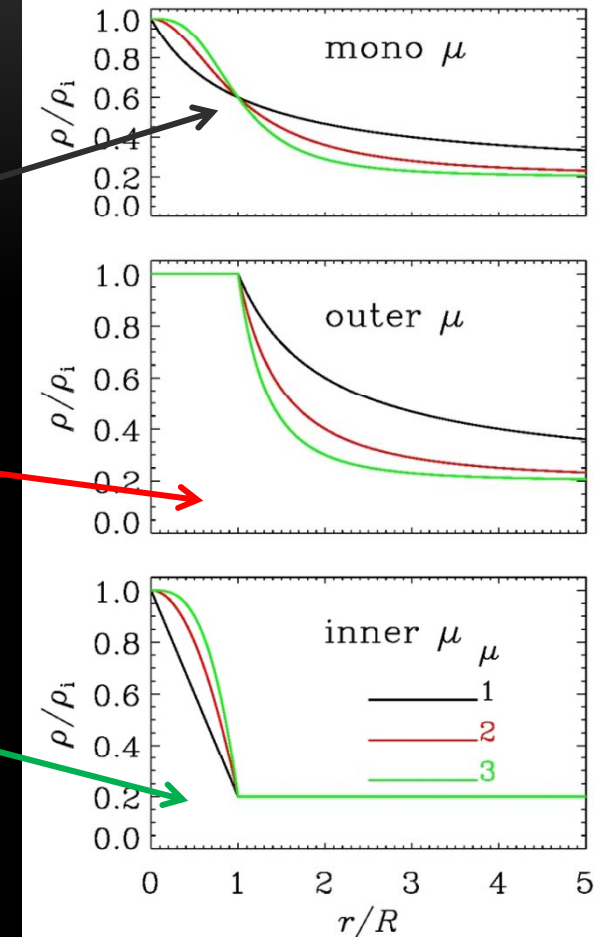
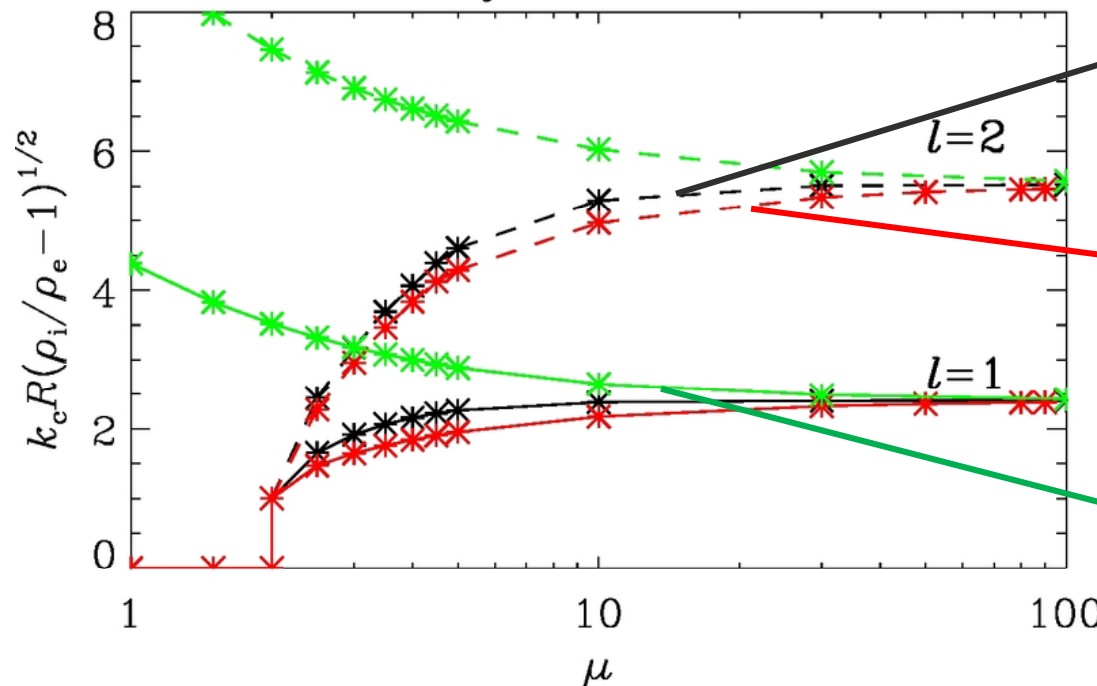




Cutoff wavenumbers

$$k_c R = \frac{d_l}{\sqrt{\rho_i/\rho_e - 1}}. \quad d_l = \text{func}(l, \mu)$$

cylindrical



- Exist only when $f(r)$ not less steep than r^{-2} (Lopin & Nagorny 15)
- When present
 - increases with l
 - but decreases with density contrast

Asymptotic k -dependence: Fact

when $kR \gg 1$

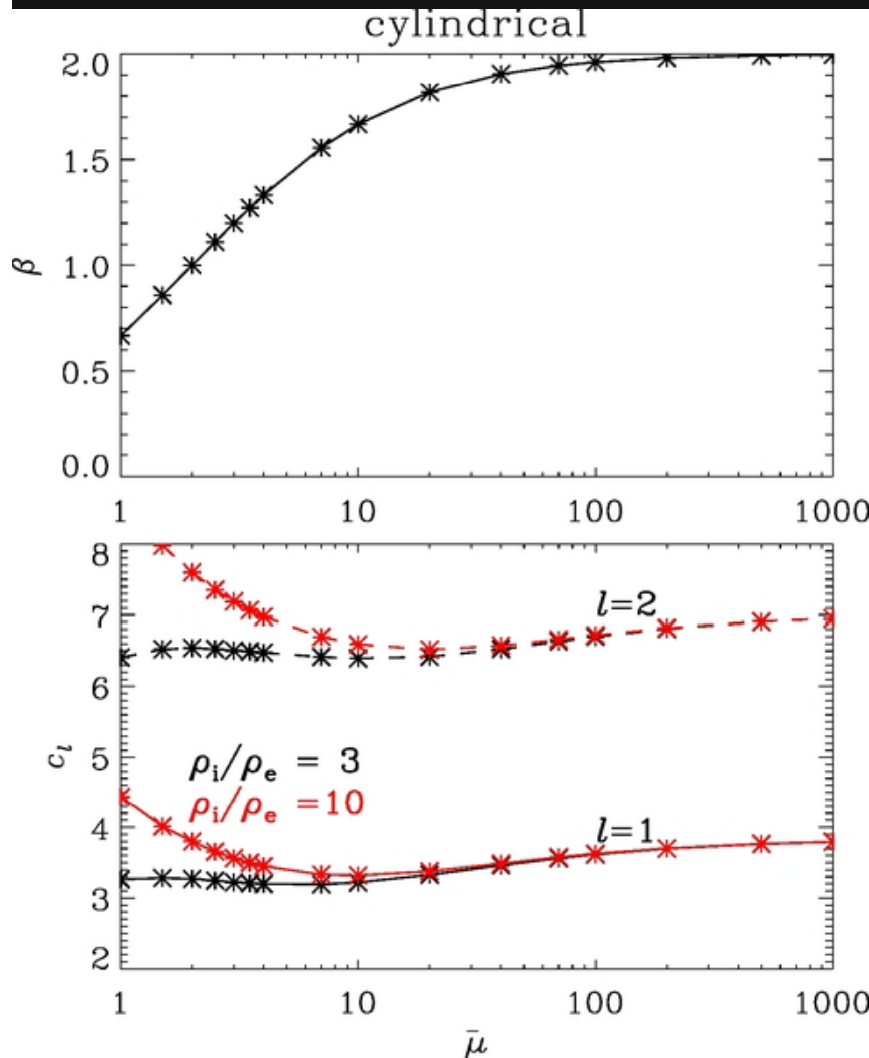
$$\frac{v_{\text{ph}}^2}{v_{\text{Ai}}^2} \approx 1 + \left(\frac{c_l}{kR}\right)^\beta, \quad \beta = \frac{2\mu}{\mu + 2}$$

$$\frac{v_{\text{gr}}^2}{v_{\text{Ai}}^2} \approx 1 + (1 - \beta) \left(\frac{c_l}{kR}\right)^\beta.$$

For both

$$f(r) = \frac{1}{1 + (r/R)^\mu}$$

$$f(r) = \begin{cases} 1 - \left(\frac{r}{R}\right)^\mu, & 0 \leq r \leq R, \\ 0, & r \geq R. \end{cases}$$

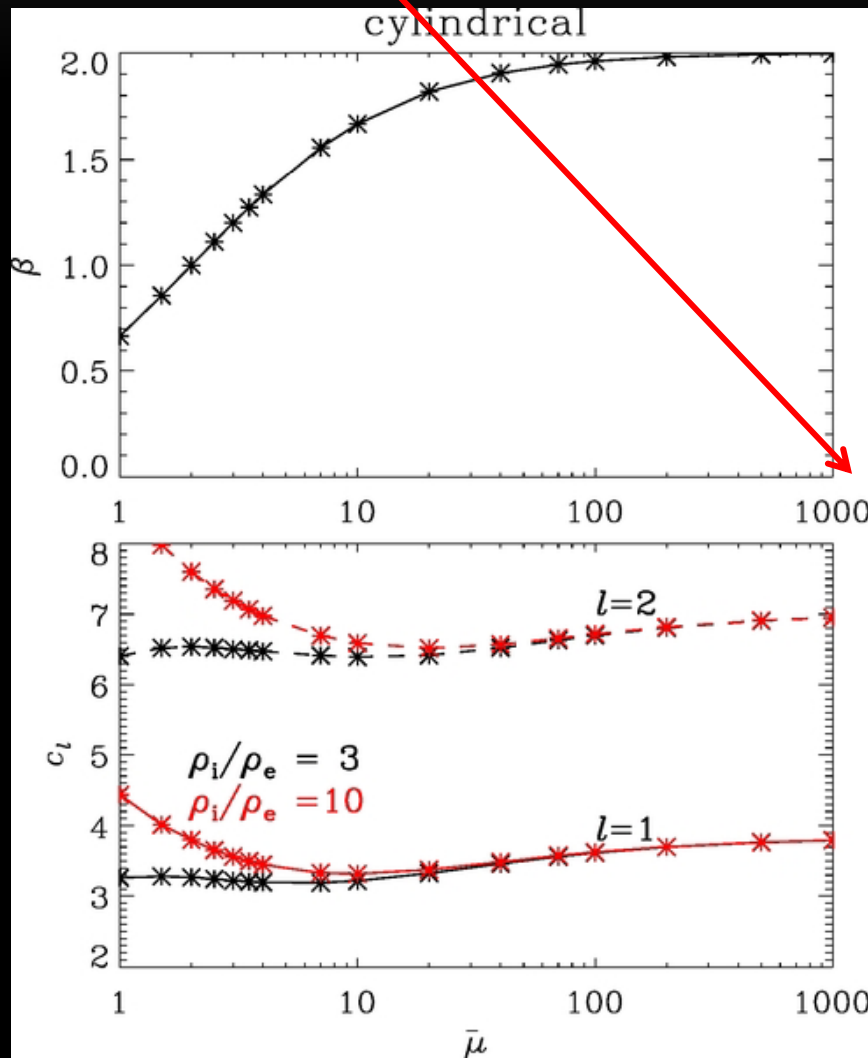


Asymptotic k -dependence: Conjecture

Conjecture 1 *When $kR \gg 1$, the phase speed for arbitrary density contrast $\rho_i/\rho_e > 1$ or transverse harmonic number l is, to leading order, given by $v_{\text{ph}} \approx 1 + [c_l/(kR)]^\beta$ for any $f(r)$ that reads, to leading order, $1 - (r/R)^{\bar{\mu}}$ when $r/R \ll 1$. Here $\beta = 2\bar{\mu}/(\bar{\mu} + 2)$, and c_l depends on ρ_i/ρ_e as well as $\bar{\mu}$.*

