Apparent cross-field superslow propagation in coronal magnetic structures

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Aim

Kaneko et al. 2015, ApJ, 812, "Apparent Cross-field Superslow Propagation of Magnetohydrodynamic Waves in Solar Plasmas"

Aim of this study

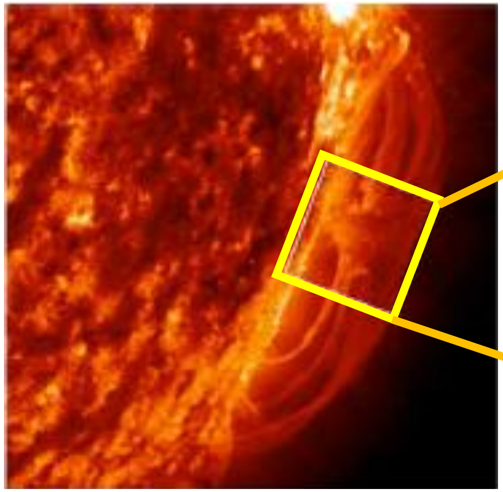
- Theoretical formulation and numerical modeling of apparent cross-field propagation by phase mixing of Alfven/slow mode
  - Apparent "superslow" propagation.
  - Application to coronal structures (prominence / coronal potential arcade)
- To show that apparent propagation can be a useful tool for prominence/coronal seismology
  - Phase speed depends on profile of Alfven frequency

Outline

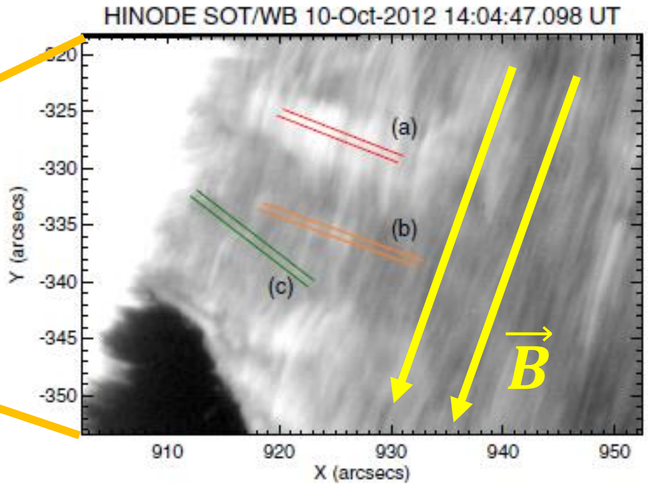
- Introduction
  - Cross-field superslow propagation
- Application of apparent propagation model
  - Formulation of apparent wave length & phase velocity
  - Application to the waves in coronal structures
    - 1: prominence in flux rope
    - 2: coronal potential arcade field
- Group velocity of apparent propagation
- Conclusion

Cross-field superslow waves: Observation

Cross-field superslow propagation in prominence
(Schmieder et al., 2013)



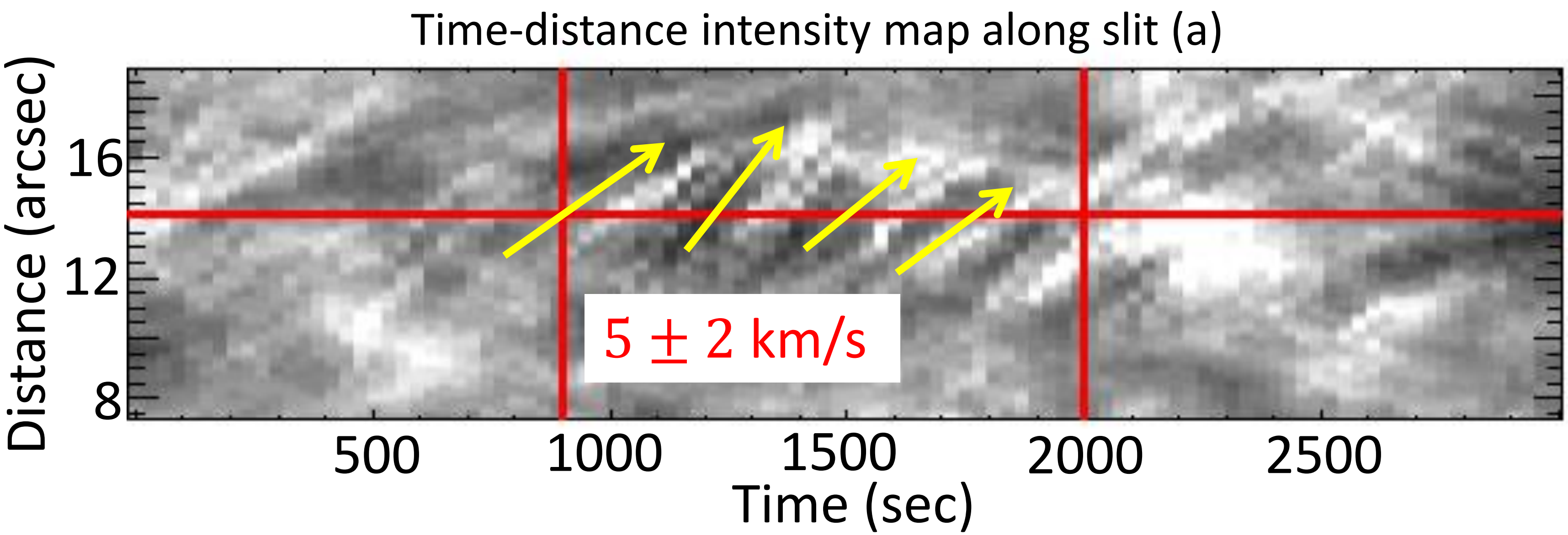
SDO/AIA 304Å

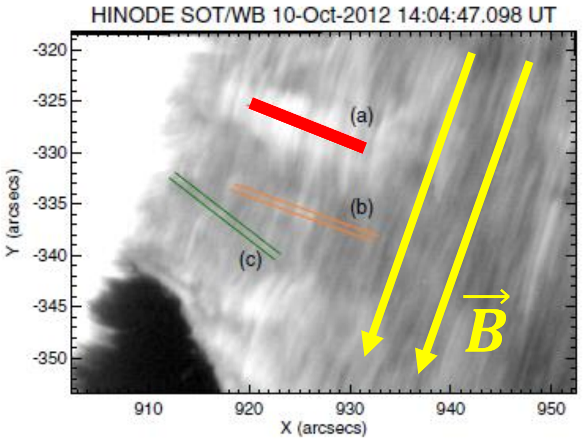


Hinode/SOT Ca II H line

- THEMIS/MTR spectropolarimeter (He D3 line) :
  $|\vec{B}| \approx 7.5$ G, $\vec{B}$ is parallel to solar limb and in the plane of sky.

Cross-field superslow waves: Observation





Hinode/SOT Ca II H line

Fast speed: $v_f \approx 75 \sim 750$ km/s

$(T = 8000\text{K}, B = 7.5\ \text{G}, n_e = 10^{9-11}\ \text{cm}^{-3})$

The propagation speed was even less than sound speed.

→ Fast mode ?

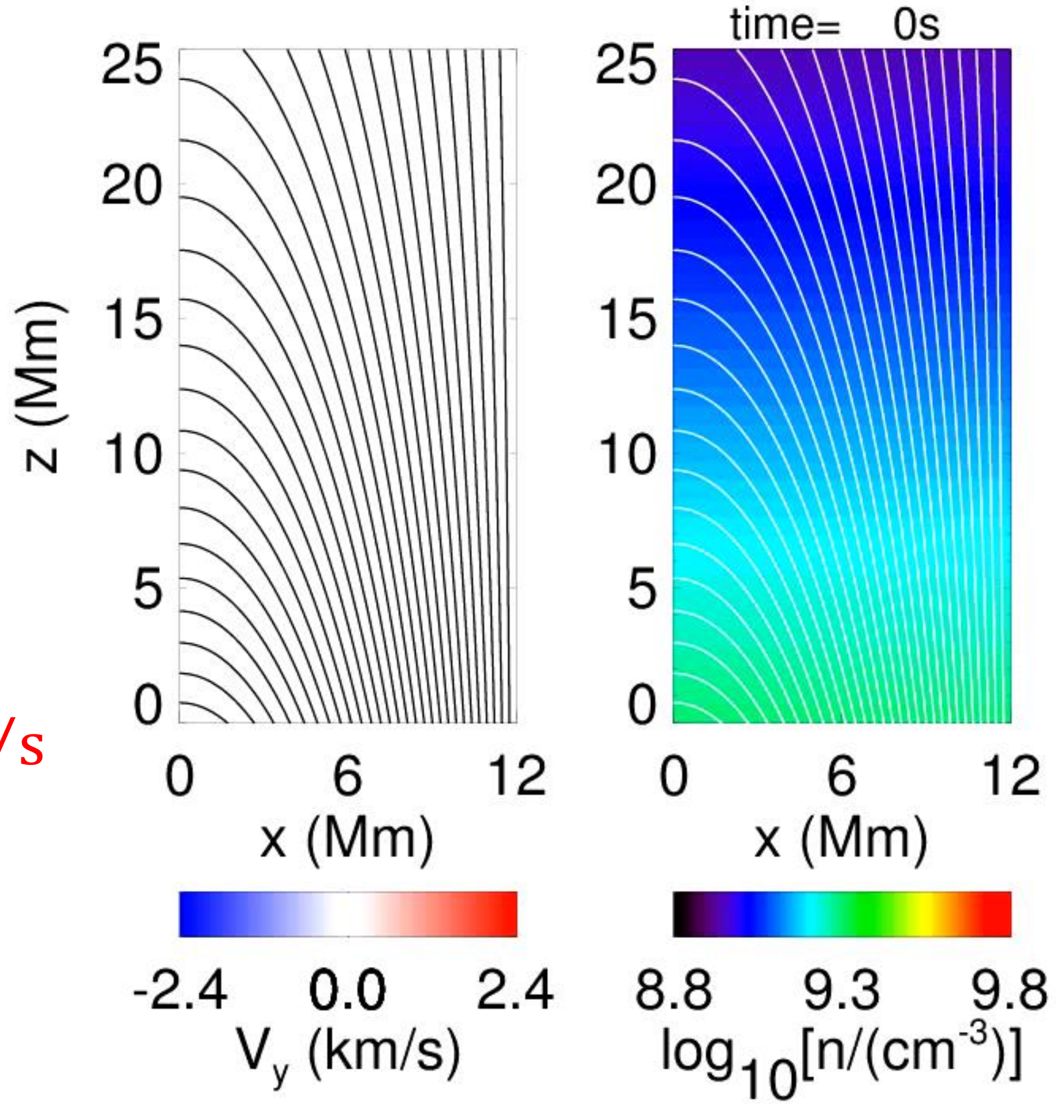
Cross-field superslow waves: Simulation

Simulation of prominence formation
(Kanekok & Yokoyama, 2015)

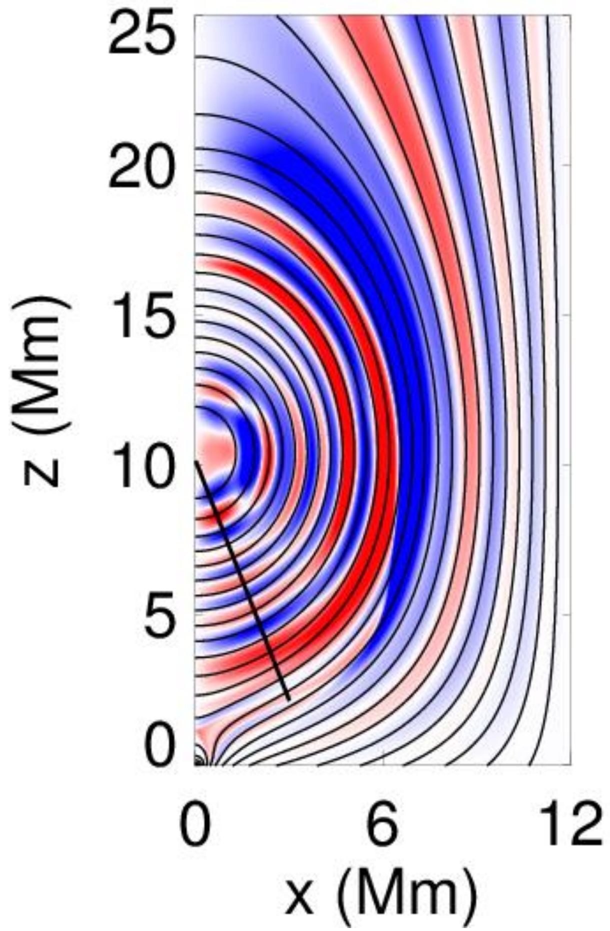
- Cross-field propagation
  → Property of fast mode
- Superslow propagation

Propagation speed 3 km/s
≪ fast mode speed 160 km/s

Fast mode ?

