CORONAL SEISMOLOGY BY MHD AUTOWAVES

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ABSTRACT

Slow magnetoacoustic waves are routinely observed in solar coronal structures. These waves can be strongly affected by non-adiabatic effects leading to self-organising compressible propagating disturbances — magnetoacoustic autowaves. Autowaves are propagating disturbances which have parameters independent of the excitation that are determined by the parameters of the medium only and, consequently, are potentially an ideal tool for coronal seismology. The influence of non-adiabatic effects is studied and magnetoacoustic autowaves are modelled with the extended Burgers equation. A numerical code is developed to study the evolution of such waves and a parametric study is carried out. Observable parameters of coronal magnetoacoustic autowaves could provide a tool for the determination of heat deposition mechanisms in the corona.

1. INTRODUCTION

Thanks to observations by satellites such as TRACE and SOHO it is now possible to observe slow magnetoacoustic waves in coronal loops and plumes. There are two main classes of observations. The first are of travelling waves, typically observed propagating up from the footpoints of coronal loops. These waves have been observed by TRACE and SOHO/EIT and are studied in [1], [2], [3], [4], [5], [6], [7], [8] and [9]. Secondly, Doppler shift oscillations observed by SUMER have revealed the existence of long wavelength standing wave oscillations in coronal loops. (References [10], [11] and [12].)

One of the important features of waves is that they carry information about the medium through which they propagate. This is the principle used in seismology and helioseismology to learn about the interior of the Earth and the sun respectively. It is hoped that the study of waves will lead to a similar theory for the corona through the science of coronal seismology. The development of this science relies on a detailed theory of wave propagation.

This paper studies magnetoacoustic autowaves. An autowave is a propagating disturbance which has parameters (such as speed, amplitude, frequency, etc.) which are independent of the initial excitation and so depend only on the properties of the medium through which it propagates. Such waves carry information only about their medium, which is what we would like to measure and not about the event that initiated them. This makes them ideal tools for coronal seismology.

Autowaves exist as the result of competition between a variety of different effects on a wave's evolution. Three effects are studied in this paper. The first is a wave amplification mechanism which acts most strongly at low frequencies. The second is non-linear steepening of the wave which moves energy from low frequencies to high frequencies. The third is high frequency dissipation which damps high frequency oscillations. It will be shown that this cycle of energy which is input at low frequencies, transferred to higher frequencies and then dissipated leads to the possible existence of magnetoacoustic autowaves.

The amplification mechanism considered in this project is thermal instability. The solar corona is continually radiating energy in addition to being heated by some unknown mechanism. Thermal instability occurs when a decrease in temperature leads to increased radiative energy losses. In this case, a section of the plasma which is initially cooler than the surrounding plasma cools more quickly. This instability has been studied extensively, [13] and used as an explanation for wide range of astronomical and solar phenomena such as prominences. It leads to condensations, regions which are cooler than their surroundings, as well as the amplification of magnetoacoustic waves.

There are a number of possible sources for high frequency dissipation such as resistivity, viscosity and thermal conductivity. According to theoretical estimations ([9] and [11]) the dominant wave damping mechanism for slow magnetoacoustic waves in coronal loops is thermal conductivity, and for this reason thermal conductivity is considered as the source of high frequency dissipation.

In this paper, an equation is derived for the evolution of magnetoacoustic waves under the influence of thermal conductivity, thermal instability and non-linearity. The existence of stationary solutions to this equation is shown, and the stability of these solutions is investigated numerically.

2. NON-ADIABATIC TERMS

2.1. Thermal Conductivity

In the solar corona magnetic conductivity is high along magnetic field lines, where heat is carried mainly by electrons and negligible across the magnetic field.

$$\kappa_{\perp} \approx 0 \tag{1}$$
$$\kappa \approx \kappa_{\parallel} \approx 10^{-11} T^{\frac{5}{2}} \ \ \mathrm{Wm^{-1}K^{-1}} \tag{2}$$

Thermal conductivity is introduced into the MHD equations as a term on the right hand side of the energy equation.

$$\frac{dp}{dt} - \frac{\gamma p}{\rho}\frac{d\rho}{dt} = (\gamma - 1)\left(\nabla \cdot \left(\kappa_{\parallel} \nabla T\right)\right) \tag{3}$$