Support for this conjecture

$$f(r) \approx 1 - (r/R)^\infty \text{ for } r/R \ll 1$$



$$f(r) = \begin{cases} 1, & 0 \leq r \leq R, \\ 0, & r \geq R. \end{cases}$$

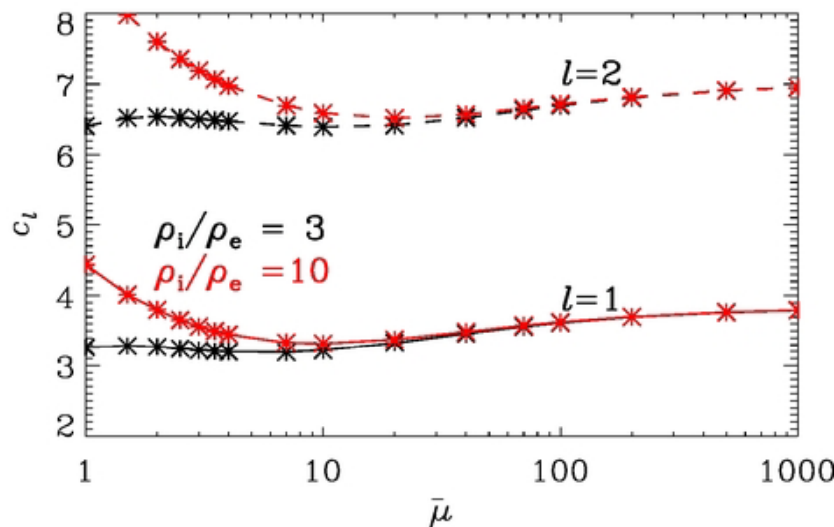
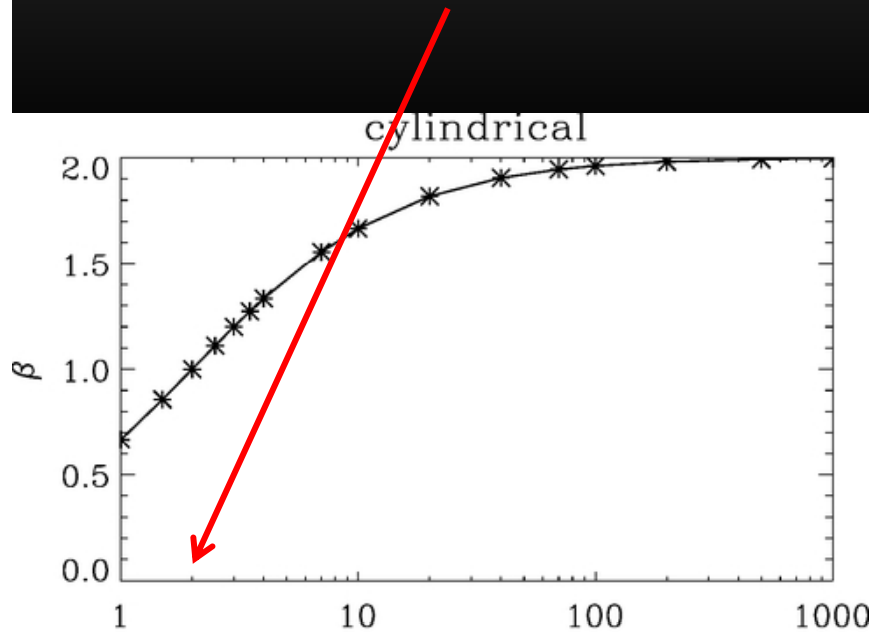
$$\frac{v_{\text{ph}}^2}{v_{\text{Ai}}^2} \approx 1 + \frac{j_{1,l}^2}{k^2 R^2}$$

also holds for (μ arbitrary)

$$f(r) = \begin{cases} 1, & 0 \leq r \leq R, \\ (r/R)^{-\mu}, & r \geq R. \end{cases}$$

Support for this conjecture

$$f(r) \approx 1 - (r/R)^2 \text{ for } r/R \ll 1$$



$$f(r) = \begin{cases} 1 - \left(\frac{r}{R}\right)^2, & 0 \leq r \leq R, \\ 0, & r \geq R. \end{cases}$$

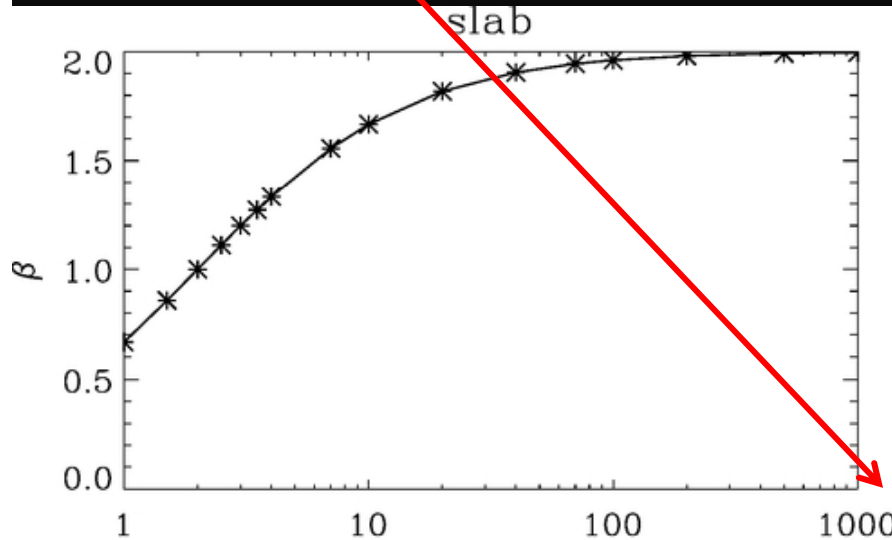
$$\frac{v_{\text{ph}}}{v_{\text{Ai}}} \approx 1 + \frac{2l\sqrt{1 - \rho_e/\rho_i}}{kR} + \frac{2l^2(1 - \rho_e/\rho_i)}{(kR)^2},$$

Further support for this conjecture

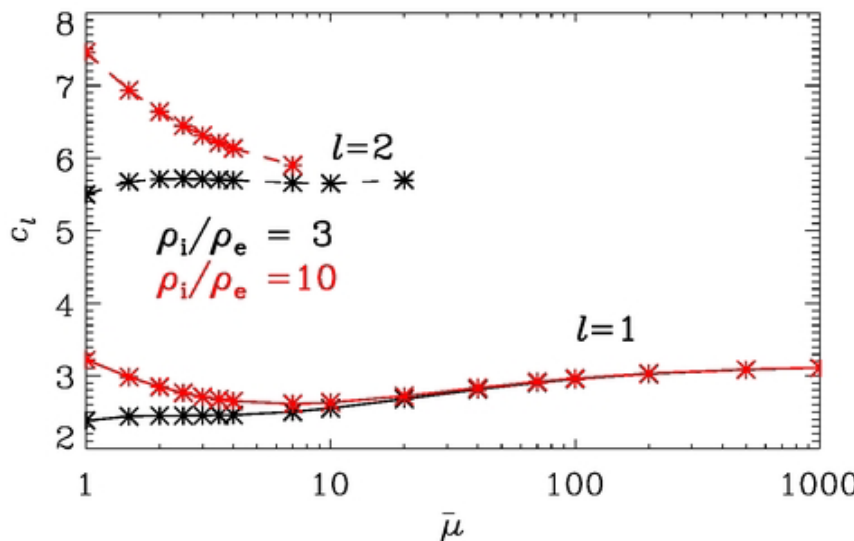
$$f(x) \approx 1 - (x/R)^\infty \text{ for } |x/R| \ll 1$$

Li et al. 2017 ApJ, to submit

$$f(x) = \begin{cases} 1, & 0 \leq x \leq R, \\ 0, & x \geq R. \end{cases}$$



$$\frac{v_{\text{ph}}^2}{v_{\text{Ai}}^2} \approx 1 + \left(\frac{l\pi}{kR} \right)^2$$