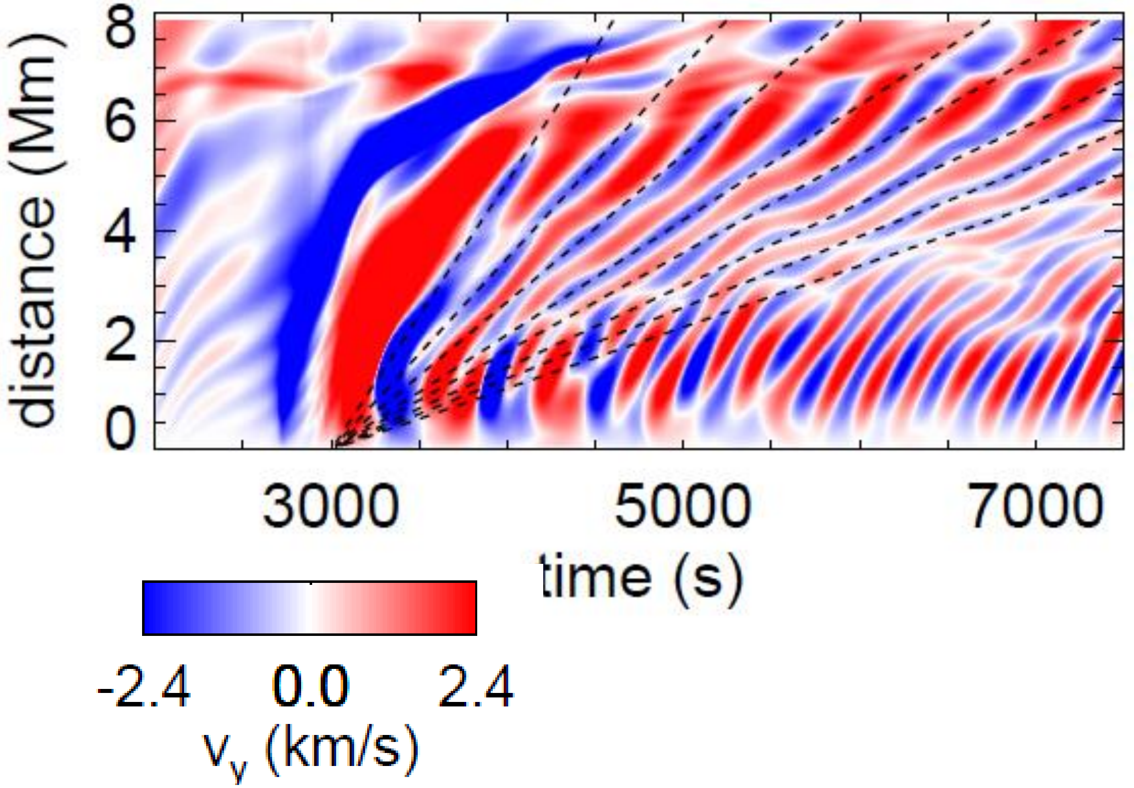


Cross-field superslow waves: Simulation



Propagation speed $3\,\mathrm{km/s}$

$\ll$ fast mode speed $160\,\mathrm{km}/s$



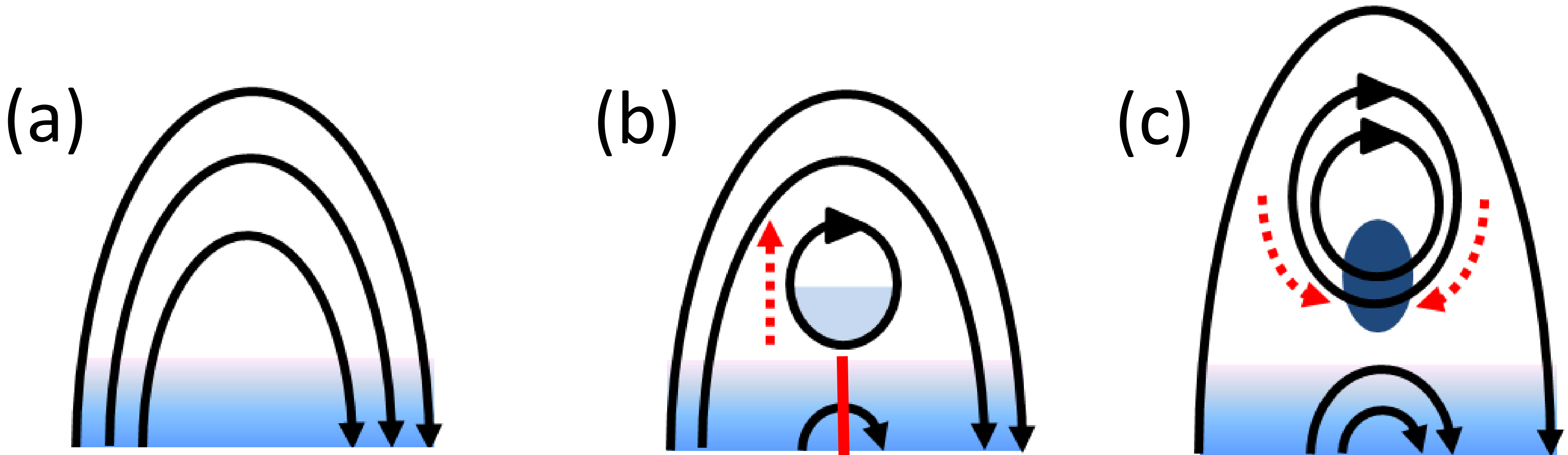
Prominence Formation

What is the origin of cool dense plasmas ?

Radiative condensation (thermal nonequilibrium)
Coronal plasmas are cooled down and condensed by radiative cooling (thermal instability).
(Karpen et al., 2007; Xia et al.,2012; Kaneko & Yokoyama,2015)

Injection, Levitaion model
Chromospheric plasmas are lifted up to coronal height by jet or emerging flux.
(Chae et al., 2003; Okamoto et al.,2007,2008; Deng et al.,2000)

Our model



Initial state

stratified atmoshere

thermal equilibrium

formation of flux rope

condensation

- **relatively dense plasmas at the bottom (strong cooling)**
- **closed field line (reduction of conduction effect)**

Numerical setting 1/4

Basic equations:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \nabla \rho = -\rho \nabla \cdot \boldsymbol{v},$$

radiative cooling

$$\frac{\partial e}{\partial t} + \boldsymbol{v} \cdot \nabla e = -(e+p)\nabla \cdot \boldsymbol{v} + \nabla \cdot \left(\kappa T^{\frac{5}{2}} \boldsymbol{b}\boldsymbol{b} \cdot \nabla T\right) - n^2 \Lambda(T) + H,$$

thermal conduction heating

$$e = \frac{p}{\gamma - 1}, \qquad T = \frac{m}{k_B}\frac{p}{\rho},$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\frac{1}{\rho}\nabla p + \frac{1}{4\pi\rho}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \mathbf{g},$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E},$$

$$\boldsymbol{E} = -\frac{1}{c}\boldsymbol{v} \times \boldsymbol{B} + \frac{4\pi\eta}{c^2}\boldsymbol{J}, \qquad \boldsymbol{J} = -\frac{c}{4\pi}\nabla \times \boldsymbol{B}.$$

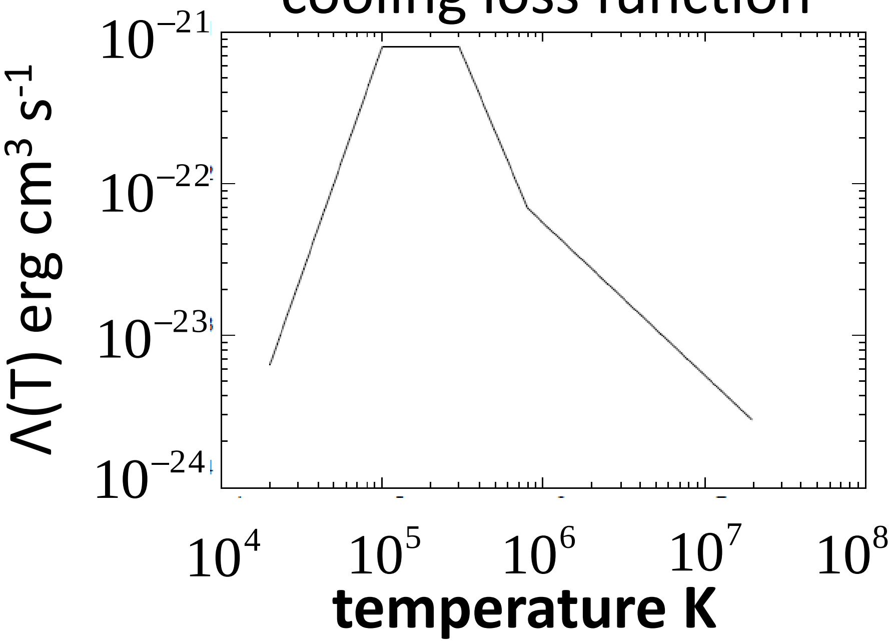
Numerical setting 2/4

Energy equation:

$$\frac{\partial e}{\partial t} + \boldsymbol{v} \cdot \nabla e = -(e+p)\nabla \cdot \boldsymbol{v} \underline{- n^2 \Lambda(T)} + H + \eta J^2 + \nabla \cdot \left(\kappa T^{\frac{5}{2}} \boldsymbol{bb} \cdot \nabla T\right)$$

radiative cooling

cooling loss function



Numerical setting 3/4

Energy equation:

$$\frac{\partial e}{\partial t} + \boldsymbol{v}\cdot\nabla e = -(e+p)\nabla\cdot\boldsymbol{v} - n^2\Lambda(T) + \underline{H} + \eta J^2 + \nabla\cdot\left(\kappa T^{\frac{5}{2}}\boldsymbol{bb}\cdot\nabla T\right)$$

