

Abstract

Localised enhancements and depressions of plasma density and magnetic field strength ("blobs") are common in the edge region of tokamaks. The existing fusion plasma literature on this subject focuses primarily on large blobs for which a fluid description is appropriate. However, the populations of blobs in the edge region may include some whose characteristic lengthscales are on the order of the ion Larmor radius. The dynamics and evolution of such blobs are unlikely to be wholly capturable by fluid models, inviting the use of a hybrid model for which we treat ions kinetically as particles and electrons as a massless fluid. Here, therefore, we report hybrid simulations of ion gyro-scale blobs with parameters characteristic of medium-size tokamaks such as MAST. We find that ion gyro-scale blobs are advected with the background flow, and develop a twin-celled vortex structure. Asymmetry then arises from finite ion Larmor radius kinetics, manifesting in the size of the internal vortices, the shape of tails forming from blob ejecta, and the growth of a Kelvin-Helmholtz instability.

Hybrid Physics

A hybrid model has numerous advantages, including the ability to capture the non-linear interaction between ion gyration and plasma inhomogeneity; cross-scale coupling between ion gyro-scale kinetic modes and fluid MHD-like modes; the ability to resolve phenomena on much shorter timescales than fluid models; and the ability to study inhomogeneities in configuration space and non-Gaussian ion velocity distributions.

The features of our hybrid code are as follows:

- It combines a particle-in-cell (PIC) algorithm for kinetic ions with an electron fluid.
- PIC codes subject weighted pseudo-particles to the Lorentz force in a continuous phase space, with E, B, J fields on a staggered grid.
- The code used is "2D3V" - a 2d simulation plane with 3 field components, e.g. $B_{x,y,z}(x, y, t)$.

For our model we make the following assumptions:

- Inertia-less electrons, " $m_e = 0$ "
- $\nabla \cdot \mathbf{E}$ is negligible on length scales of interest, implying charge neutrality $en_e = q_i n_i$
- Collisionless plasma, $\mu = 0$
- Ideal, isothermal electron gas, $\nabla \cdot \underline{P}_e = \nabla p = kT_e \nabla n$
- Low frequency limit, so neglect displacement current in Ampère's Law.

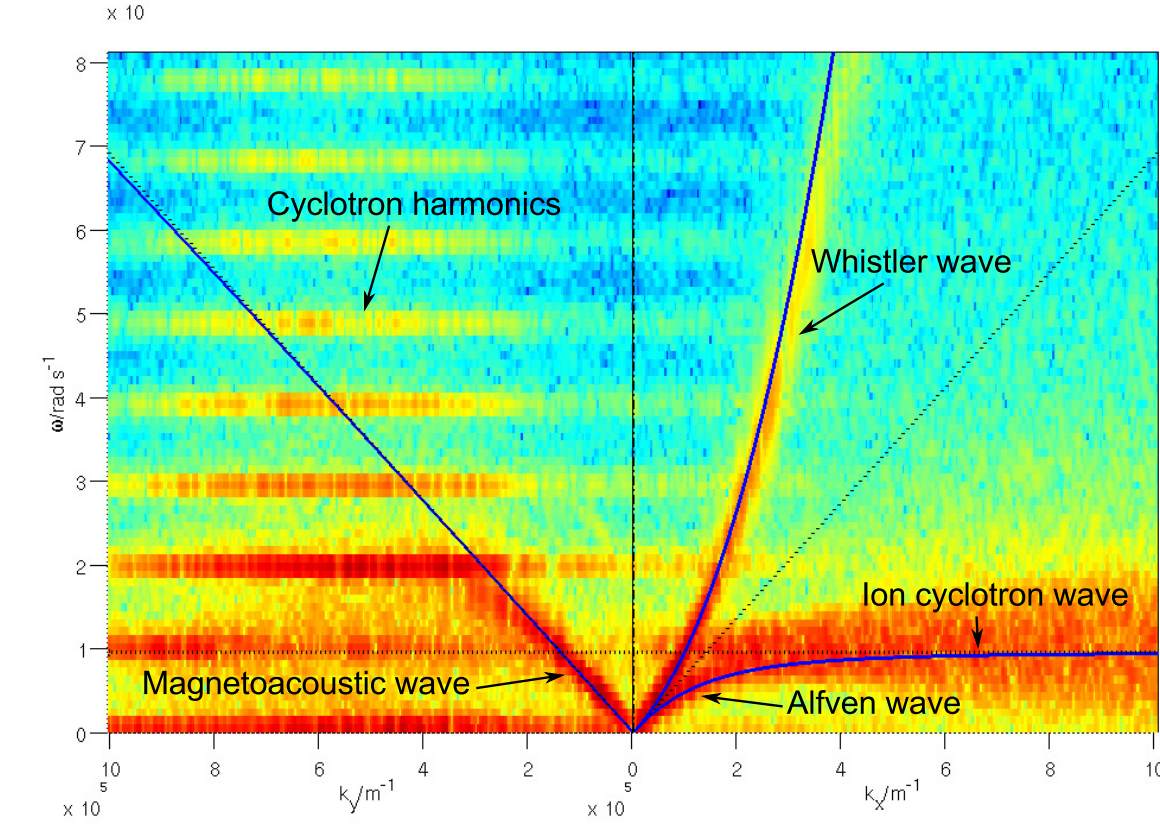
From the electron fluid momentum equation,