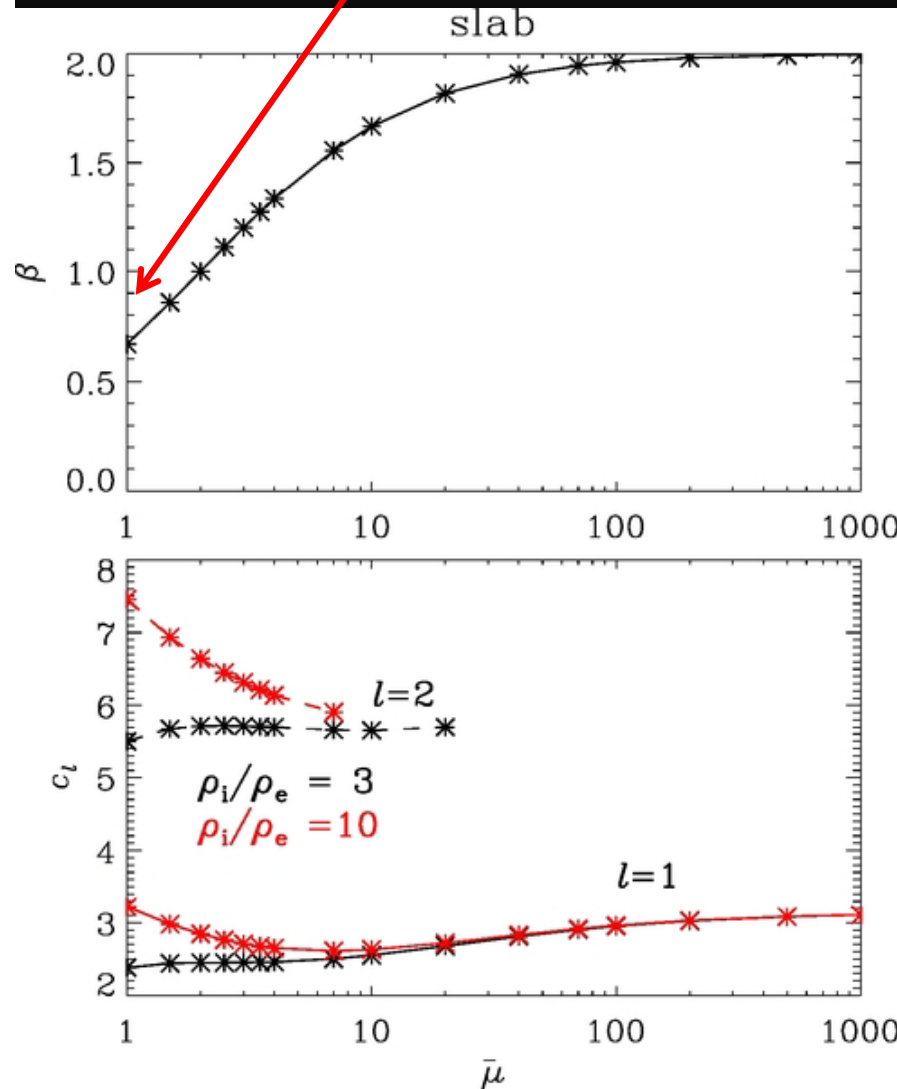


also holds for (mu arbitrary)

$$f(x) = \begin{cases} 1, & 0 \leq x \leq R, \\ (x/R)^{-\mu}, & x \geq R. \end{cases}$$

Further support for this conjecture

$f(x) \approx 1 - (x/R)$ for $|x/R| \ll 1$ Li et al. 2017 ApJ, to submit



$$f(x) = \frac{1}{1 + (x/R)}$$

$$f(x) = \begin{cases} 1 - \left(\frac{x}{R}\right), & 0 \leq x \leq R, \\ 0, & x \geq R. \end{cases}$$

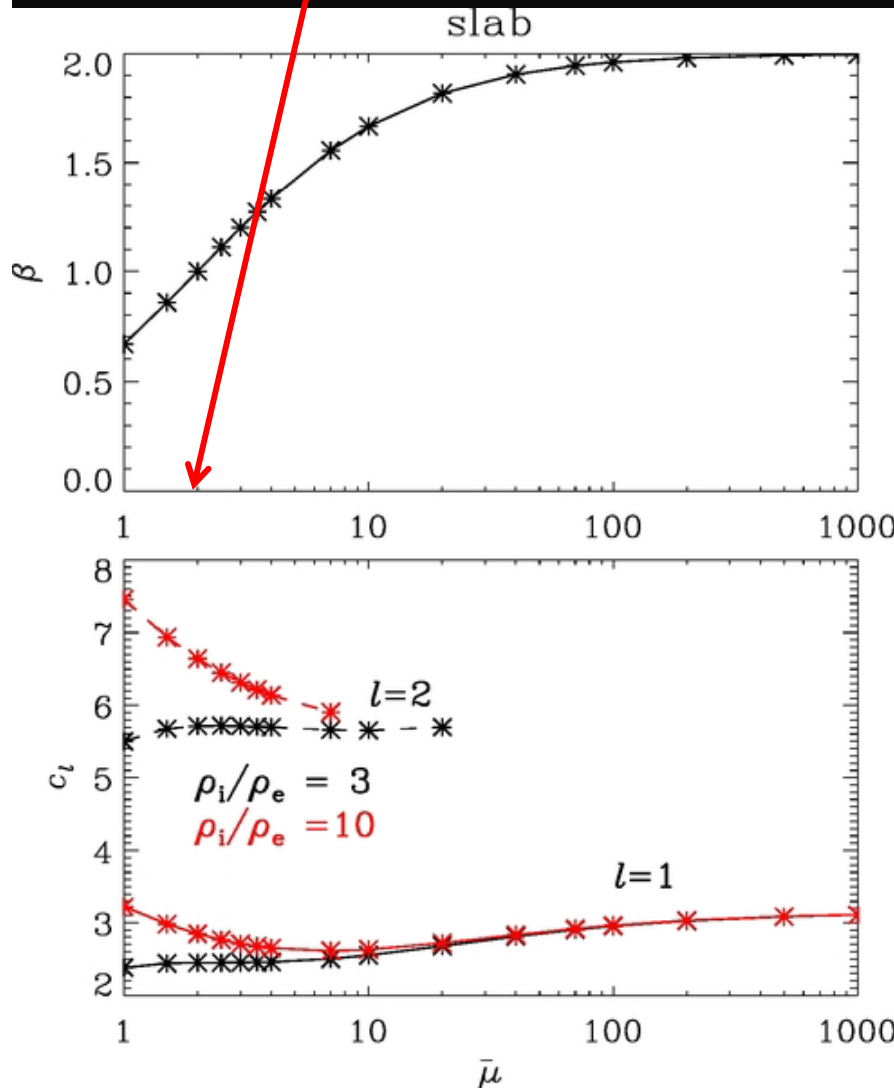
$$f(x) = \exp\left(-\frac{x}{R}\right)$$

$$\frac{v_{\text{ph}}^2}{v_{\text{Ai}}^2} \approx 1 + \left[\frac{3(4l - 1)\pi(1 - \rho_e/\rho_i)}{8kR} \right]^{2/3}$$

Further support for this conjecture

$$f(x) \approx 1 - (x/R)^2 \text{ for } |x/R| \ll 1$$

Li et al. 2017 ApJ, to submit

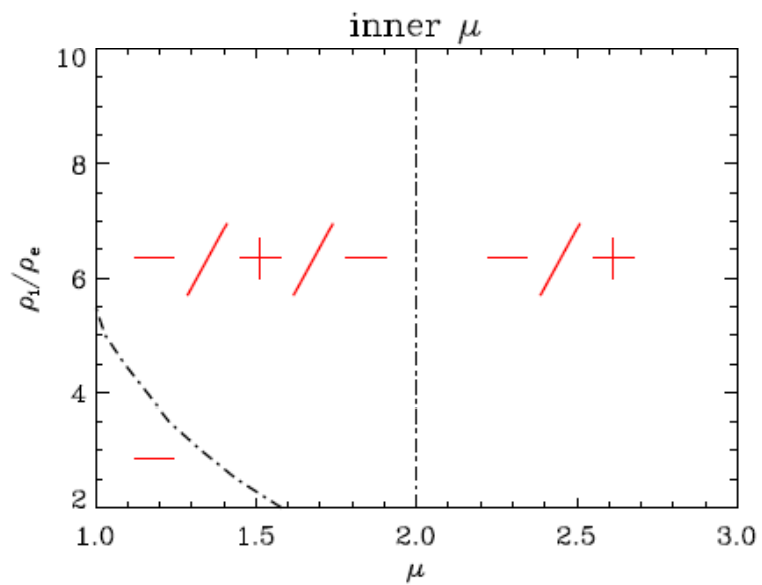
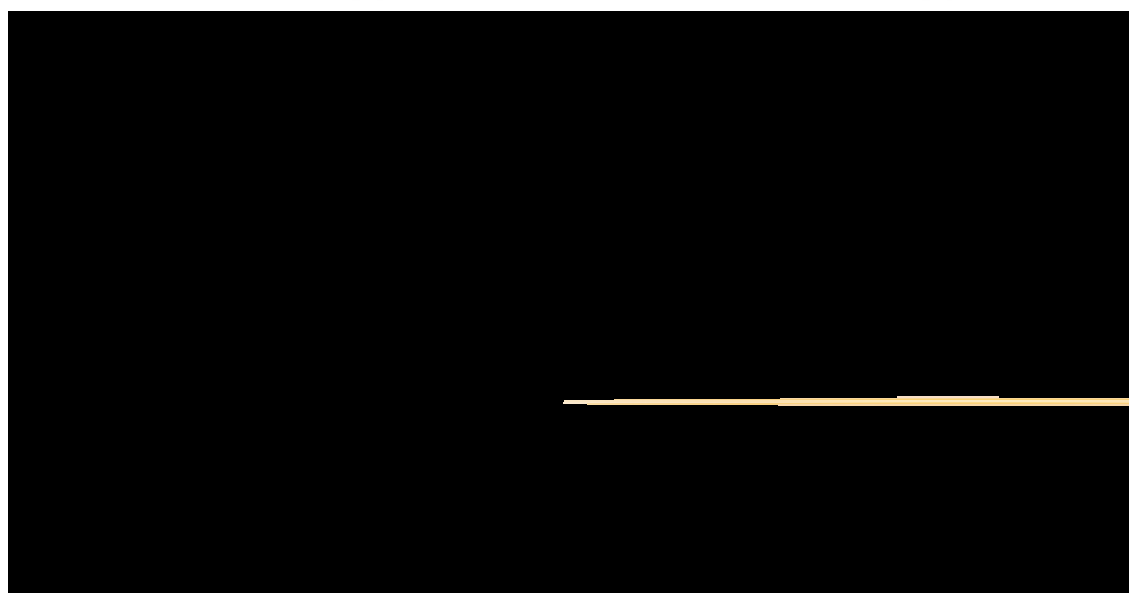
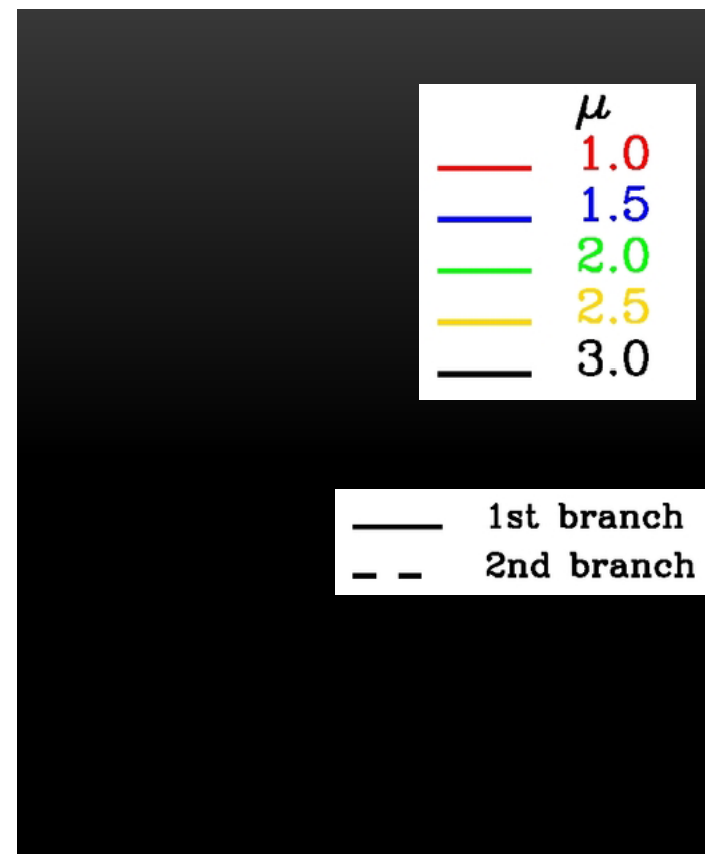
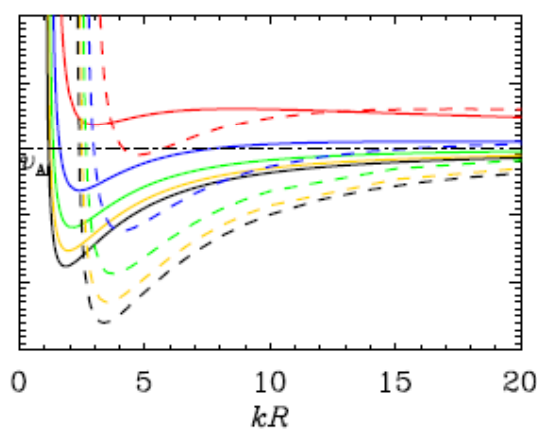
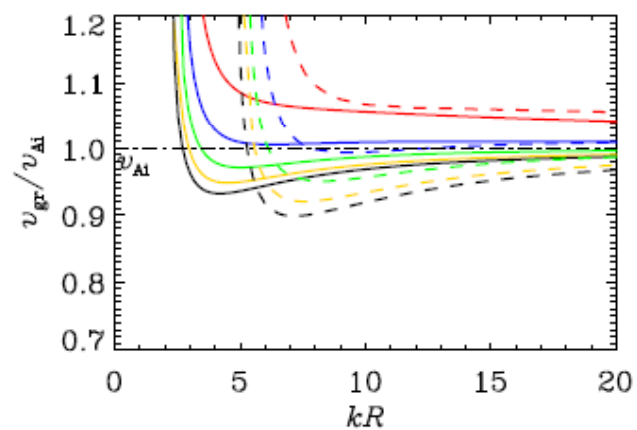
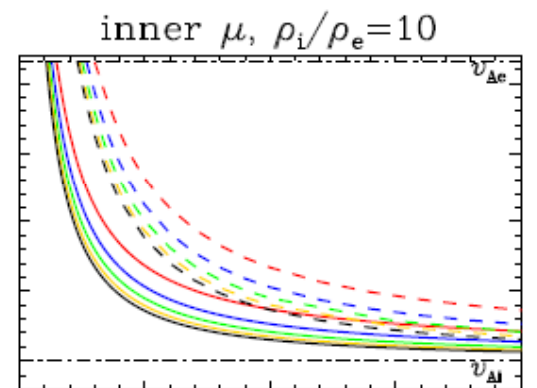
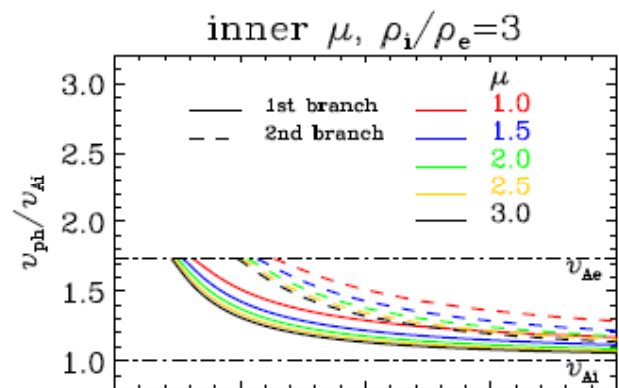