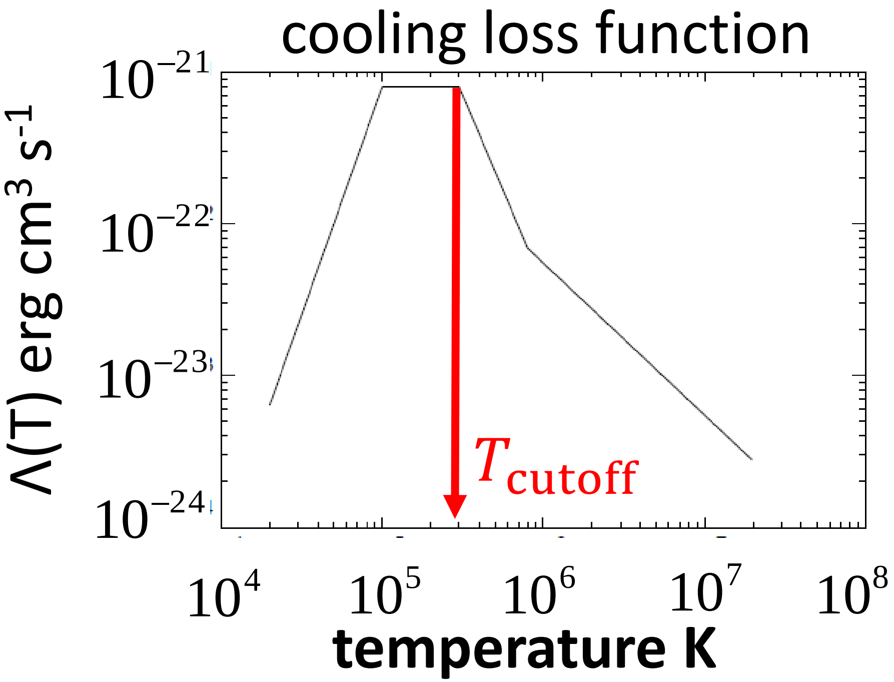
background heating

$$H \propto P_m$$

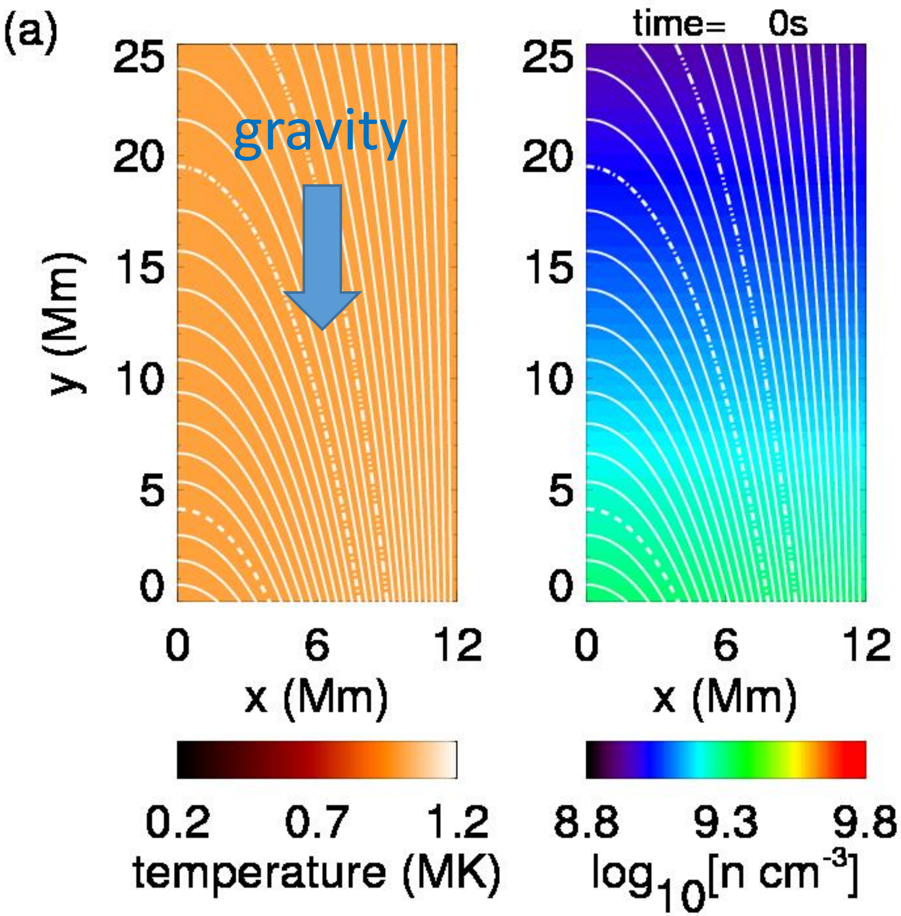
Assumption:
coronal heating related to magnetic energy

when $T < T_{\mathrm{cutoff}}$

$$H = n^2\Lambda(T)$$



Numerical setting 4/4



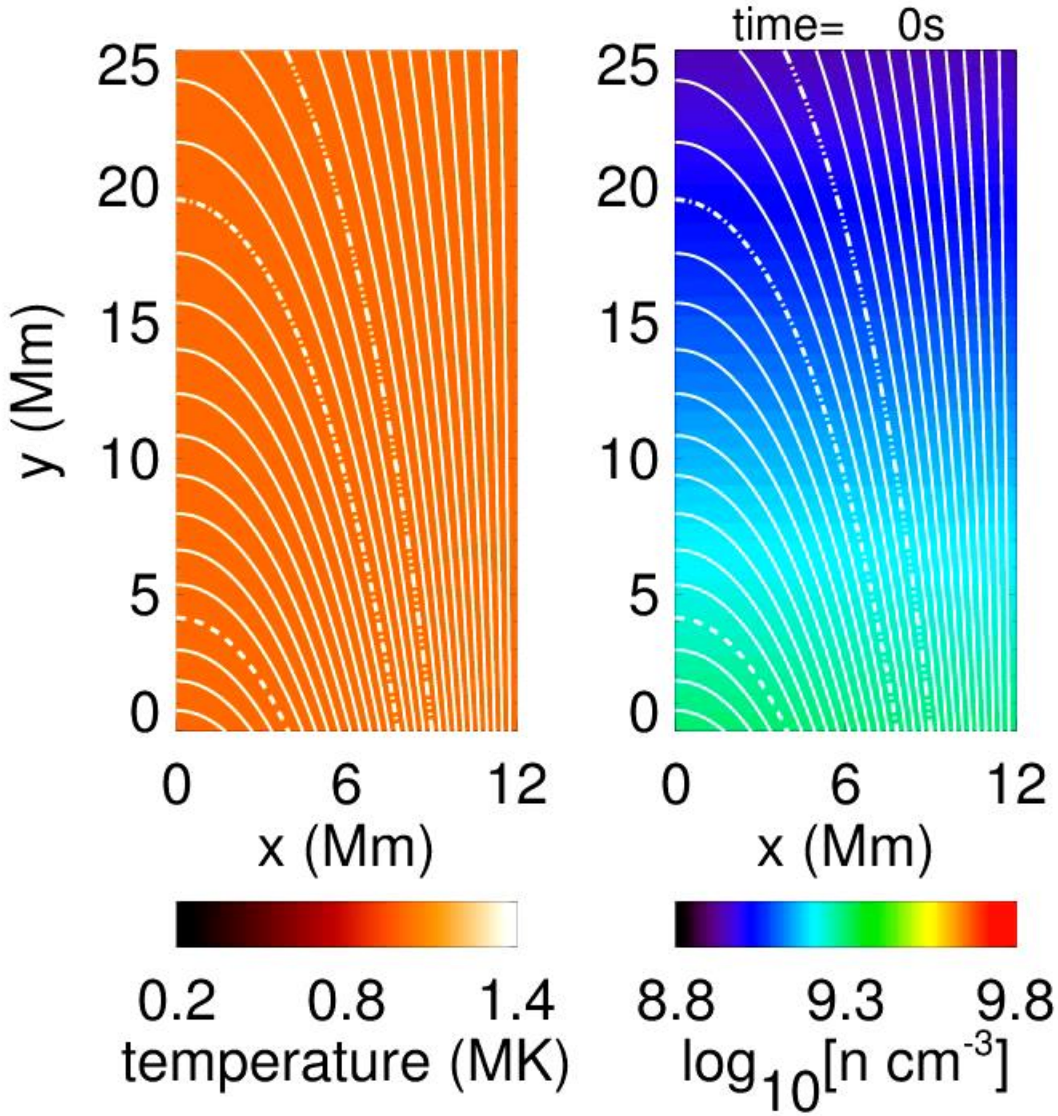
<u>Initial condition</u>

temperature: 10^6K (uniform)
density:$2 \times 10^9 \mathrm{cm}^{-3}$(stratified)
field strength: 3G
mechanical & thermal equilibrium

<u>Numerical scheme</u>

MHD part: RCIP-MOCCT
(Kudoh et al.,1999; Xiao et al., 1996)
Anisotropic thermal conduction:
Slope-limiting method
(Sharma & Hammett 2007)

Result



flux rope formation

thermal imbalance
in **thermally isolated**
closed loops

radiative condensation

Cross-field Superslow Waves: Simulation

Radiative condensation

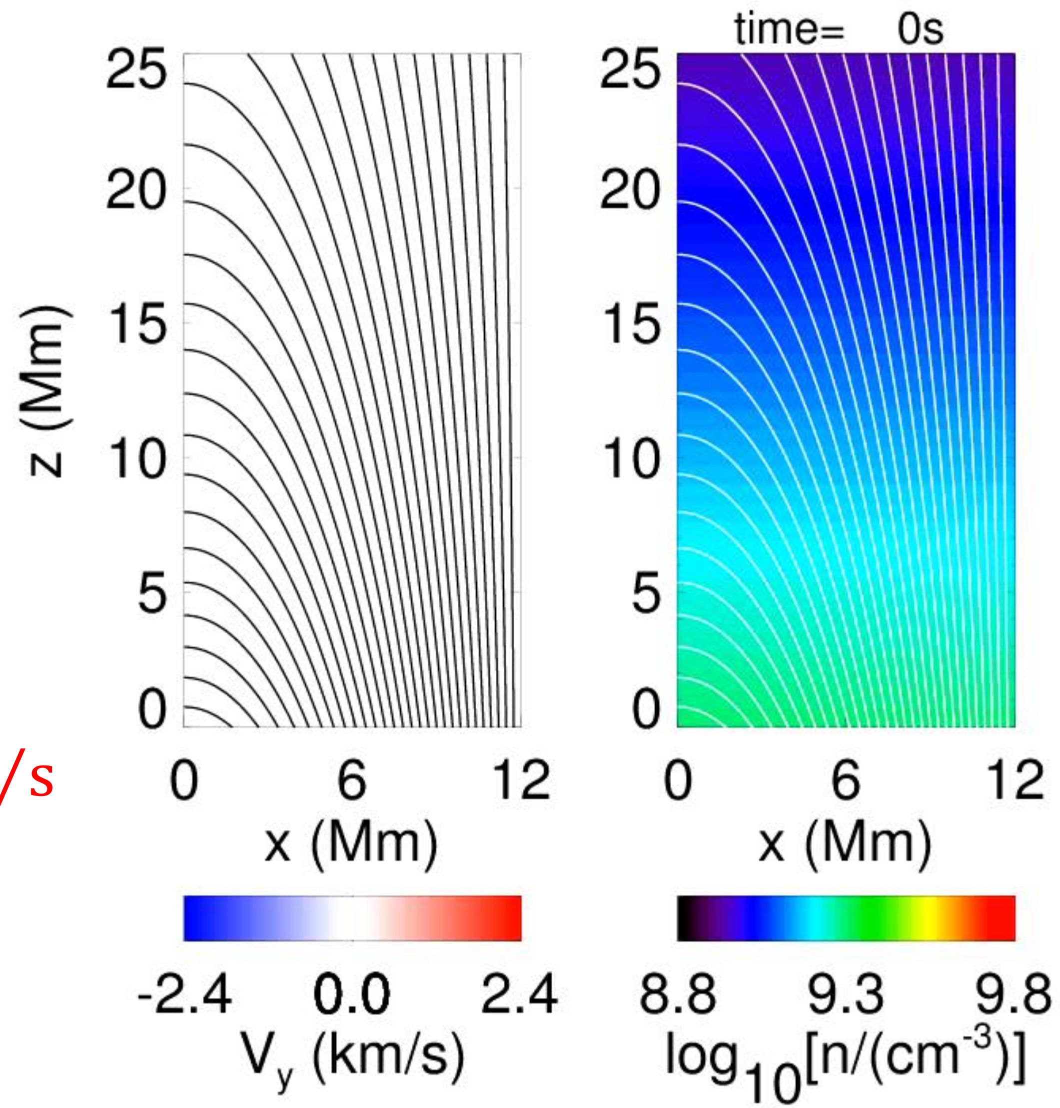
Excitation of waves

- Cross-field propagation

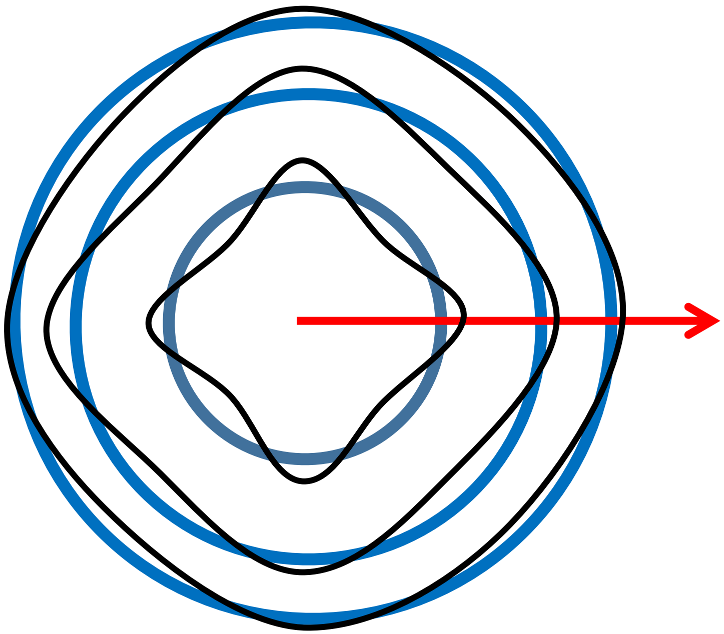
→ Property of fast mode

- Propagation speed $3\ \mathrm{km/s}$

$\ll$ fast mode speed $160\ \mathrm{km/s}$



Mechanism of Apparent Propagation 1/2



Each magnetic loop oscillates independently.