$$n_e m_e \frac{d\mathbf{v}_e}{dt} = -en_e(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \nabla \cdot \underline{P}_e + \mu \quad (1)$$

and Ampère's Law, we arrive at the equation we use to update the electric field in the code:

$$\mathbf{E} = -\frac{1}{en} \left(\frac{q_i}{e} kT_e \nabla n + \frac{\nabla B^2}{2\mu_0} - \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{\mu_0} + \mathbf{J}_i \times \mathbf{B} \right), \quad (2)$$

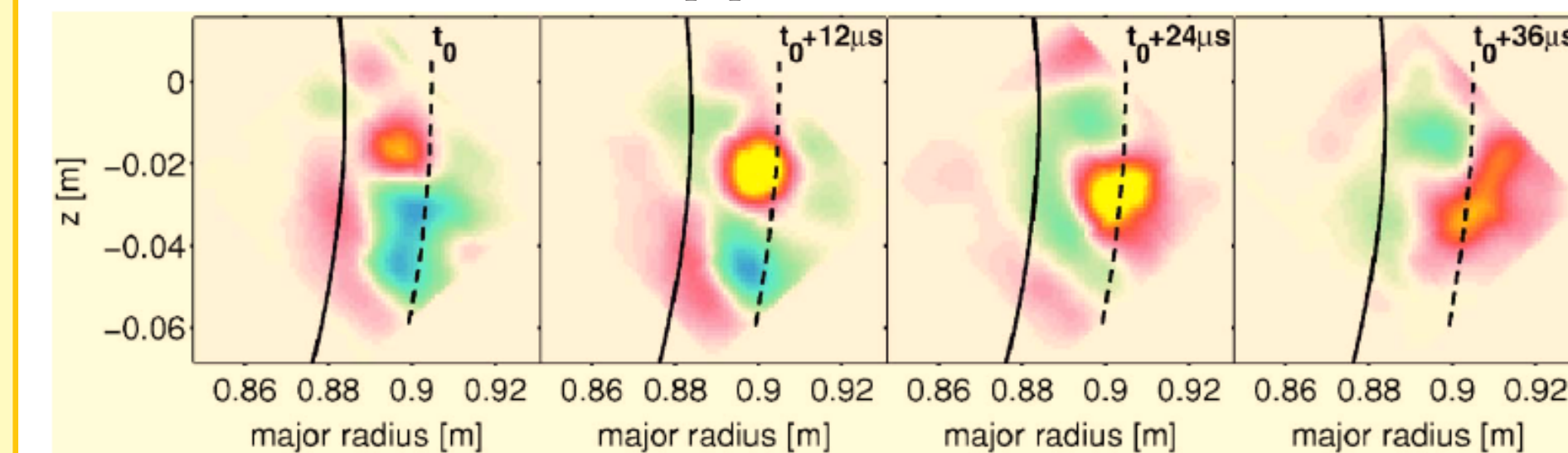
Under these assumptions electric fields cannot arise from charge separation; hence electric fields are purely motional in origin. This level of model description allows for the propagation of Alfvén and magnetoacoustic waves, ion cyclotron waves, lower hybrid waves and whistler waves. However, since whistler waves are undamped at the electron gyrofrequency, numerical damping must be applied to the electromagnetic fields to prevent their unstable growth.