$$f(x) = \begin{cases} 1 - \left(\frac{x}{R}\right)^2, & 0 \leq x \leq R, \\ 0, & x \geq R. \end{cases}$$

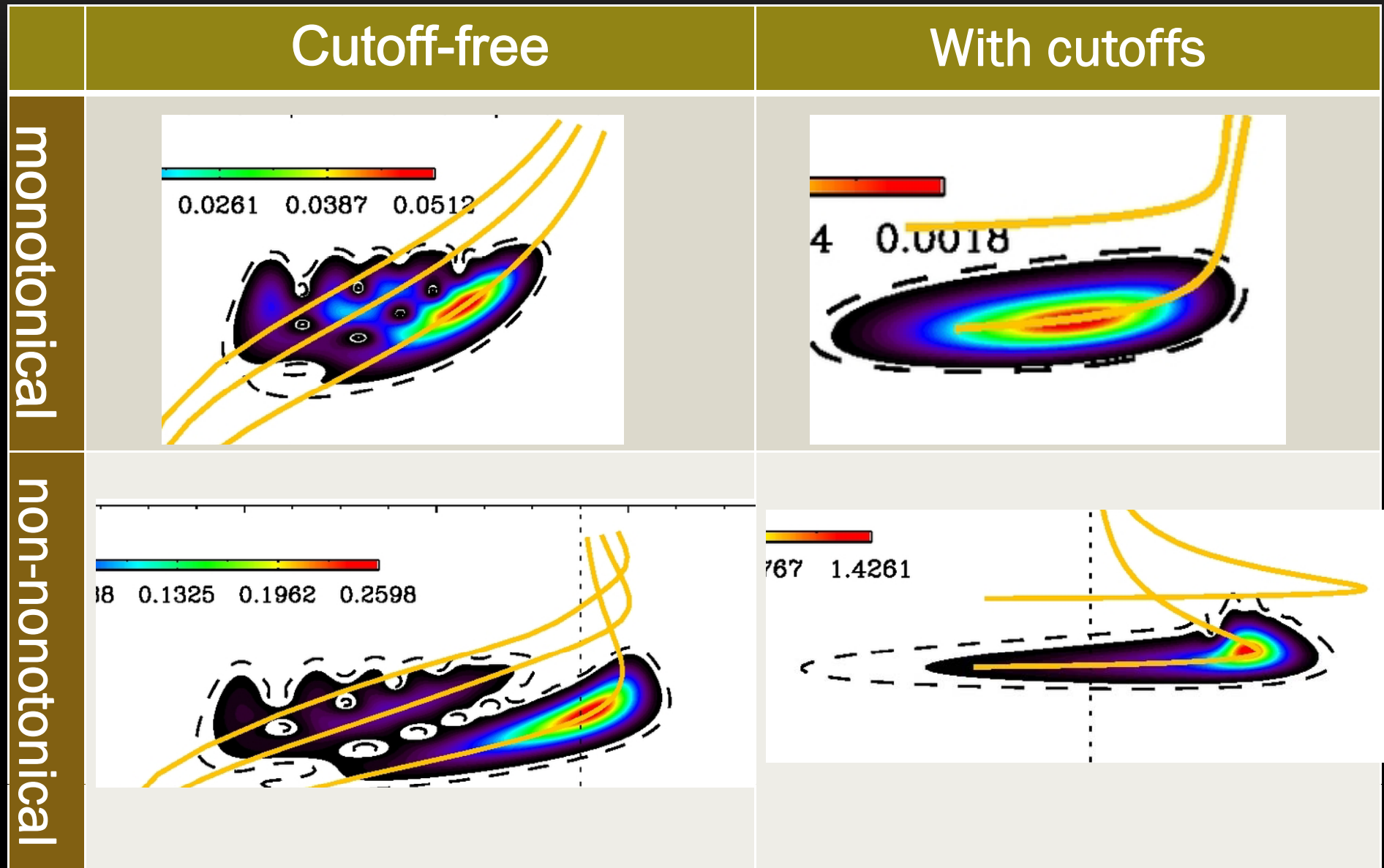
$$\frac{v_{\text{ph}}}{v_{\text{Ai}}} \approx 1 + \frac{4l-1}{2} \frac{\sqrt{1-\rho_e/\rho_i}}{kR} + \frac{(4l-1)^2}{8} \frac{(1-\rho_e/\rho_i)}{(kR)^2}$$

$$f(x) = \text{sech}^2\left(\frac{x}{R}\right)$$

$$\frac{v_{\text{ph}}}{v_{\text{Ai}}} \approx 1 + \frac{4l-1}{2} \frac{\sqrt{1-\rho_e/\rho_i}}{kR} - \frac{(2l-1/2)^2(1+\rho_e/\rho_i)/2 - l(2l-1)}{k^2 R^2}$$