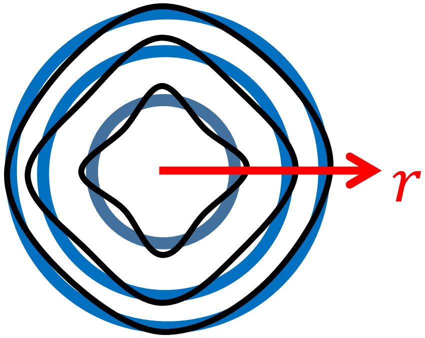
Alfven/slow standing wave on each magnetic loop with individual frequency

e.g.
Alfven velocity: $v_A = \text{const.}$
Alfven frequency: σ_A

$$\sigma_A = n \frac{v_A}{2\pi r}$$

$$\sigma(r - \Delta r) > \sigma(r) > \sigma(r + \Delta r)$$

Mechanism of Apparent Propagation 2/2

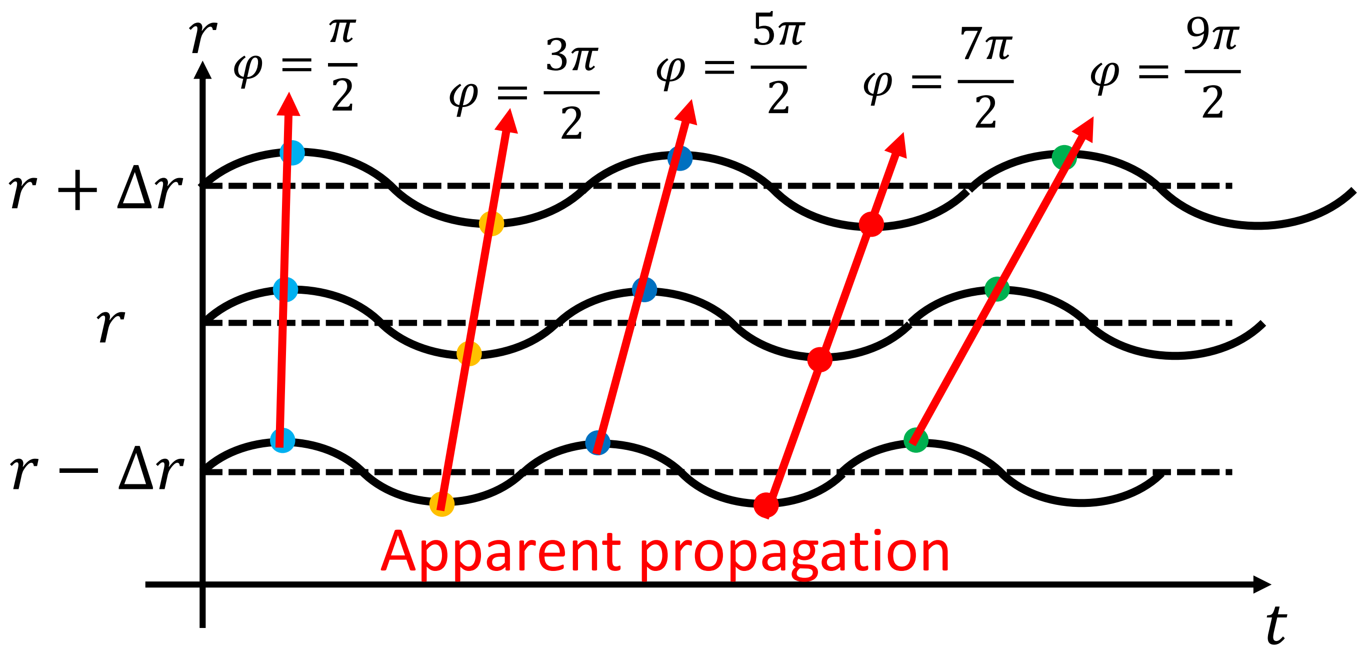


Each magnetic loop oscillates independently.

e.g. $\sigma(r-\Delta r) > \sigma(r) > \sigma(r+\Delta r)$

Phase lag is generated among magnetic surfaces.

➡ phase mixing



Apparent wavelength, phase speed

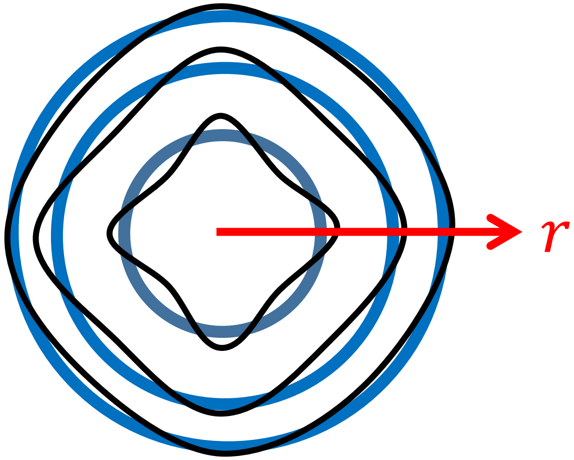
Apparent propagation in the r-direction

$$\xi_{y,n}(t,r) = \exp(\sigma_A(r)t) \longrightarrow \text{phase: } \varphi = \sigma_A(r)t$$

$$\sigma_{ap} = \frac{\partial\varphi}{\partial t} = \sigma_A(r),$$

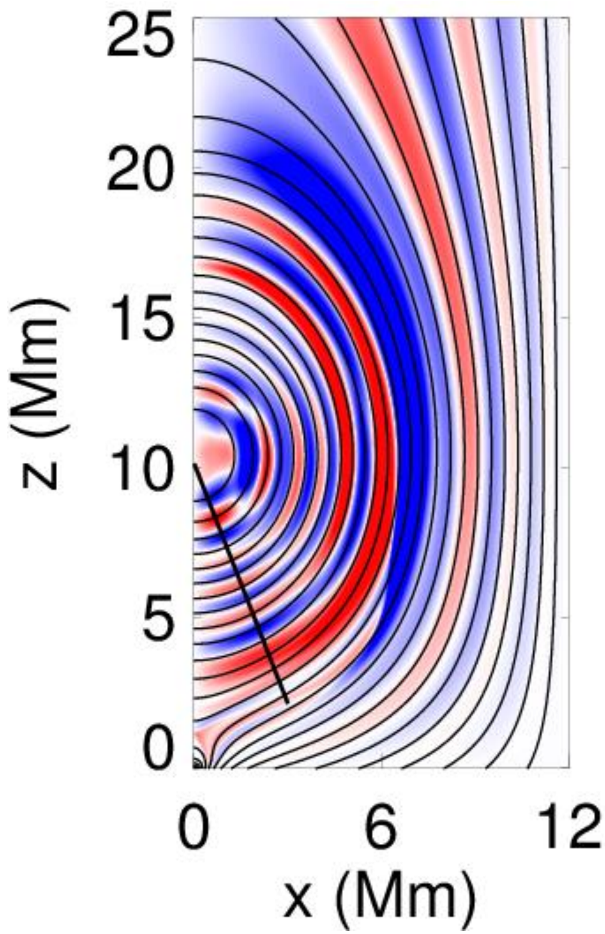
$$k_{r,ap} = -\frac{\partial\varphi}{\partial r} = -t\frac{d\sigma_A}{dr}$$

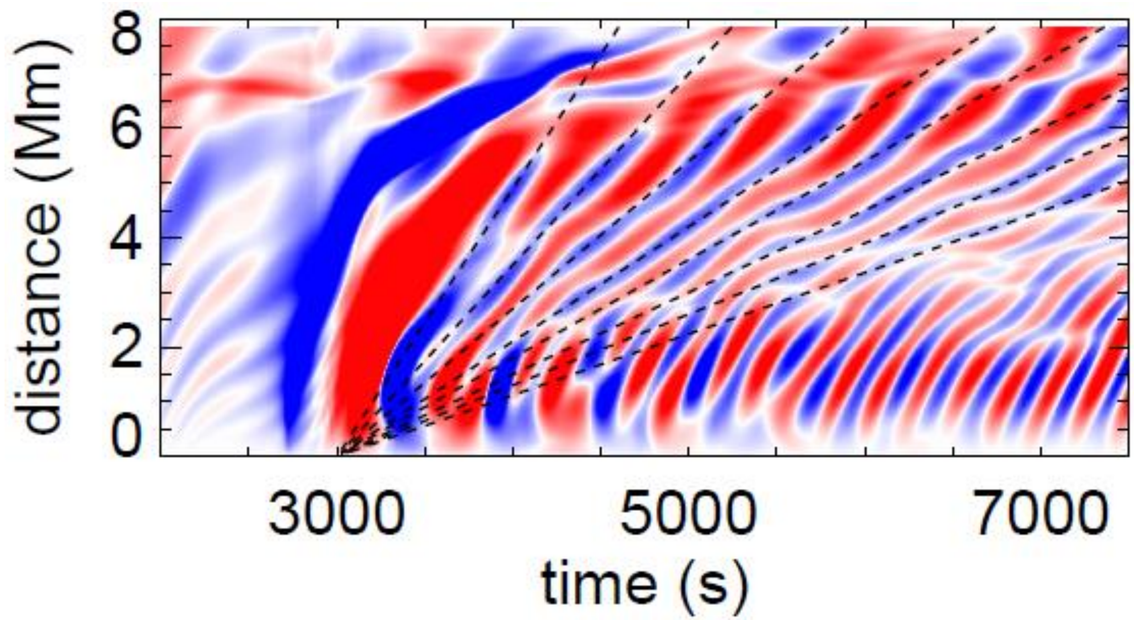
$$v_{ph,ap} = \frac{\sigma_{ap}}{k_{r,ap}} = -\frac{\sigma_A(r)}{t\frac{d\sigma_A}{dr}}$$



general formulation
→Kaneko et al. 2015

Application to Simulation Result 1/5





$$v_{ap} = 1 - 5 \text{ km/s}$$

Application to Simulation Result 2/5

1. The flux rope is regarded as a concentric cylinder.

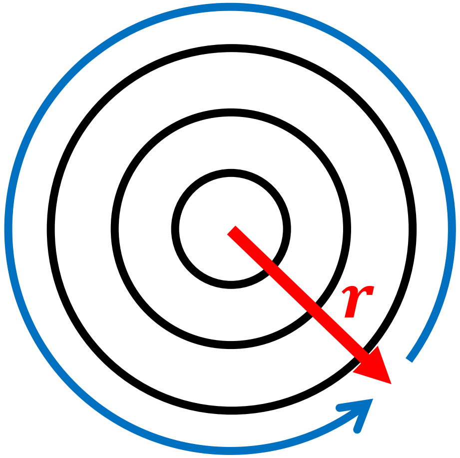
$$k_{ap}(r,t) = -t\frac{d\sigma_A}{dr}$$