Thermal conductivity can damp acoustic waves. As one increases κ, the coefficient of thermal conductivity, however, this wave damping reaches a maximum and begins to decrease again. In the limit of infinite thermal conductivity, there is no damping. This behaviour creates two regimes. In the adiabatic (low thermal conductivity) regime, sound waves propagate adiabatically at the adiabatic sound speed $(\gamma p/\rho)$ with small attenuation. In the isothermal regime, sound waves propagate isothermally at the isothermal sound speed (p/ρ). In the transition between these two regimes the attenuation becomes extremely large, and can damp away waves within a couple of wavelengths. According to theory, slow magnetoacoustic waves in the corona are strongly damped by thermal conductivity. Observations show strong damping, which is consistent with this theory. The amount of damping also depends on the wavelength, short wavelength waves are more strongly affected by thermal conductivity. Figure 1 shows how the the damping length depends upon the plasma temperature and the wavelength.

In the analysis that follows, it is assumed that thermal conductivity is weak, and therefore that we are within the quasi-adiabatic regime. In this regime damping due to thermal conductivity always increases at higher frequency, and so thermal conductivity acts as high frequency dissipation.

2.2. Radiative Instability

The second important non-adiabatic term is the radiative loss and heating term, which may lead to wave amplification due to thermal instability. The optically thin coronal plasma is continually losing energy due to radiation. These radiative losses depend upon temperature in complex manner that is determined by the atomic physics of the plasma. One approximation to this cooling function is that given by [14] which is shown in Figure 2.

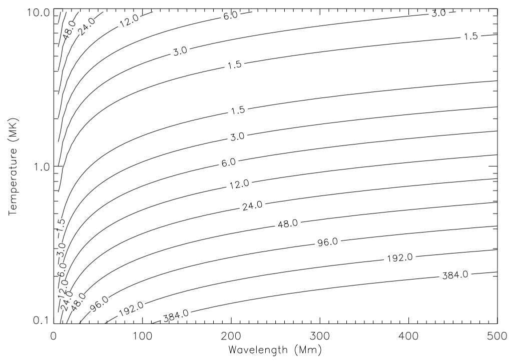

Figure 1. A contour plot showing the damping length due to thermal conductivity in wavelengths for various temperatures and velocities. This plot is generated for a density $\rho = 8 \times 10^{-13}$ kg m^{-3}.

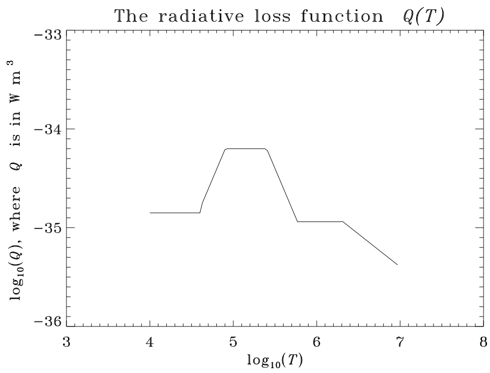

Figure 2. The dependence of the loss function on temperature as estimated by [14]

These radiative losses are represented by a function $\mathcal{L}(\rho, T)$ on the right hand side of the energy equation. This function is a combination of the the radiative losses described by [14] and a heating function, $\mathcal{H}$. The dependence of the heating function upon the parameters of the plasma is unknown. $\mathcal{L}$ is expressed as:

$$\mathcal{L} = n_e^2 \mathcal{Q}(T) - \mathcal{H} \tag{4}$$

Thermal instability may occur in any region where $\frac{d\mathcal{L}}{dT}$ is negative. In this case, a section of the plasma which is cooler than its surroundings will lose heat faster and hence continue to cool. The plasma is thermally unstable and this effect leads to wave amplification. [13]

3. EXTENDED BURGERS EQUATION

The goal is to derive an evolutionary equation for magnetoacoustic waves starting with the MHD equations. This derivation extends that carried out by [15].

$$\rho\frac{d\mathbf{V}}{dt} = -\nabla p - \frac{1}{\mu_0}\mathbf{B}\times\nabla\times\mathbf{B} \quad (5)$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{V}\times\mathbf{B}) \quad (6)$$

$$\frac{\partial p}{\partial t} + \nabla\cdot(\rho\mathbf{V}) = 0 \quad (7)$$

$$\frac{dp}{dt} - \frac{\gamma p}{\rho}\frac{d\rho}{dt} = (\gamma-1)\left(\nabla\cdot\left(\kappa_{\|}\nabla T\right) - \mathcal{L}\right) \quad (8)$$

The magnetic field is taken to be in the (x, z)-plane, $\mathbf{B_0} = B_0\sin\alpha\mathbf{x_0} + B_0\cos\alpha\mathbf{z_0}$, where α is the angle between the magnetic field and the z-axis. The dynamics of waves propagating along the z-axis are considered, so dependences upon x and y are ignored. Only quadratic non-linearity is considered, and the effect of thermal conductivity and cooling is assumed to be small.