Morlet spectra may look different from tadpoles



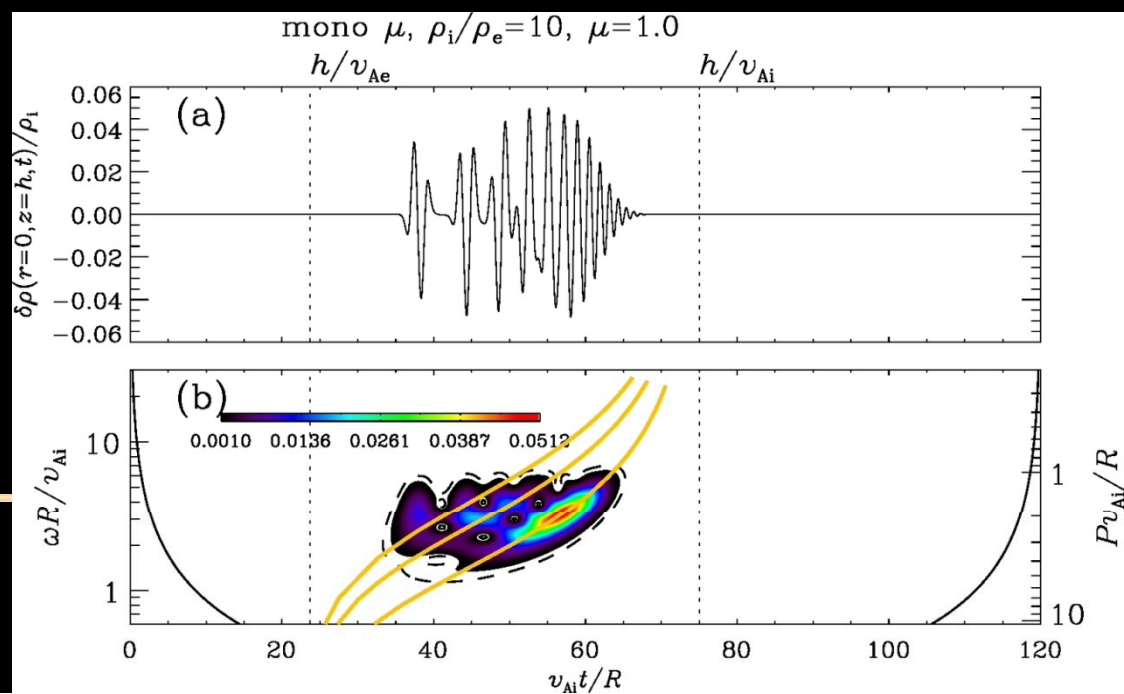
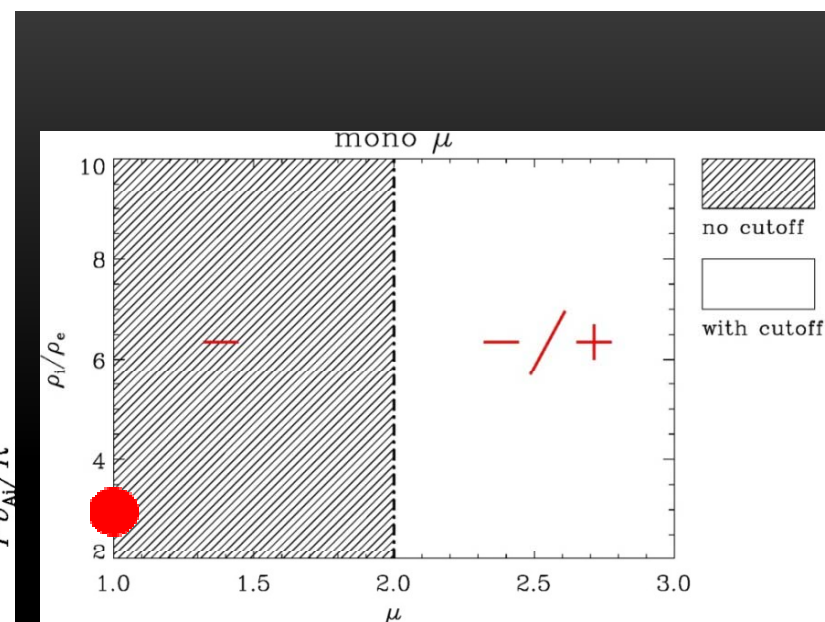
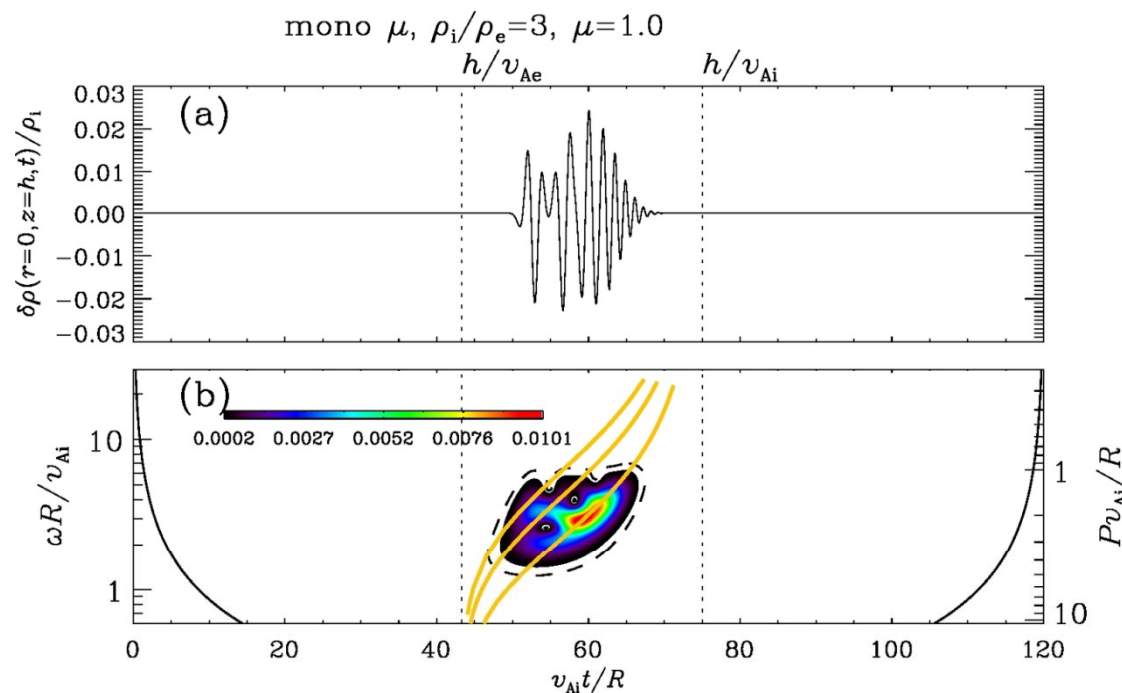
Summary

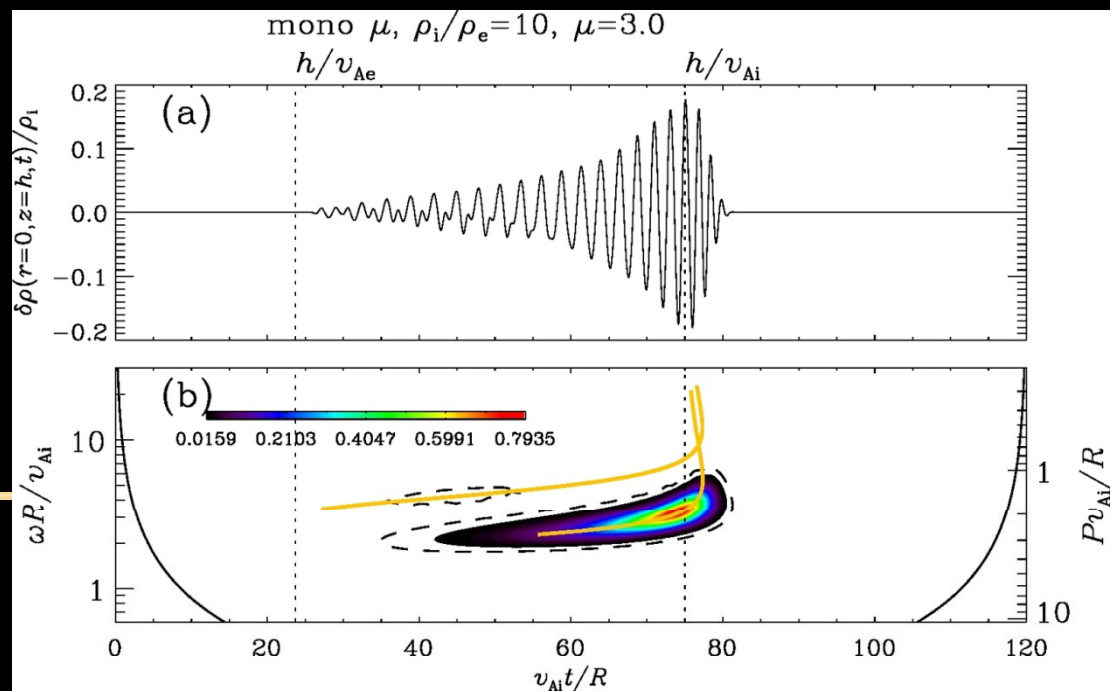
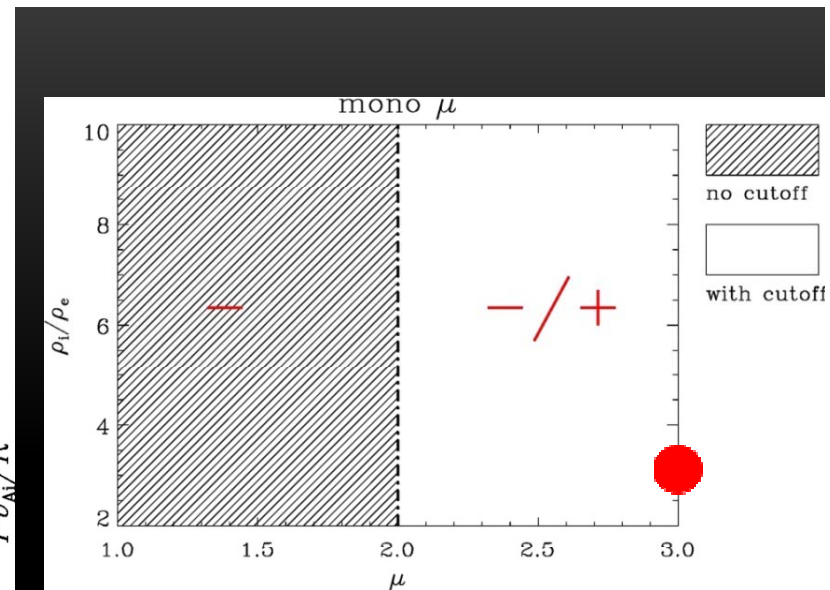
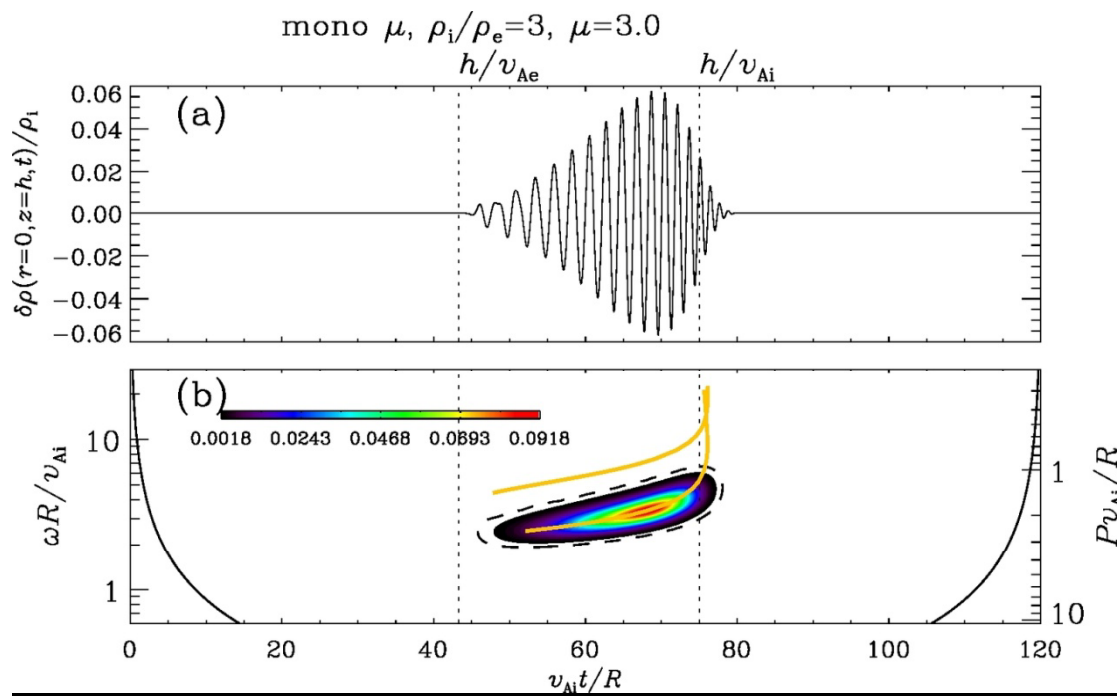
- Fast sausage waves in coronal tubes
 - axisymmetric, do not displace tube axis
 - (quasi-)periodic contractions & expansions
 - Strong compressibility & strong dispersion
- Fundamental, standing, fast sausage modes in flare loops
 - often invoked to interpret QPPs with periods of order seconds
 - uncertainties in density distribution leads to considerable uncertainties in the deduced transverse Alfvén time
 - **Gas pressure not important for periods → cold MHD theory can be used, but deduced Alfvén time is actually transverse fast time**
- Impulsively generated sausage wave trains in coronal tubes
 - invoked to interpret rapid oscillatory behavior
 - uncertainties in density distribution → **Morlet spectra that look quite different from “crazy tadpoles” → worth digging into high-cadence data in optical and radio passbands**