$$v_{ap}(r,t) = -\frac{\sigma_A(r)}{t\frac{d\sigma_A}{dr}}$$

$$\sigma_A = 2\pi v_A/L = v_A/r$$
$$(L = 2\pi r)$$

$$k_{ap}(r,t) = \frac{t}{r}\left(\frac{v_A}{r} - \frac{dv_A}{dr}\right)$$

$$v_{ap}(r,t) = \frac{1}{t}\frac{v_A(r)}{\frac{v_A}{r} - \frac{dv_A}{dr}}$$



$L(r) = 2\pi r$

Application to Simulation Result 3/5

2. Gradient of Alfven velocities is negligible.

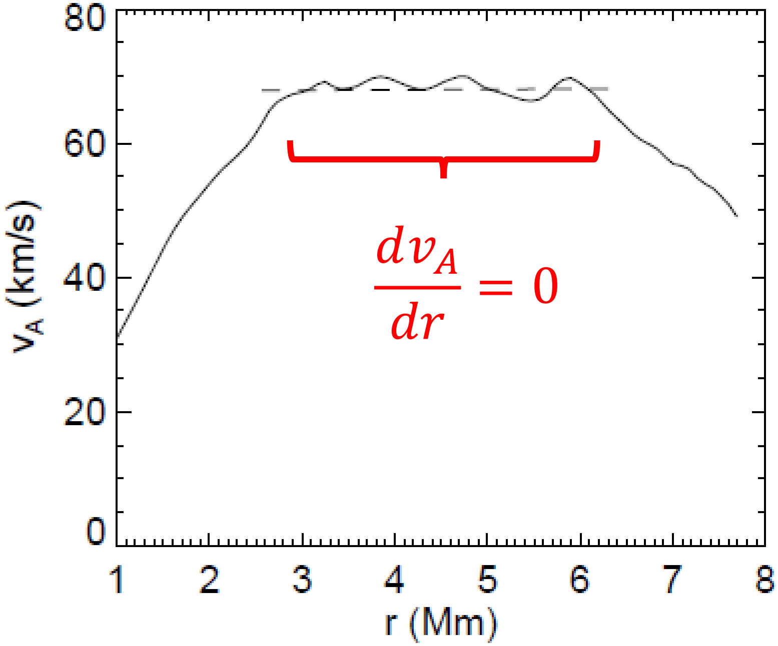
$$k_{ap}(r,t) = \frac{t}{r}\left(\frac{v_A}{r} - \frac{dv_A}{dr}\right)$$

$$v_{ap}(r,t) = \frac{1}{t}\frac{v_A(r)}{\frac{v_A}{r} - \frac{dv_A}{dr}}$$

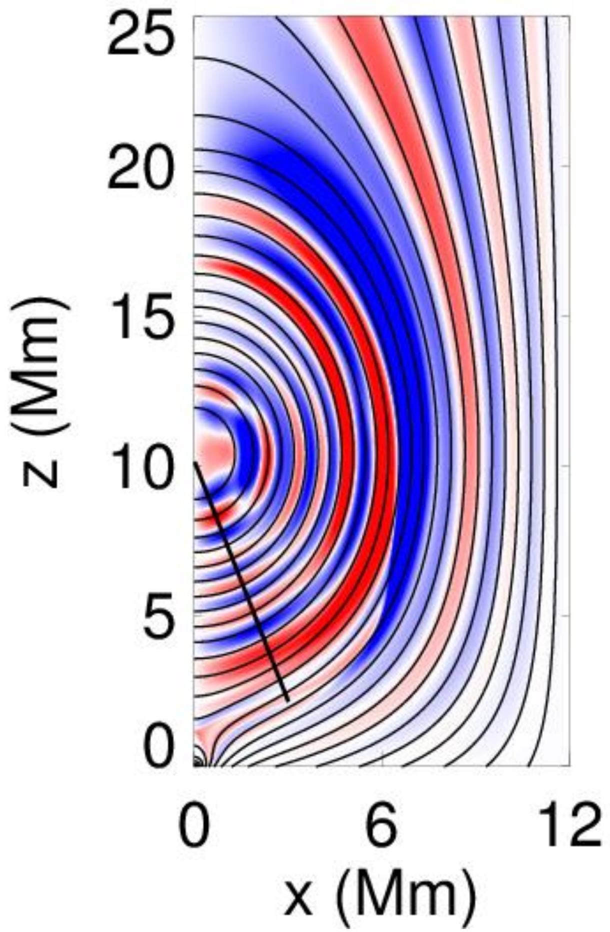
$$\Downarrow \quad \frac{dv_A}{dr} = 0$$

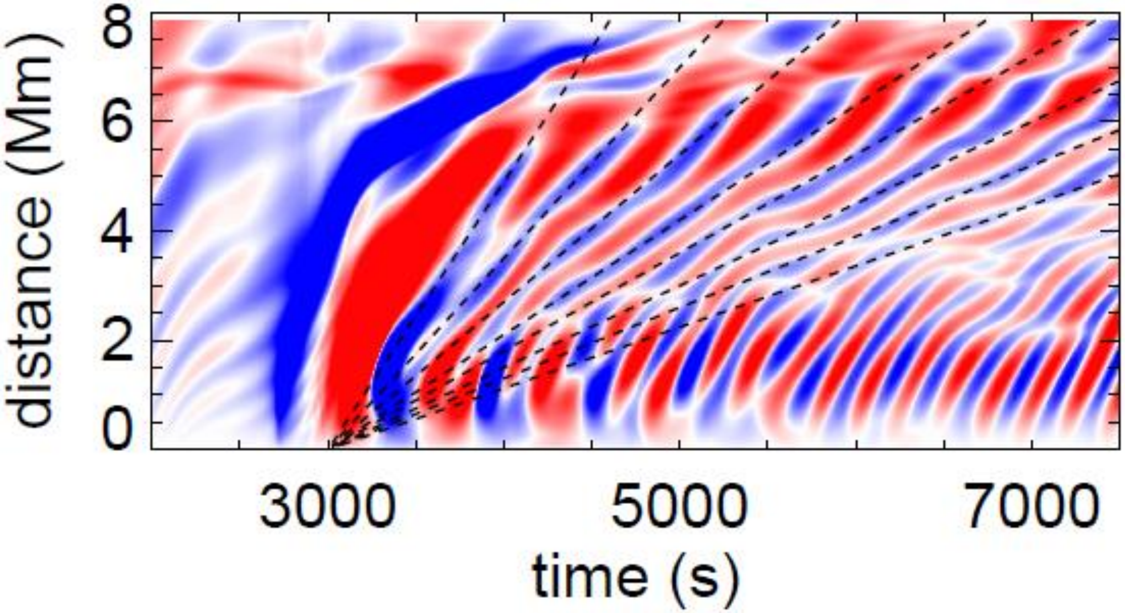
$$\boxed{k_{ap}(r,t) = t\frac{v_A(r)}{r^2} \qquad v_{ap}(r,t) = \frac{r}{t}}$$

harmonic mean Alfven velocity of each mag. surface along r



Application to Simulation Result 4/5



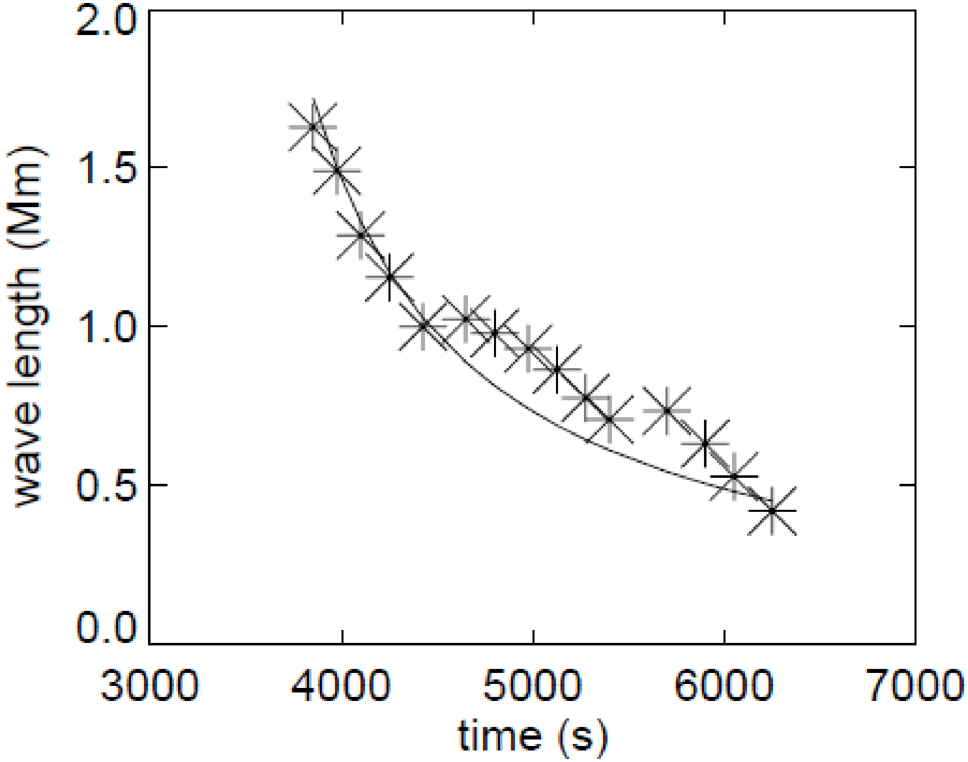


dashed lines: $\dfrac{r}{t - t_i} = v_{ap},$

$v_{ap} = 1 - 5\ \mathrm{km/s} \qquad t_i = 3000\ \mathrm{s}$

Application to Simulation Result 5/5

wavelength @ r=4 Mm



Dots:
wave length in r-direction

Solid line:
apparent wavelength computed by

$$\lambda_{ap}(r,t) = \frac{2\pi}{k_{ap}} = \frac{1}{t - t_i}\frac{2\pi r^2}{v_A(r)}$$

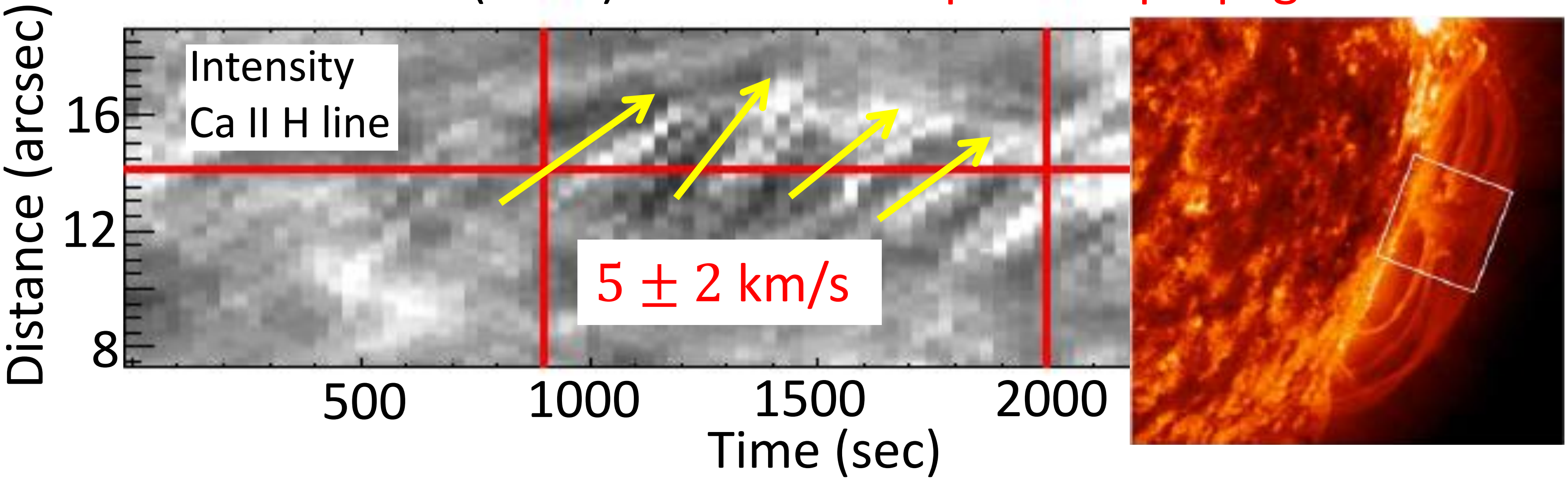
where

$$r = 4\ \mathrm{Mm}, \qquad t_i = 3000\ \mathrm{s},$$

$$v_A = 70\ \mathrm{km/s}$$

Applicability to Observation

Schmieder et al. (2013): cross-field superslow propagation



Fast mode model:

$$v_f = \sqrt{C_s^2 + V_A^2} \approx 75 \sim 750 \text{ km/s}$$

$$C_s = 10 \text{ km/s} \quad (T = 8000\text{K})$$

$$V_A = 70 \sim 750 \text{ km/s}$$
$$(B = 7.5 \text{ G}, n_e = 10^{9-11} \text{ cm}^{-3})$$

Apparent propagation model:

$$v_{ap} = \frac{\sigma}{t\frac{d\sigma}{dr}} \approx \frac{r}{t} = 3 \sim 6 \text{ km/s}$$

$$r = 4 \text{ arcsec}, \quad t = 500 \sim 1000 \text{ s}$$

Order-of-estimate is O.K.