Dispersions relation for a cold, magnetised plasma (blue line) parallel (right) and perpendicular (left) to the background magnetic field, over the FFT of thermal noise in a uniform plasma run of the code.

Blobs

The outer regions of tokamak plasmas, near the last closed flux surface, are observed to generate coherent propagating blobs [1]. These undergo radial transport, and can break free of the confinement region. Blobs with sufficient radial velocity can strike the walls of the tokamak, increasing local particle and energy fluxes, reducing divertor efficiency, and increasing impurity levels. Polarising curvature drifts of ions and electrons, giving rise to an electric field that may not be shorted out by parallel motion under tokamak edge conditions, give rise in the classic picture [2] to $\mathbf{E} \times \mathbf{B}$ radial blob motion. Models for blob transport and evolution have principally focused on fluid and multi-fluid descriptions, approached both analytically and numerically; see, for example, Refs. [3, 4, 5]. For a recent review of edge plasma physics in tokamaks we refer to Ref. [6].

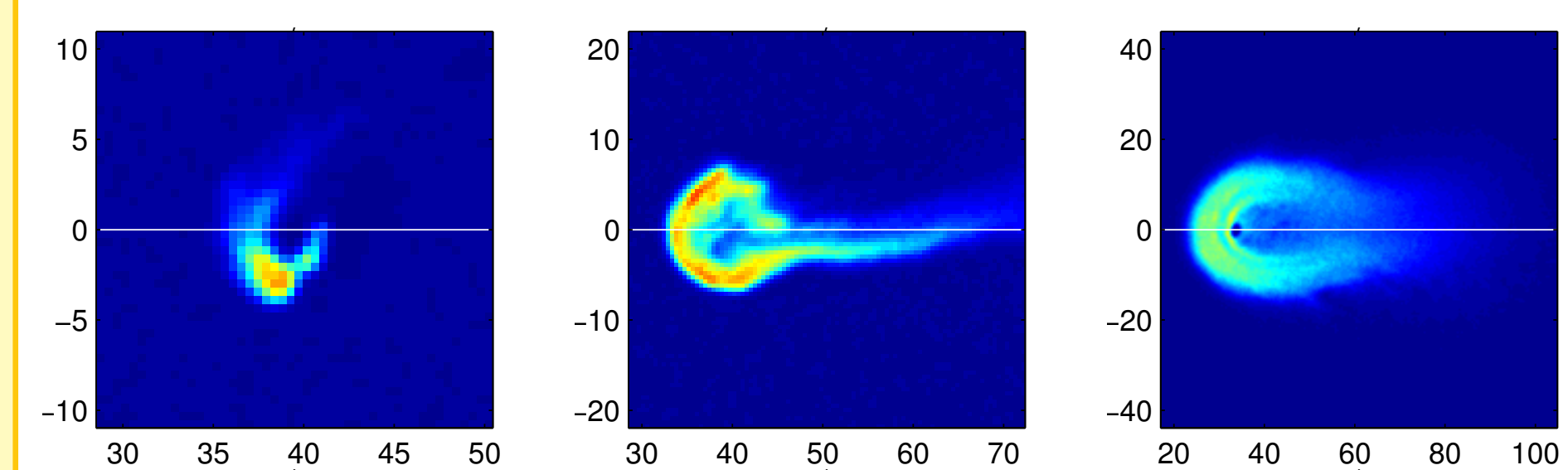


Blobs imaged in Alcator C-mod [1]