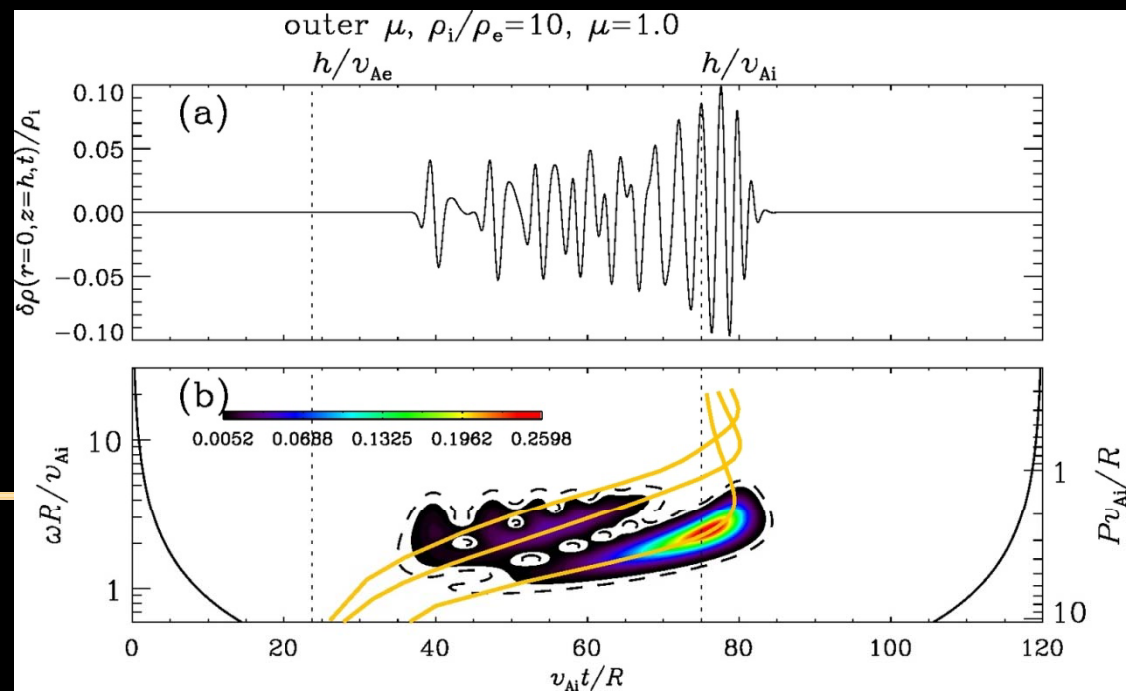
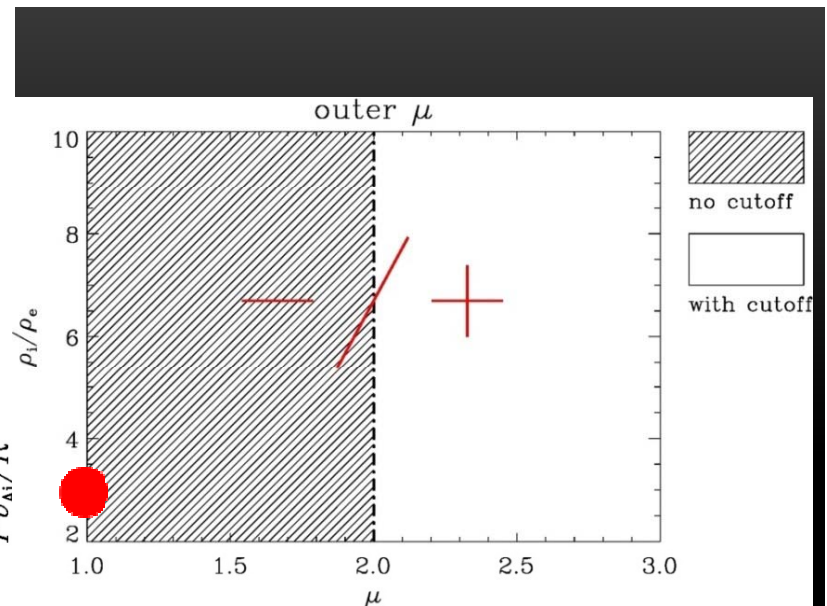
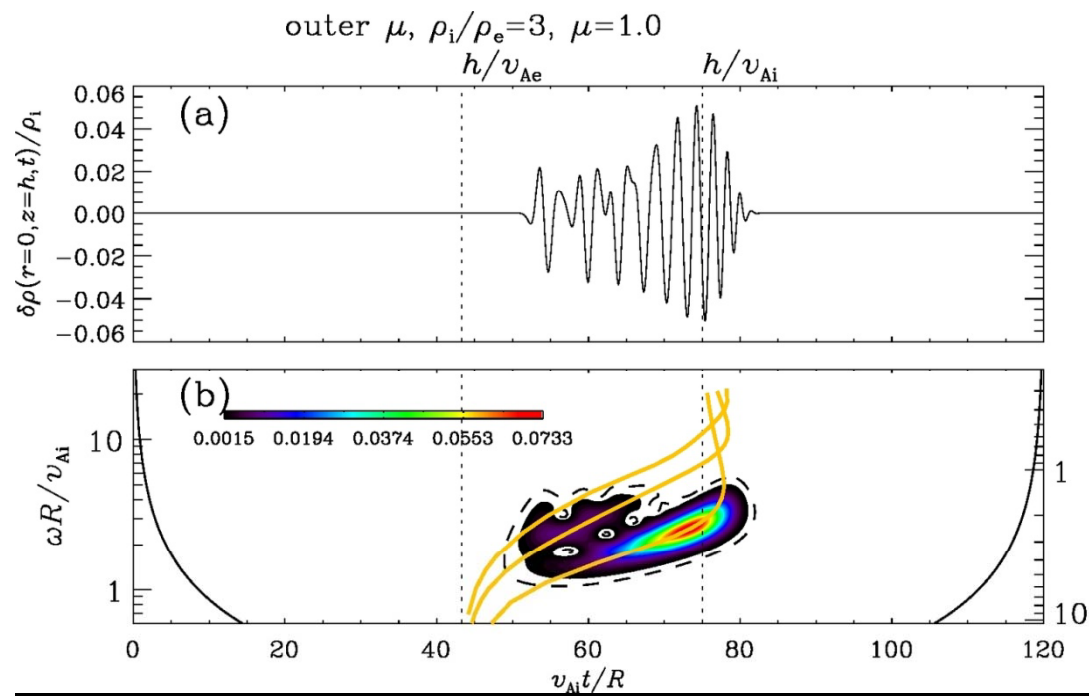
谢谢
Thanks.