Application to coronal potential arcade

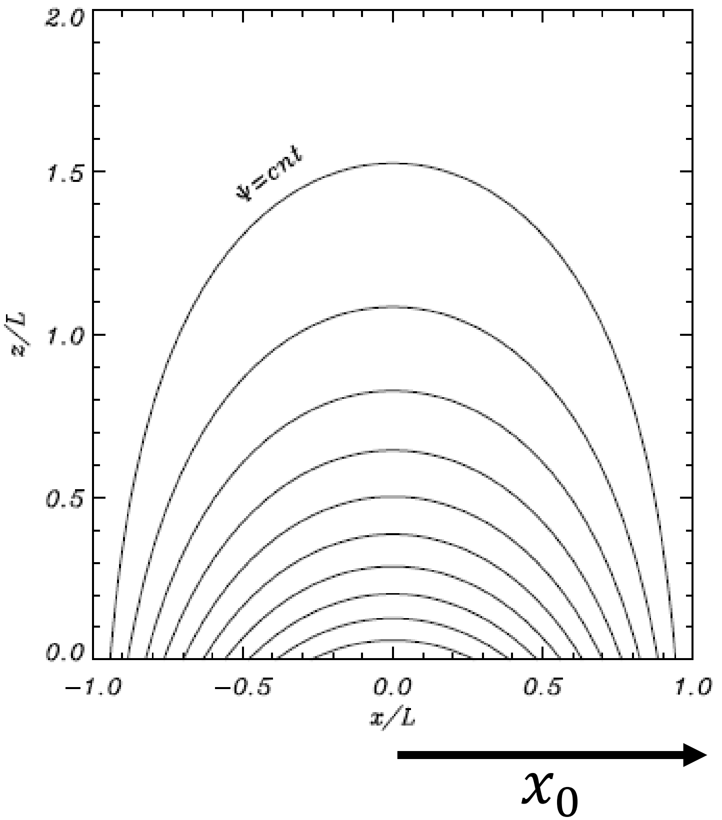
Potential arcade field
in stratified atmosphere

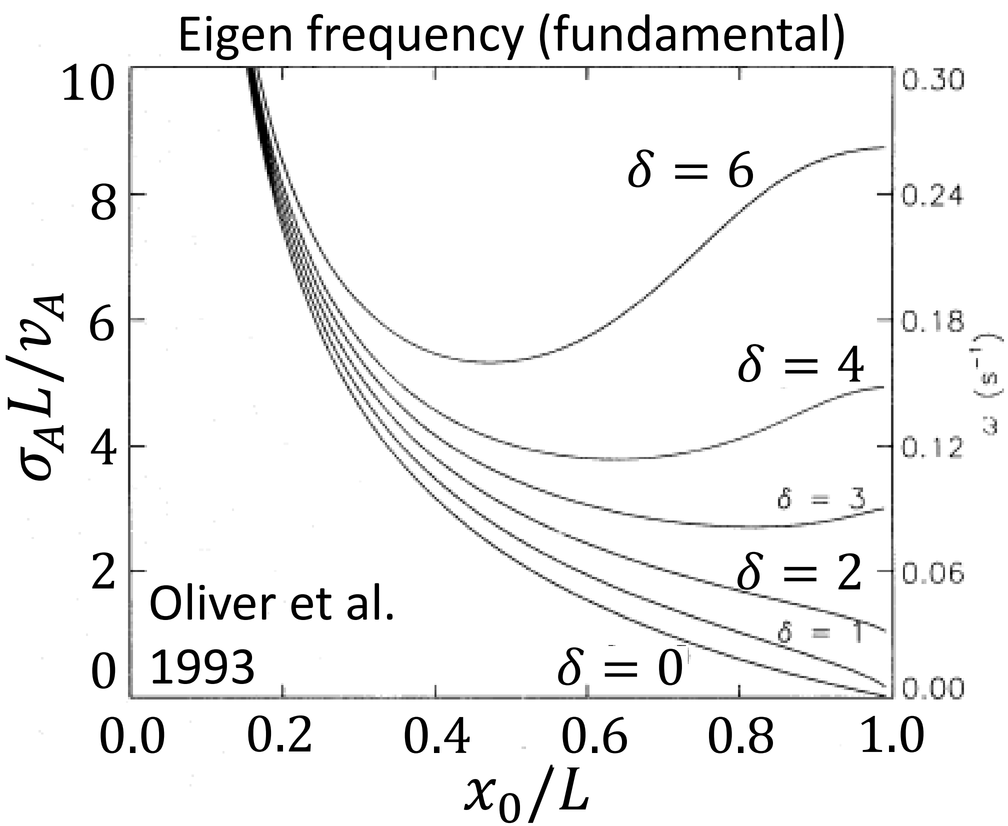
$$\rho = \rho_0 \exp(-y/\Lambda)$$

$$B = B_0 \exp(-y/\Lambda_B)$$

parameter

$$\delta = \frac{\Lambda_B}{\Lambda}$$





Application to coronal potential arcade

phase speed:

$$v_{ph,ap} = -\frac{\sigma_A(x_0)}{t\,\frac{d\sigma_A}{dx_0}}$$

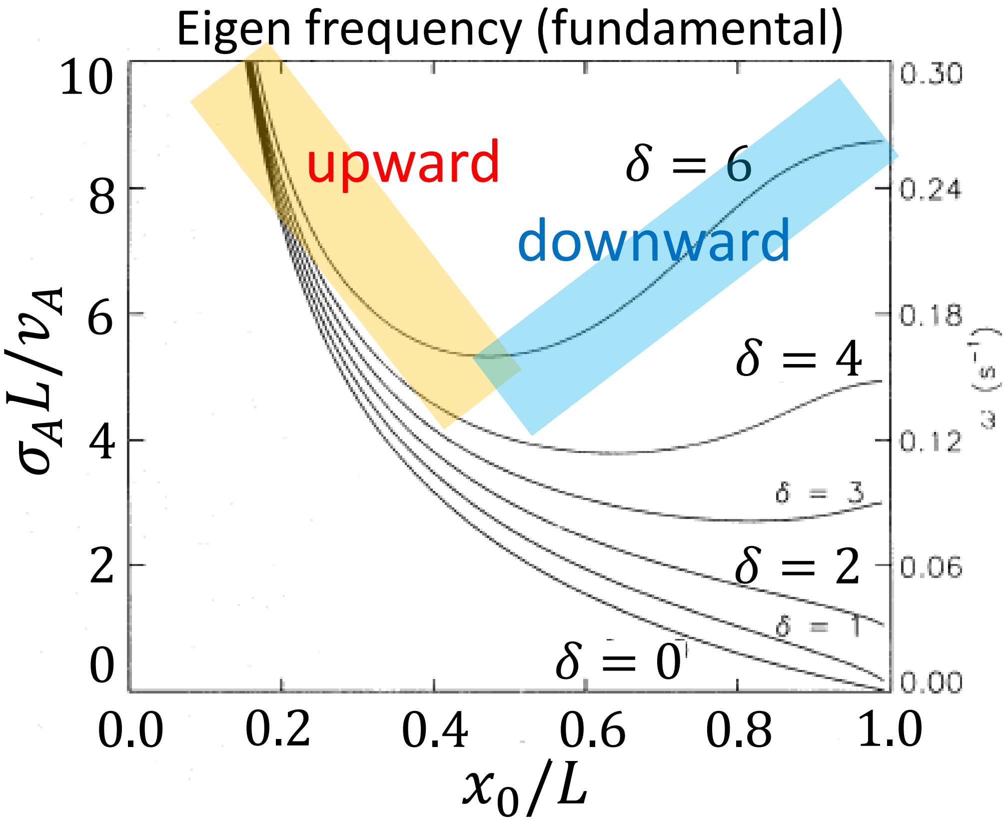
$\delta < 2$

Everywhere upward

$\delta > 3$

Upward and **downward**

<u>coexist.</u>



Demonstration (δ=1)

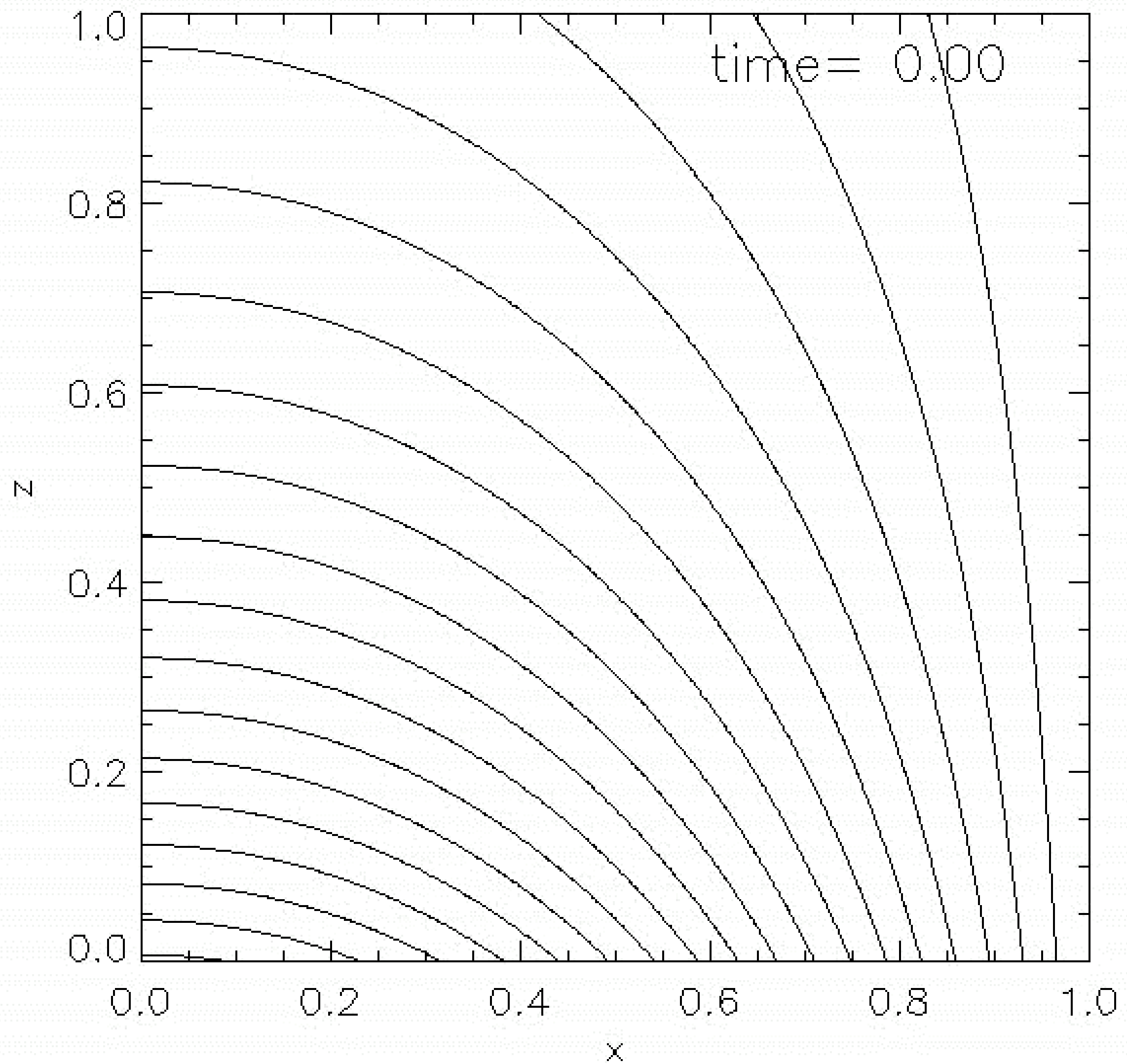
$$\delta = 1$$

Everywhere upward

Alfven mode (Oliver et al.,1999)

$$\frac{d^2 v_y}{dx^2} + \frac{\omega^2}{v_{A0}} K v_y = 0$$

$$K = \left[\frac{\cos\left(\frac{x_0}{\Lambda_B}\right)}{\cos\left(\frac{x}{\Lambda_B}\right)}\right]^{\delta} \cos^{-2}\left(\frac{x_0}{\Lambda_B}\right)$$