Results

Notable features of the evolution include:

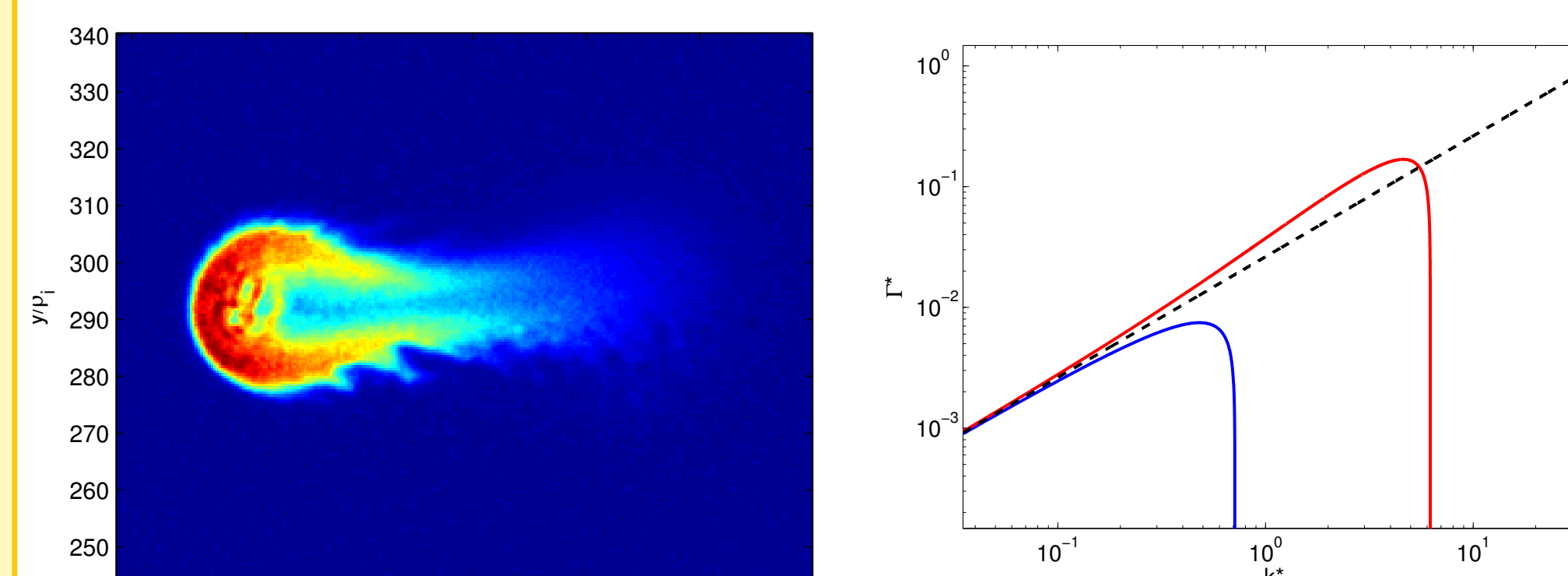
- Structures and gradients forming on the ion gyro-scale,
- Advection of the blob in the flow direction,
- Formation of twin-cell internal vortices,
- Growth of the K-H instability in a tail formed from blob ejecta,
- Asymmetry in all of the above, which becomes more pronounced for the smallest blobs.



Number density for 1, 4 and $10\rho_i$ blobs (left to right) after approximately 5 gyroperiods.