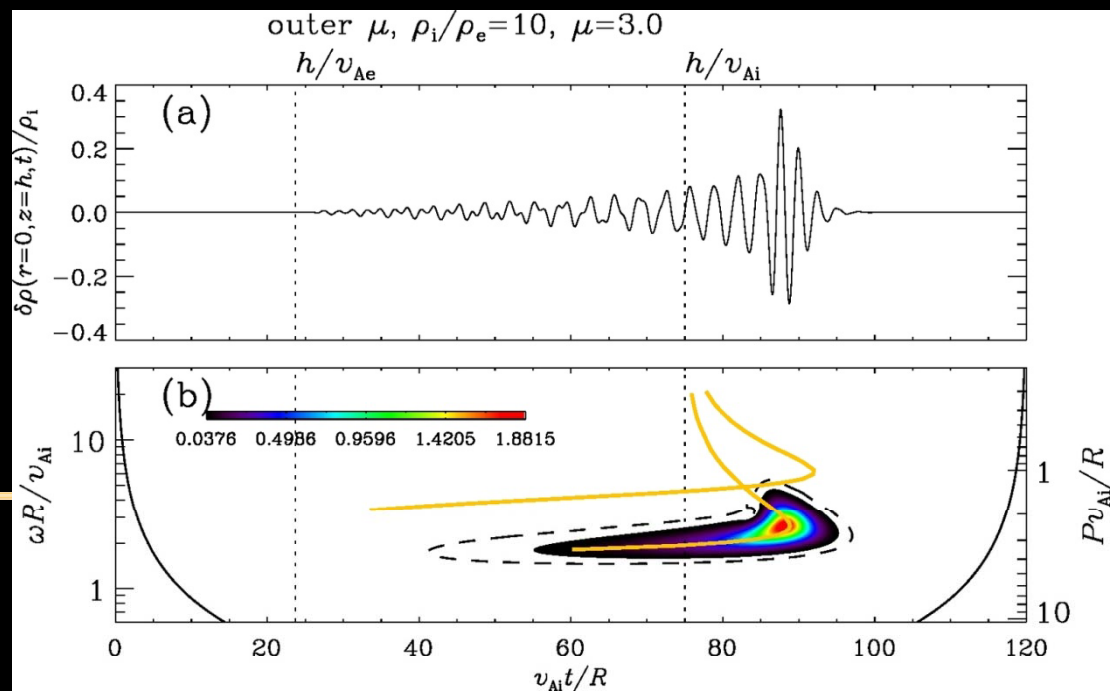
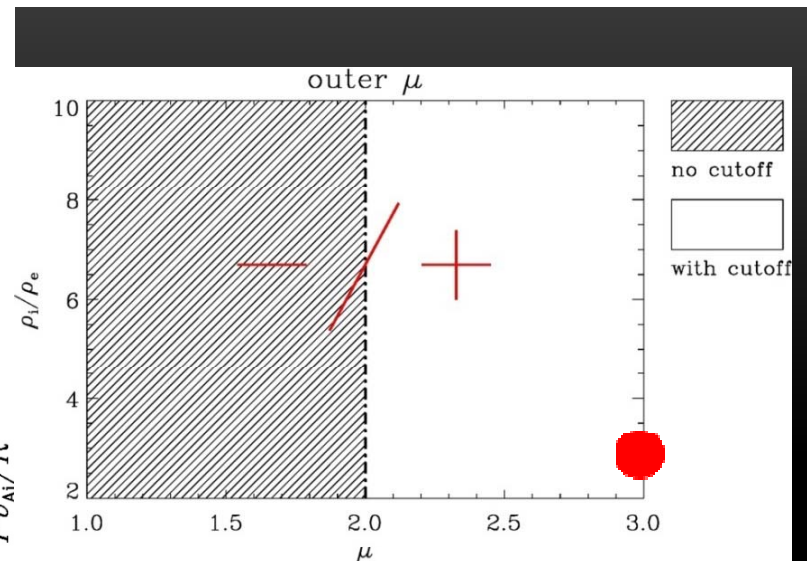
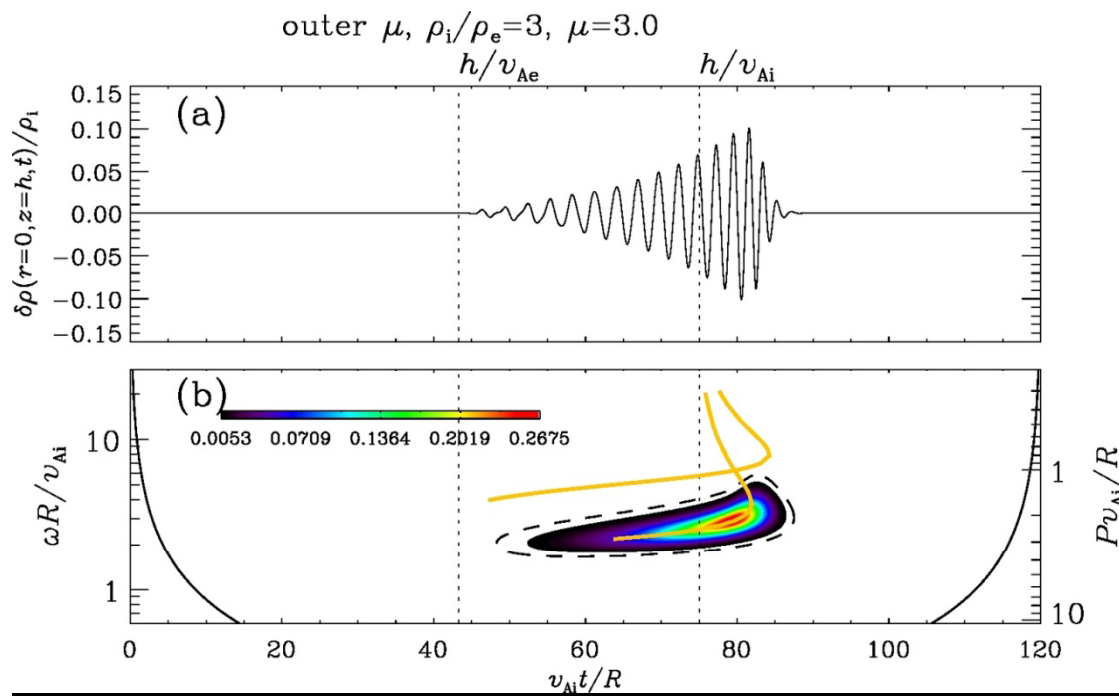
Chen, Li, et al.	2015 SoPh, 290, 2231
Chen, Li, et al.	2015 ApJ, 812, 22
Guo, Chen, Li, et al.	2016 SoPh, 291, 877
Chen, Li, et al.	2016 ApJ, 833, 114
Yu, Li, et al.	2016 ApJ, 833, 51
Yu, Li, et al.	<u>2017 ApJ, in press, arXiv:161209479Y</u>
Li, et al.	2017 ApJ, to be submitted soon

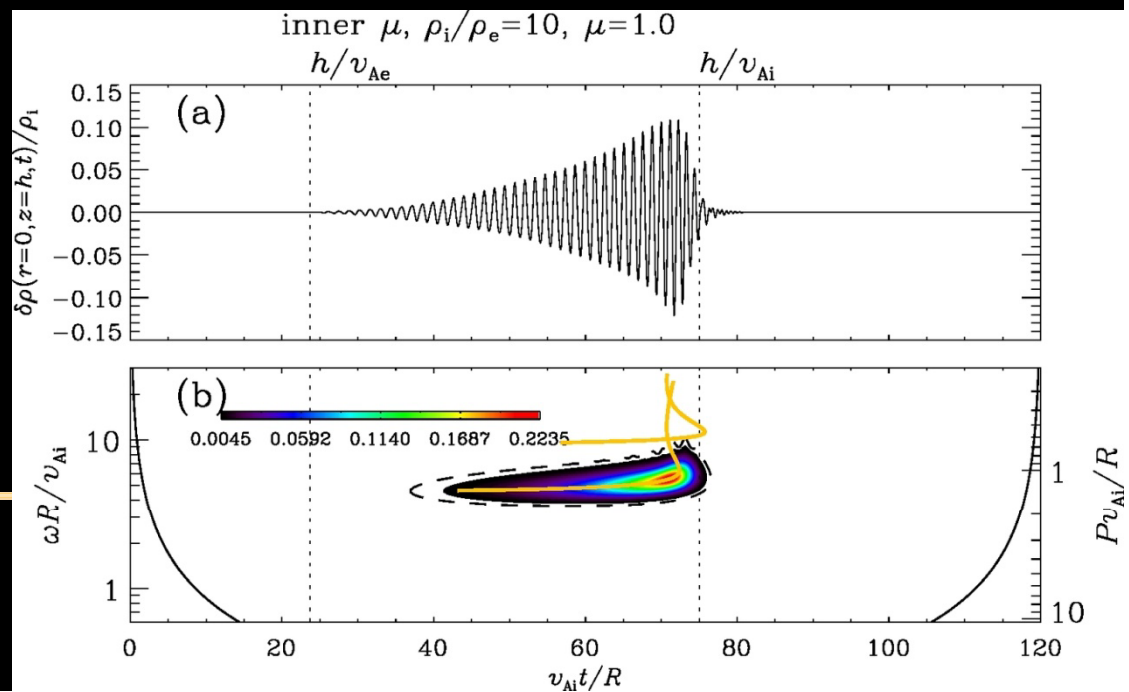
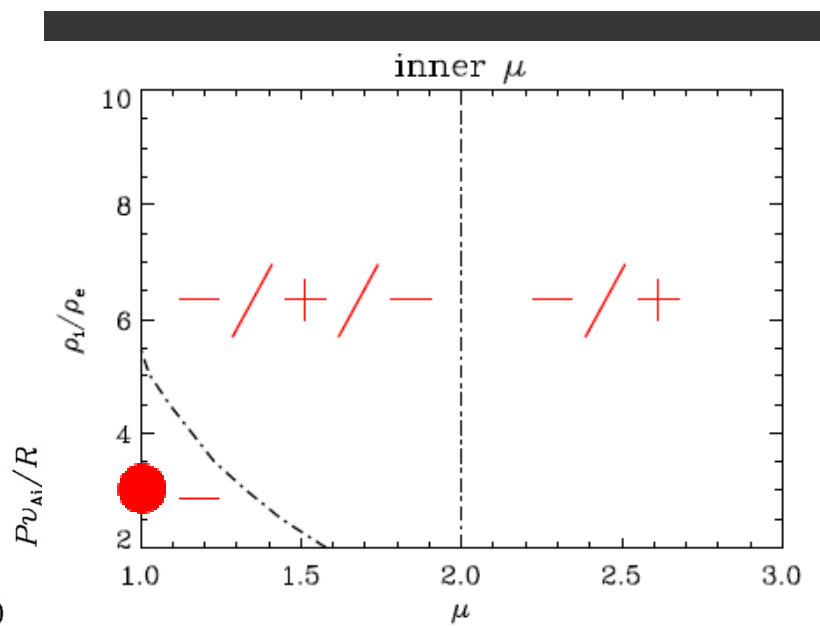
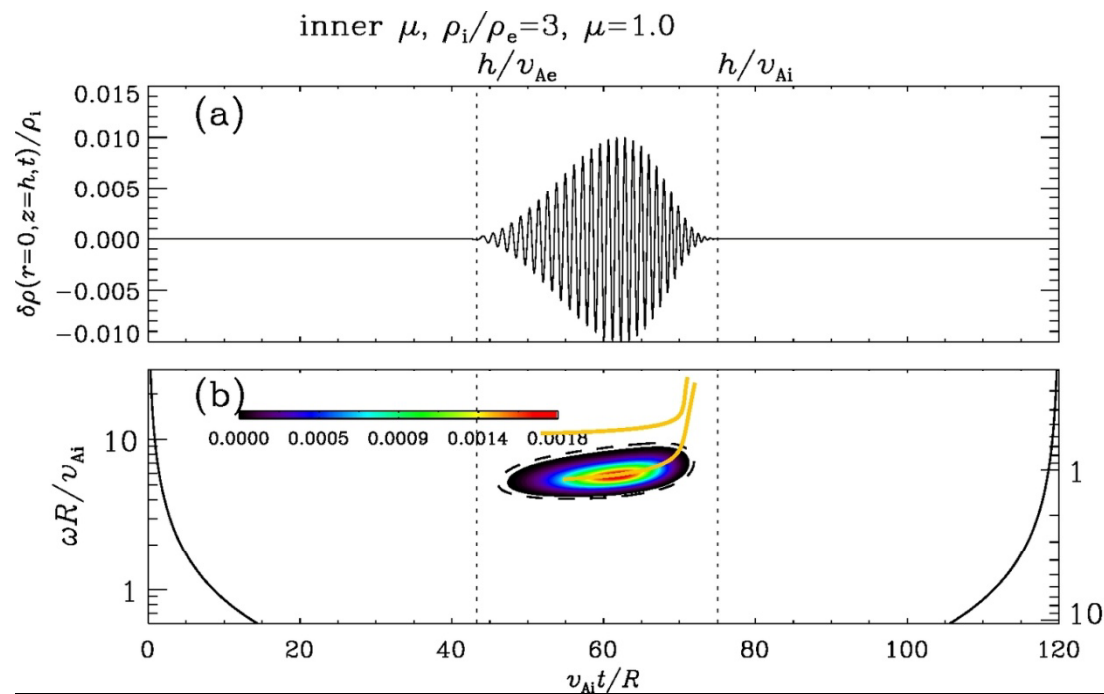
BACKUP SLIDES