Demonstration (δ=6)

$$\delta = 6$$

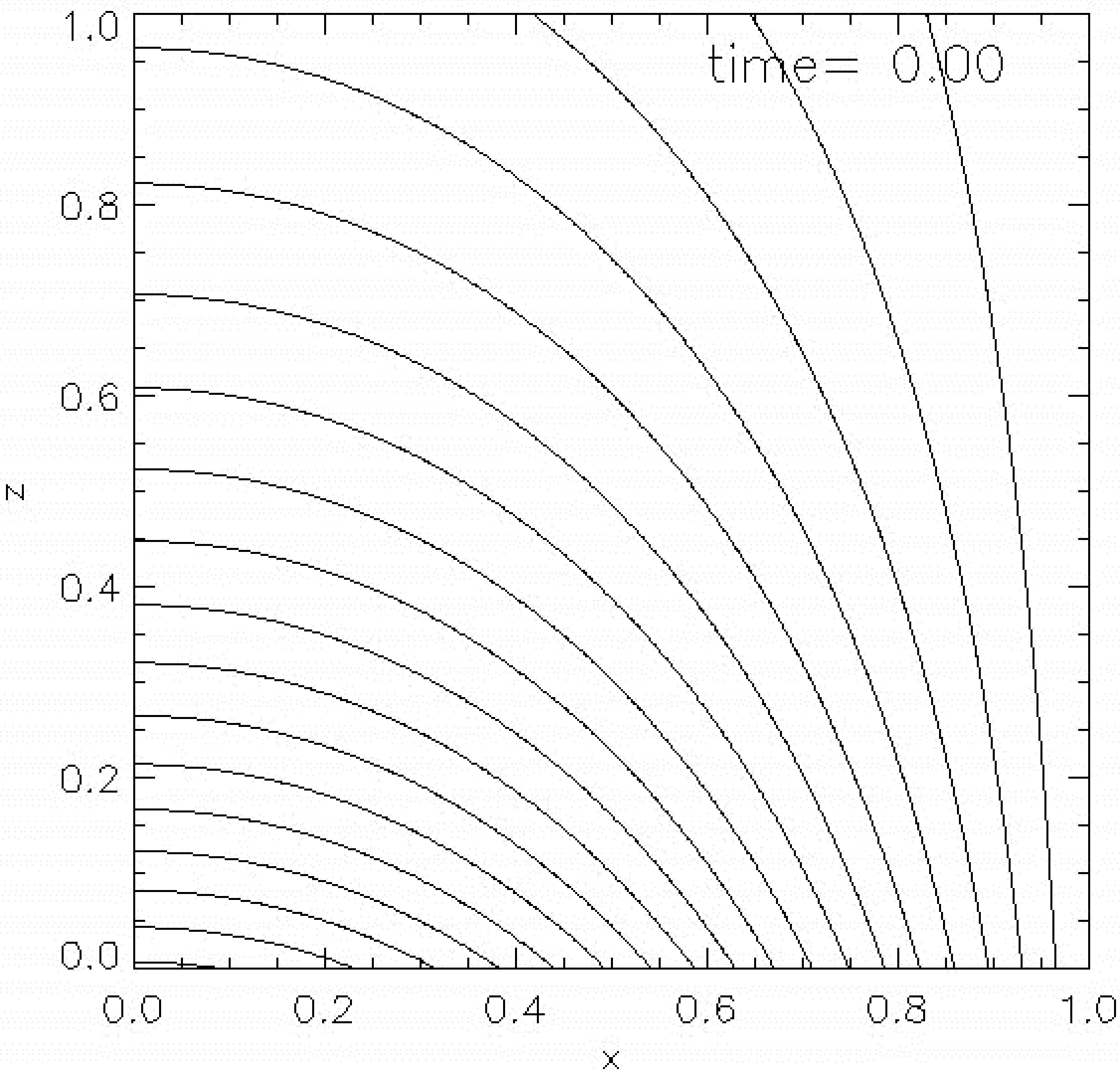
Upward and **downward**

coexist.

Alfven mode (Oliver et al.,1999)

$$\frac{d^2 v_y}{dx^2} + \frac{\omega^2}{v_{A0}} K v_y = 0$$

$$K = \left[\frac{\cos\left(\frac{x_0}{\Lambda_B}\right)}{\cos\left(\frac{x}{\Lambda_B}\right)} \right]^{\delta} \cos^{-2}\left(\frac{x_0}{\Lambda_B}\right)$$



Discussion

$$\Lambda_B = \frac{\delta}{\Lambda}$$

δ ⟵ derived by apparent phase speed

Λ ⟵ derived by temperature

where Λ:pressure scale height

Λ_B:magnetic pressure scale height

We can know magnetic pressure scale height Λ_B by the analysis of phase speed of apparent propagation.

What can we do by knowing Λ_B?

Discussion

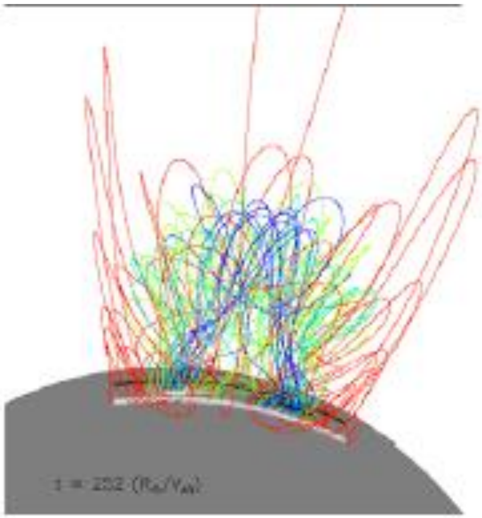
- Torus instability

criteria: $n_{cr} < n = -\dfrac{d \log B_p}{d \log h} = \dfrac{y}{\Lambda_B}$

where $n_{cr} = 1 - 2$

(e.g. Kliem & Torok, 2006; Fan & Gibson, 2007)

$\boxed{y > y_{cr} = n_{cr}\Lambda_B \quad \text{:critical height}}$



Fan 2010

If Λ_B is derived from the apparent phase speed, we can know critical height of torus instability.

$\boxed{\text{Flare/CME prediction is possible ?}}$

Summary for application to potential arcade

- Phase velocity of apparent propagation depends on $\delta\ (= \Lambda_B/\Lambda)$.
- We can know δ or Λ_B from phase speed of apparent propagation.
- Only from the direction of propagation, we can estimate how large δ is (whether lager than 2 or not).
- Hopefully, applicable to flare/CME prediction

(Apparent) Group velocity

$$\sigma_{ap} = \frac{\partial \varphi}{\partial t} = \sigma(z), \quad k_{ap} = -t\frac{d\sigma}{dz}$$

$$dk_{ap} = -\frac{d\sigma}{dz}dt - t\frac{d^2\sigma}{dz^2}dz \Rightarrow \frac{\partial z}{\partial k_{ap}} = -\left(t\frac{d^2\sigma}{dz^2}\right)^{-1}$$

$$v_g = \frac{\partial \sigma_{ap}}{\partial k_{ap}} = \cancelto{0}{\frac{\partial \sigma_{ap}}{\partial t}}\frac{\partial t}{\partial k_{ap}} + \frac{\partial \sigma_{ap}}{\partial z}\frac{\partial z}{\partial k_{ap}} = -\frac{\frac{d\sigma}{dz}}{t\frac{d^2\sigma}{dz^2}} \neq 0$$

- We can mathematically derive group velocity.
- The group velocity is not zero across mag. surfaces…

(Apparent) Group velocity

Questions:

- Does the group velocity have physical meaning?

→ Can we find physical variables propagating with the group velocity?

→ Answer: Yes. Shown later.

- Does it mean energy propagation across mag. surfaces?

→ Answer: ?? (depends on interpretation?)

Example

example:

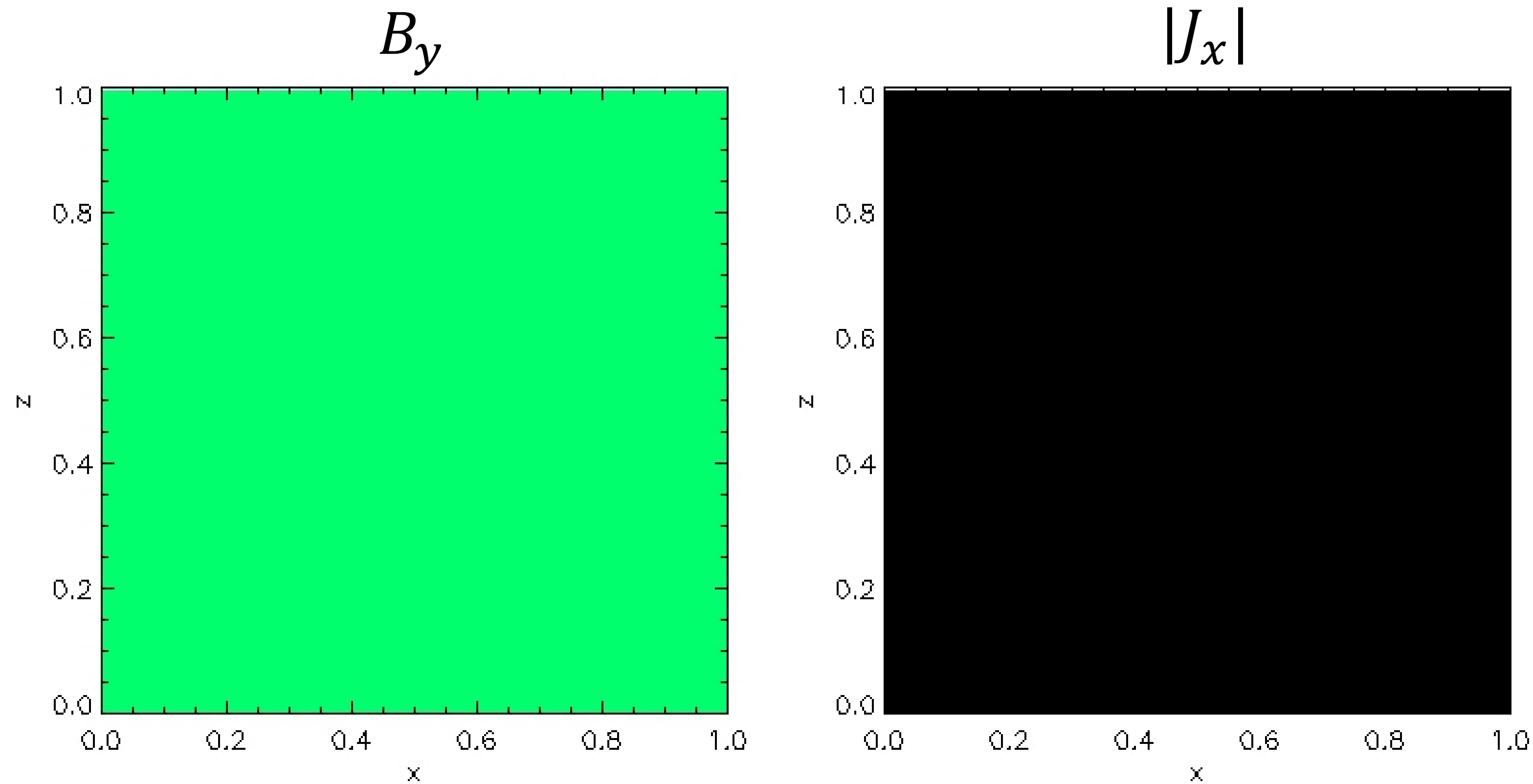
$$\sigma(z) = 5\left(1 + \cos\left(\frac{\pi}{2}z\right)\right)$$

$$\frac{d\sigma}{dz}(z) = -\frac{5\pi}{2}\sin\left(\frac{\pi}{2}z\right) \qquad \frac{d^2\sigma}{dz^2}(z) = -\frac{5\pi^2}{4}\cos\left(\frac{\pi}{2}z\right)$$

$$\Rightarrow \quad v_{ph} = -\frac{\sigma(z)}{t\,\dfrac{d\sigma}{dz}} > 0, \qquad v_g = -\frac{\dfrac{d\sigma}{dz}}{t\,\dfrac{d^2\sigma}{dz^2}} < 0$$

In this particular case, the phase velocity and group velocity are easily distinguished by the direction of propagation.