Kelvin-Helmholtz Instability

A Kelvin-Helmholtz (K-H) instability forms at the shear boundary at the edges of blobs. The instability allows for momentum transfer between the two sheared fluids, contributing to the internal convection pattern described above. The instability becomes most apparent in the tail of the larger blobs, where a clear asymmetry is visible. Both the growth rate and fastest growing mode scale size are seen to differ between the upper and lower edge of the blob as a consequence of FLR symmetry breaking.

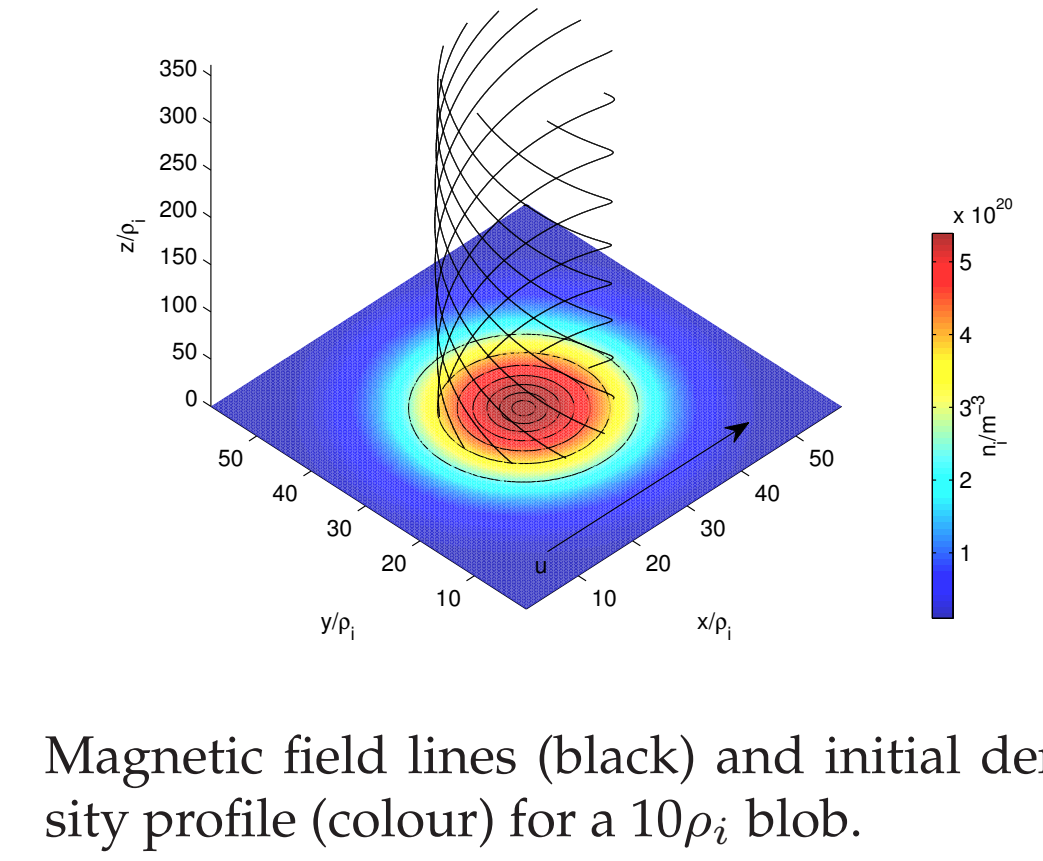


Number density plot showing K-H instability growing more quickly on the lower edge of a $10\rho_i$ blob after approximately 10 gyroperiods.

Theoretical growth rate Γ^* of the K-H instability on the upper (blue) and lower (red) edges, assuming an incompressible plasma with a finite Larmor radius. [8].

Simulations

Blobs are set up as flux ropes with Gaussian spatial profiles for enhancements of particle number density, and depressions of magnetic field strength out of the simulation plane, B_z . The magnitudes of the Gaussian enhancements and depressions are chosen such that blobs are set up in pressure equilibrium with the background flowing plasma, balancing combined gas and magnetic pressures.