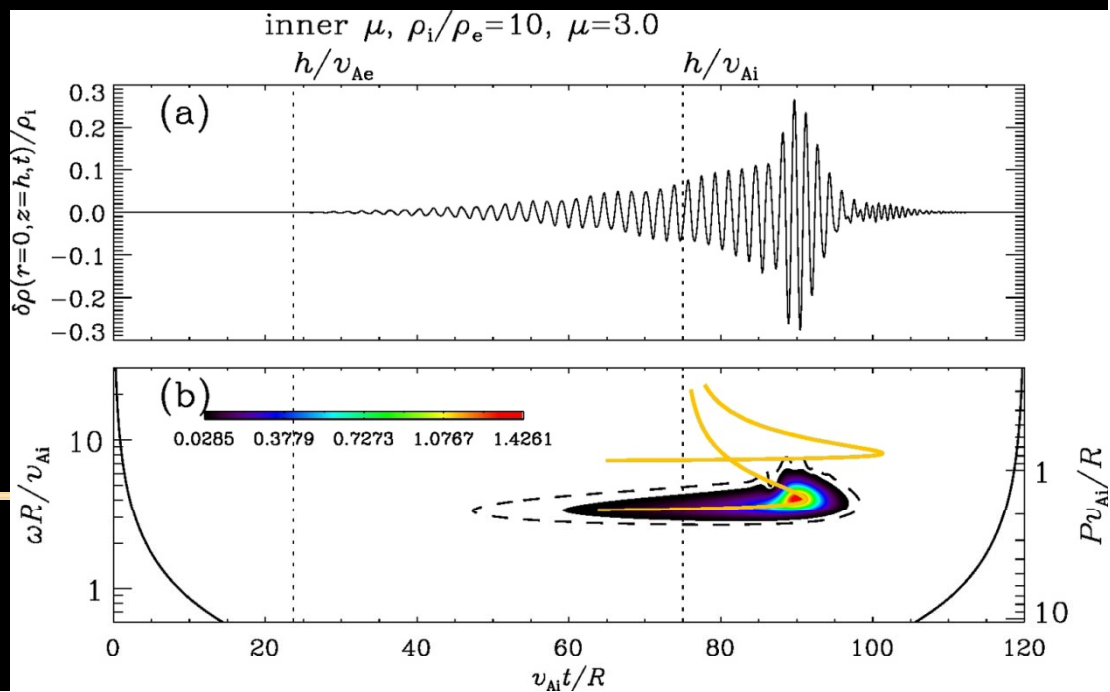
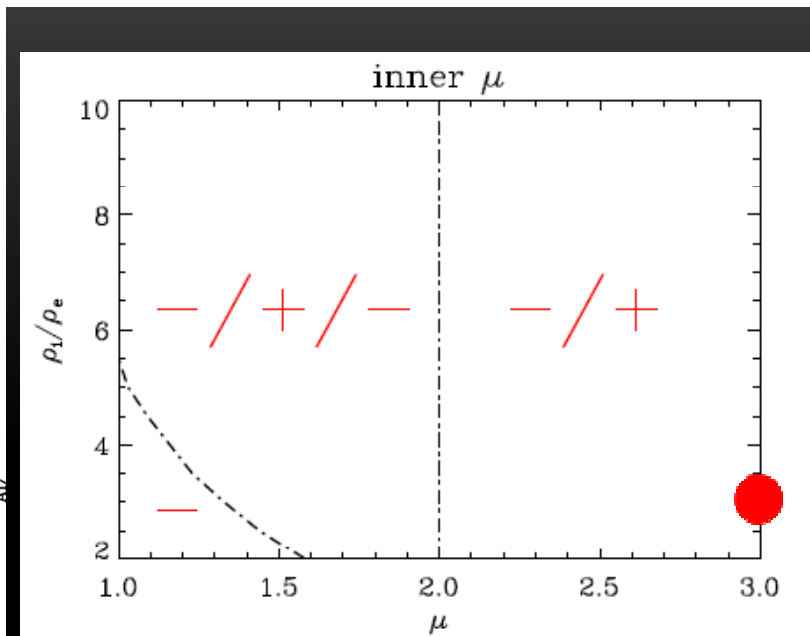
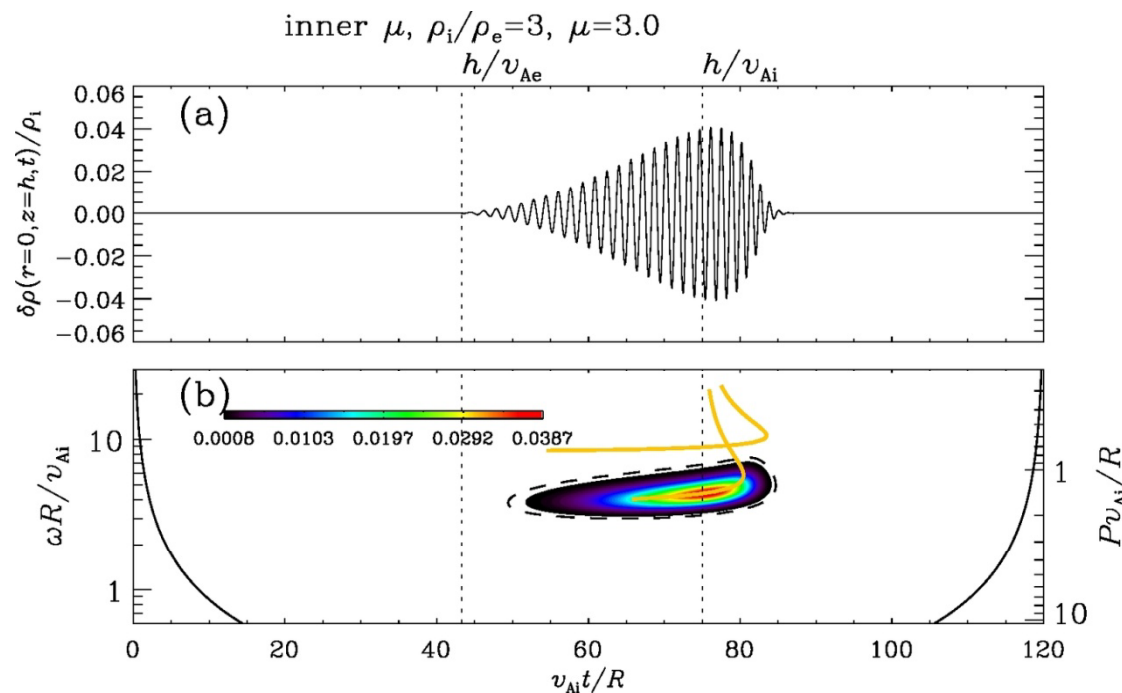




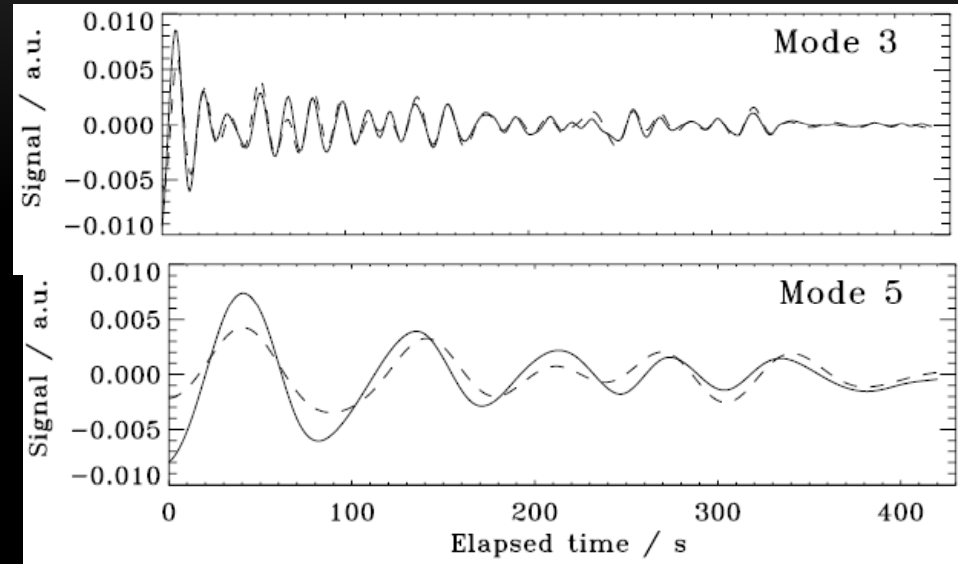
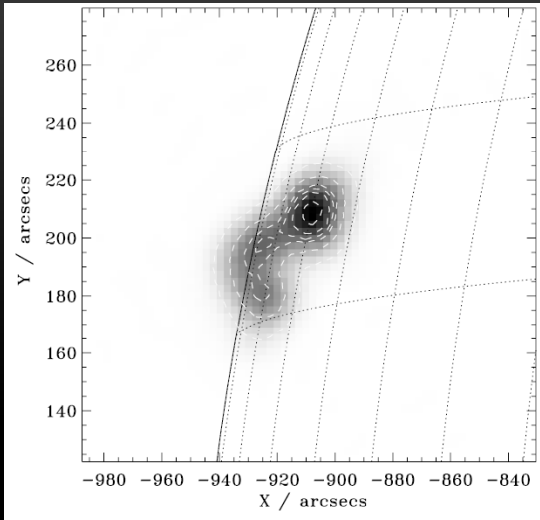








NoRH QPP: 14 May 2014 [Kolotkov+15]



profile	l/R	ρ_i/ρ_e	v_{Ai} (km/s)	$P_{\text{kink,theory}}$ (s)
linear	0.167	28.5	653.8	91.5
parabolic	0.240	28.4	657.7	89.2
inverse-parabolic	0.277	31.1	593.7	102.5
sine	0.284	29.9	620.5	95.9

Guo et al. 2016 SoPh, 291, 877