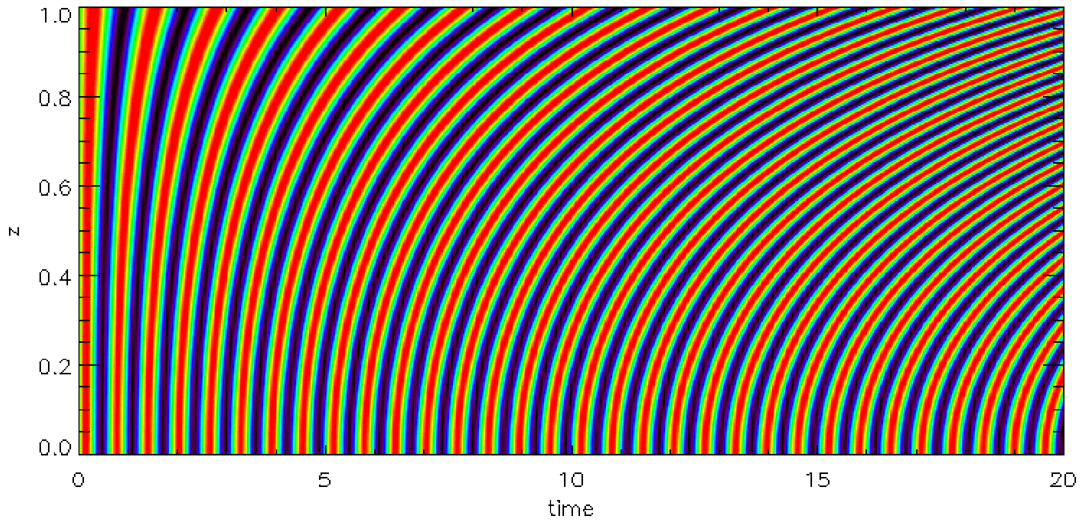
Example: current wave



$$v_{ph} = -\frac{\sigma(z)}{t\,\frac{d\sigma}{dz}} > 0$$

$$v_g = -\frac{\frac{d\sigma}{dz}}{t\,\frac{d^2\sigma}{dz^2}} < 0$$

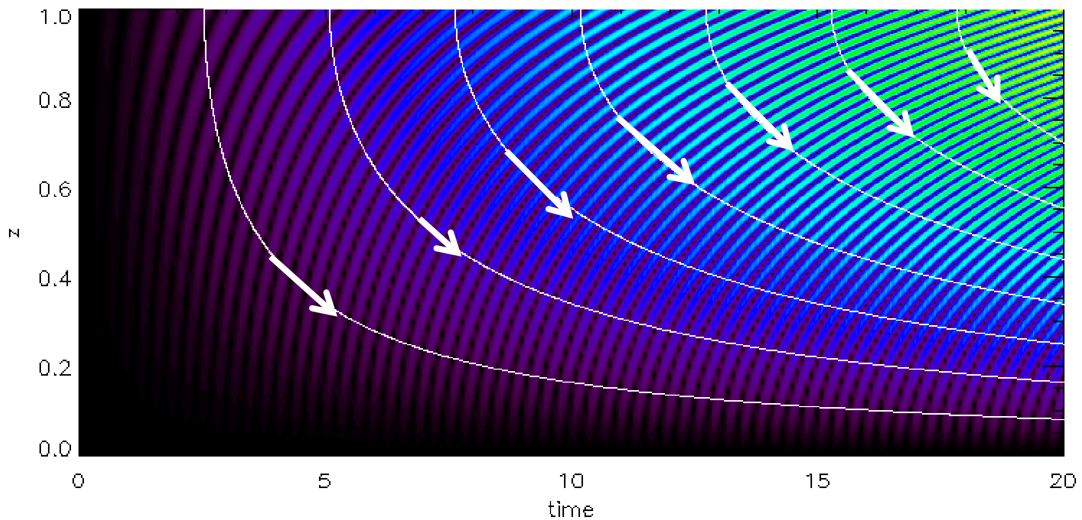
Example:current wave



B_y

$$v_{ph} = -\frac{\sigma(z)}{t\,\frac{d\sigma}{dz}} > 0$$

Phase propagates upward.



J_x

$$v_g = -\frac{\frac{d\sigma}{dz}}{t\,\frac{d^2\sigma}{dz^2}} < 0$$

Isocontour of $|J_x|$ propagates downward

Discussion 1/3

- Proof: current wave propagates with the group velocity.

$$k_{ap} = -t\frac{d\sigma}{dz} \Rightarrow dk_{ap} = -\frac{d\sigma}{dz}dt - t\frac{d^2\sigma}{dz^2}dz$$

$$dk_{ap} = 0 \leftrightarrow \frac{dz}{dt} = -\frac{\frac{d\sigma}{dz}}{t\frac{d^2\sigma}{dz^2}} = v_g$$

The group velocity represents the propagation speed of constant apparent wave number.

Discussion 2/3

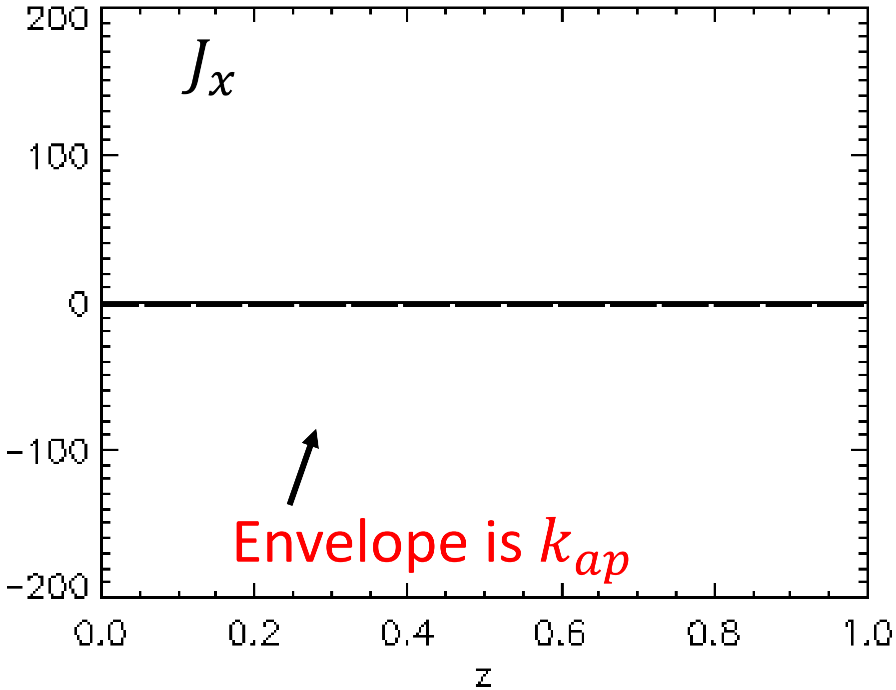
□ Proof: current wave propagates with the group velocity.

$$B_y = \sin(\sigma(z)t)$$

$$J_x = \frac{\partial B_y}{\partial z} = -t\frac{d\sigma}{dz}\cos(\sigma(z)t)$$

$$= \underline{k_{ap}}\cos(\sigma(z)t)$$

Envelope of current is proportional to k_{ap}.



Envelope of current moves with the group velocity v_g.

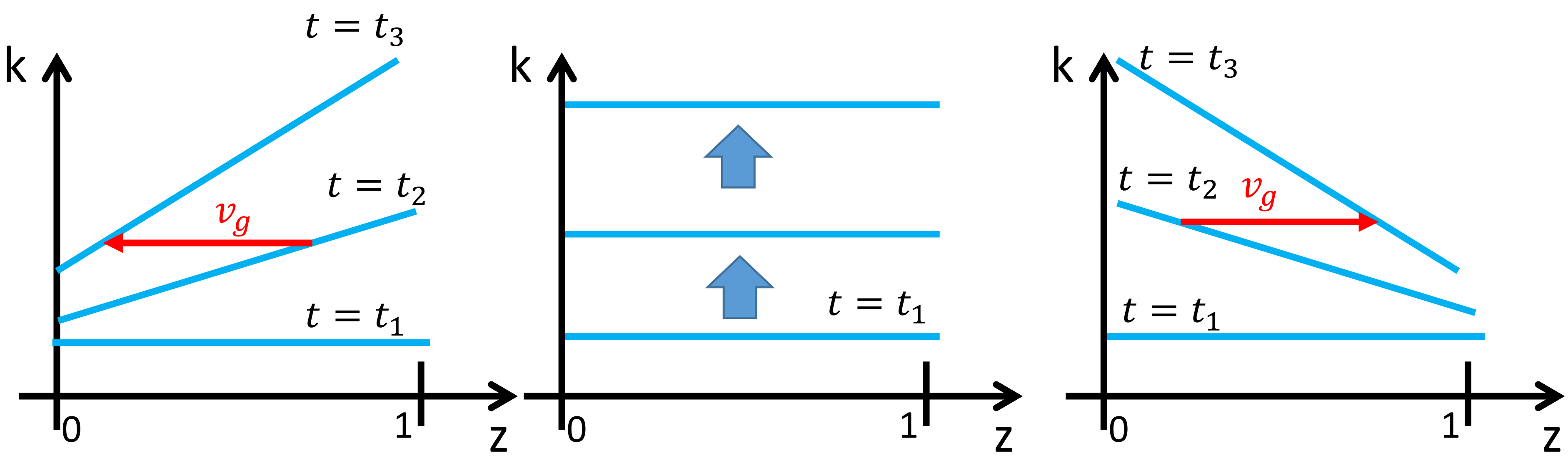
Discussion 3/3

$$\frac{\partial k_{ap}}{\partial t} = -\frac{d\sigma}{dz} \qquad v_g = -\frac{\frac{d\sigma}{dz}}{t\,\frac{d^2\sigma}{dz^2}}$$

$\left|\frac{d\sigma}{dz}\right|$ monotonically increase

$\left|\frac{d\sigma}{dz}\right|$ constant

$\left|\frac{d\sigma}{dz}\right|$ monotonically decrease



Conclusion

- Phase mixing of Alfven/slow mode wave can be observed as cross-field superslow propagation.
  - Apparent wavelength and phase velocity depends on gradient of Alfven frequencies.
- In our prominence model, apparent wavelength and phase velocity agrees with theoretical values computed by in-plane Alfven velocity.
- In the case of coronal potential arcades, the phase velocity depends on the ratio between mag. scale height and pressure scale height.
- Group velocity exists. It corresponds to apparent propagation speed of current density.