Magnetic field lines (black) and initial density profile (colour) for a $10\rho_i$ blob.

The background plasma parameters are chosen to be approximately characteristic of edge conditions in a medium-size tokamak such as MAST:

- Number density, $n_0 = 10^{19} \text{m}^{-3}$
- Magnetic fields strength, $B_{z,0} = 0.4 \text{T}$
- Temperature, $T_0 = 4 \times 10^6 \text{K}$
- Thermal gyro-radius, $\rho_i = 8 \times 10^{-3} \text{m}$
- Background flow speed, $u_0 = 0.2v_A$

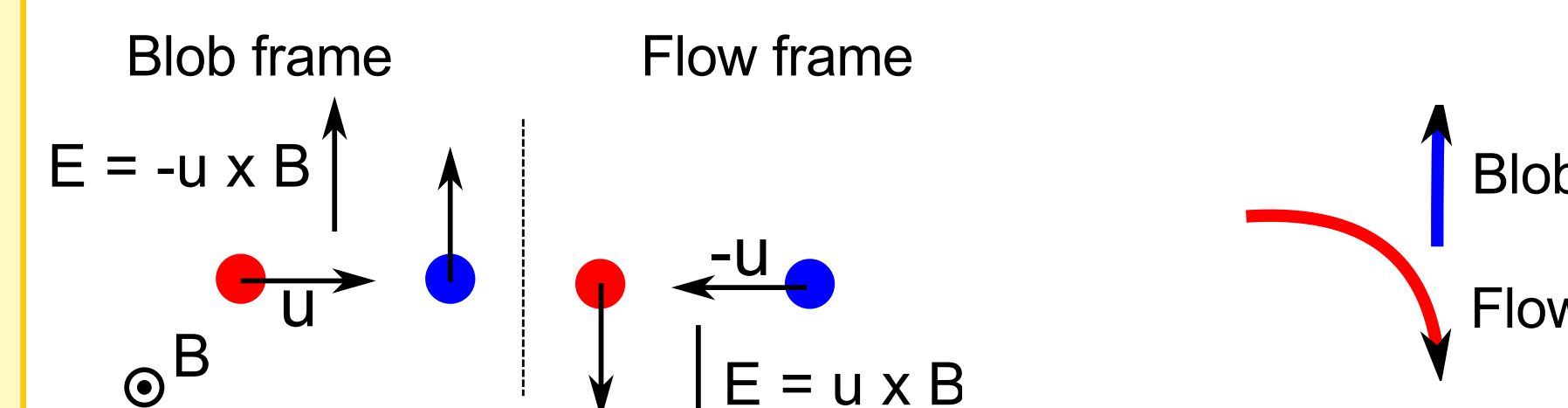
Finite Larmor Radius Symmetry Breaking

Since an ion traverses much of the blob as it gyrates in the magnetic field, we expect gyromotion to play a significant role in the dynamics and evolution of both ion populations. The chirality of ion gyration can thus break the symmetry of the evolution of an otherwise symmetric blob.

Substituting the electric field derived from the electron momentum equation into an ion momentum equation, we arrive at the following [7]:

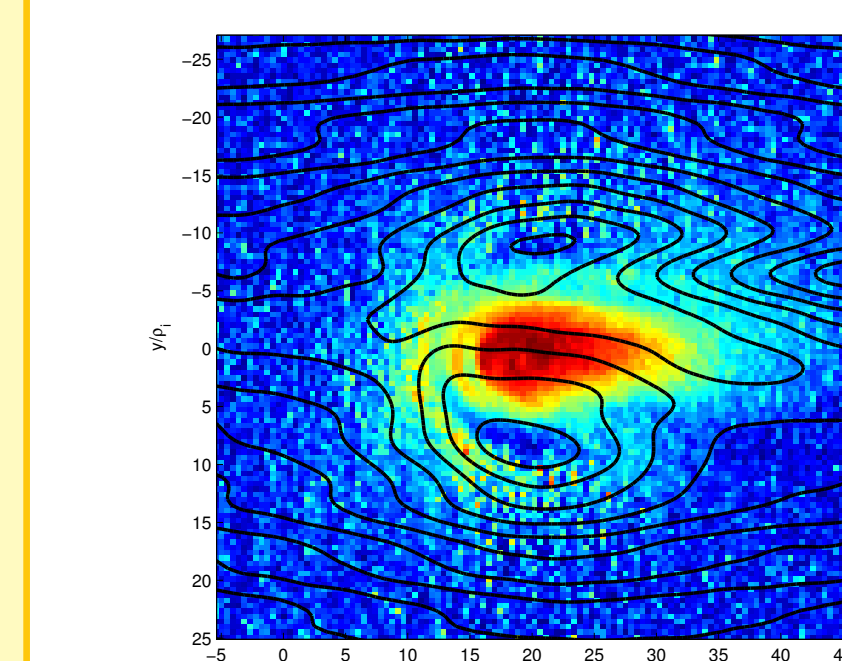
$$m_\alpha n_\alpha \frac{D\mathbf{v}_\alpha}{Dt} = en_\alpha \left(\mathbf{v}_\alpha - \frac{1}{n} \sum_{\alpha'} n_{\alpha'} \mathbf{v}_{\alpha'} \right) \times \mathbf{B} \quad (3) + \frac{n_\alpha}{n} (\mathbf{J} \times \mathbf{B} - \nabla p_e) - \nabla p_\alpha$$

The first term on the RHS represents an asymmetry due to the finite Larmor radius, while the following terms are usual for MHD. This asymmetry leads to momentum transfer between populations of ions with different velocities, in our case the background flow and blob ions. The form of this asymmetry is as follows:



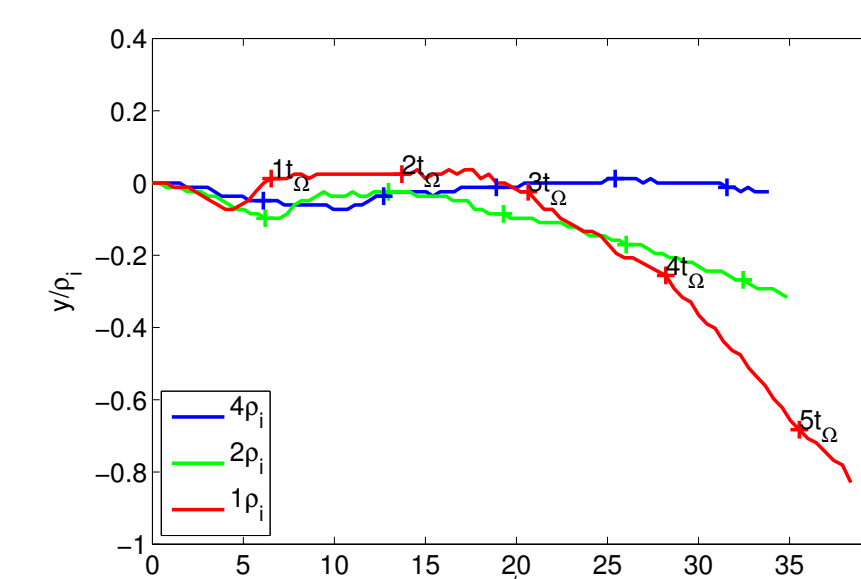
Blob particles (blue) feel the $\mathbf{E} = -\mathbf{u} \times \mathbf{B}$ electric field of the flow particles (red), causing them to move up, while the reverse happens for the flow particles.

Bulk Asymmetries



Streamlines (black) over bulk velocity (colour) for a $4\rho_i$ blob.

The twin-vortices deviate from the symmetric solution that would emerge from a purely fluid description. The difference in flow speeds between the upper and lower sides of the blob causes the vorticity cells to grow asymmetrically.



Trajectories of the centres of mass for blobs of radius $1, 2, 4\rho_i$

Blobs with $R_b \sim \rho_i$ exhibit a small but measurable downward deflection of up to a full gyroradius over 5.5 gyroperiods, an order of magnitude greater than the negligible deflection for larger blobs. This is a result of the downward deflection of the background flow by the FLR symmetry breaking mechanism.

Conclusions

The preceding results demonstrate how the hybrid model (fluid electrons and fully kinetic ions) embodied in the code captures key elements of the plasma dynamics and the evolution of the fields in ion gyro-scale blobs. The parameter set chosen approximates to edge plasma conditions in a medium-size tokamak such as MAST. Many of the phenomena described above are common to a purely fluid approach to blob simulation, including:

- Advection in the direction of the background flow
- Development of a twin-celled internal vortex pattern
- Growth of a Kelvin-Helmholtz instability in the tail of the blob

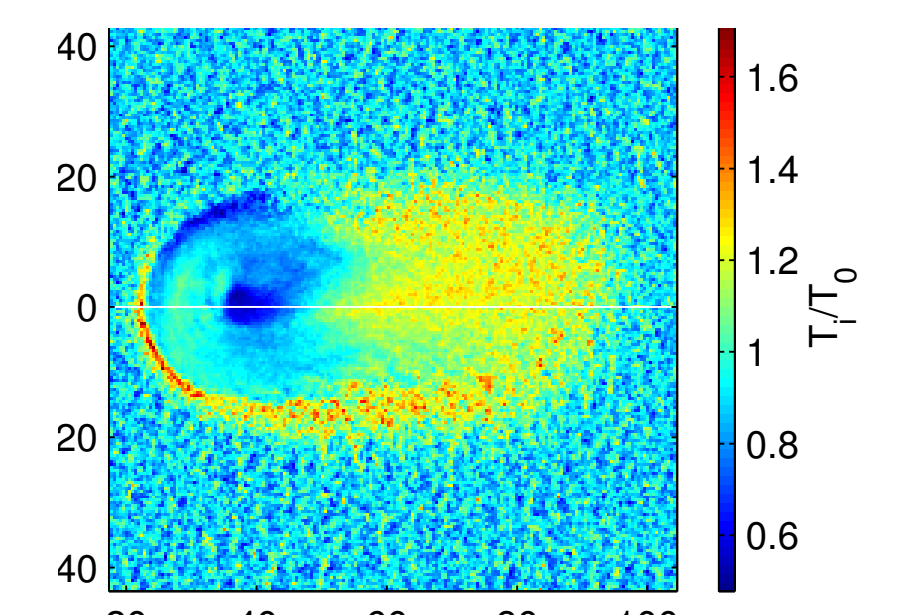
The substantial asymmetry which we find develops in these evolving features, which strengthens for smaller blobs, underlines the need to study small scale blob propagation using a hybrid method. The emergence of structures and gradients on ion gyro lengthscales, both internal to the blobs and at the boundary between the blob and external flowing plasma, further demonstrates the value of such a model.

We also conclude that FLR symmetry breaking plays a significant role in the evolution of blobs, manifesting as: a poloidal deflection of the blob plasma in the $\mathbf{u} \times \mathbf{B}$ direction; a difference in size between the twin vortices generated internal to the blob; and a difference in the growth rate of the Kelvin-Helmholtz instability between the upper and lower edges of the blob.

Future Work

From the results collected so far, two avenues for further research immediately present themselves for study in both the short and long term. These include:

- Quantitative study of the ability of smaller blobs, which cannot be detected, to heat the bulk plasma. Simulations have been carried out to compare the effect of many smaller blobs with a fewer larger blobs of the same total mass.
- Replacing single species blobs and background with a D-T plasma with α impurities, to examine the effect of populations with differing Larmor radii within a single blob.



Temperature plot showing asymmetric heating in the tail of a $8\rho_i$ blob.

Acknowledgements

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