Combining these equations gives an equation taking into account thermal conductivity, radiative loss/gain terms and quadratic non-linearity.

$$\mathcal{D}_{Az}\mathcal{D}_S V_z - C_{Ax}^2\frac{\partial^4 V_z}{\partial z^2\partial t^2} - \mathcal{A}\mathcal{D}_{Az}\int\frac{\partial^2 V_z}{\partial z^2}\,dt = \frac{1}{\rho_0}\left\{\mathcal{D}_{Az}\left[\frac{\partial N_6}{\partial t} - \frac{\partial}{\partial z}\left(C_S^2 N_8 + \frac{N_7}{\rho_0}\right)\right] - \frac{B_{0x}}{\mu_0}\frac{\partial^2}{\partial z\partial t}\left(\frac{\partial N_1}{\partial t} + \frac{B_{0z}}{\rho_0}\frac{\partial N_4}{\partial z}\right)\right\}. \quad (9)$$

This equation introduces the non-linear terms N_1, N_4, N_6 and N_7 and the new operators and variables given by:

$$\mathcal{D}_S = \frac{\partial^2}{\partial t^2} - C_S^2\frac{\partial^2}{\partial z^2} \quad (10)$$

$$\mathcal{D}_{Az} = \frac{\partial^2}{\partial t^2} - C_{Az}^2\frac{\partial^2}{\partial z^2} \quad (11)$$

$$\mathcal{A} = K\frac{\partial^2}{\partial z^2} - A \quad (12)$$

$$K = \kappa\cos^2\alpha\,\frac{(\gamma-1)\tilde{\mu}C_S^2}{R\gamma\rho_0} \quad (13)$$

$$A = \rho_0(\gamma-1)\left(\mathcal{L}'_\rho + \frac{(\gamma-1)\tilde{\mu}C_S^2}{R\gamma\rho_0}\mathcal{L}'_T\right) \quad (14)$$

In the weakly non-adiabatic, non-linear case, waves propagate approximately at the slow or fast wave speed.

$$C_{\mathrm{fast,slow}}^2 = \frac{1}{2}\left[C_S^2 + C_A^2 \pm\sqrt{(C_S^2+C_A^2)^2 - 4\cos^2\alpha\, C_S^2C_A^2}\right] \quad (15)$$

In order to follow the evolution of a single wave mode we move into a frame of reference which travels at the speed of the wave. This is done by performing the change of variables

$$\xi = z - Ct \ \text{ and } \ \tau = t. \quad (16)$$

After some manipulation an extended Burgers equation for the evolution of the wave, taking into account non-linearity, thermal instability and thermal conductivity is obtained:

$$\frac{\partial V_z}{\partial\tau} + \mu V_z + \nu\frac{\partial^2 V_z}{\partial\xi^2} + \epsilon V_z\frac{\partial V_z}{\partial\xi} = 0 \quad (17)$$

Three new coefficients are introduced ν, μ and ϵ, which represent radiative losses, thermal conductivity and non-linearity respectively:

$$\nu = -K\frac{C^2 - C_A^2\cos^2\alpha}{2C^2(2C^2 - C_S^2 - C_A^2)} \quad (18)$$

$$\mu = A\frac{C^2 - C_A^2\cos^2\alpha}{2C^2(2C^2 - C_S^2 - C_A^2)} \quad (19)$$

$$\epsilon = \frac{3C_A^2C^4\sin^2\alpha + C_S^2(\gamma+1)(C^2 - C_A^2\cos^2\alpha)^2}{2C^2(C^2 - C_A^2\cos^2\alpha)(2C^2 - C_S^2 - C_A^2)} \quad (20)$$

These coefficients could be changed to include different terms, such as viscosity instead of thermal conductivity or stratification instead of radiative losses without changing the general form of the extended Burgers equation. [9] By looking for solutions to Eq. 17 we can study in a general sense the result of competition between an amplification term, high frequency dissipation and non-linearity. The coefficient ϵ is always positive while ν is always negative. μ can be positive or negative depending upon the signs of $\mathcal{L}'_\rho$ and $\mathcal{L}'_T$, with negative μ corresponding to amplification of the wave. The first step in analysing Eq. 17 is to look for stationary solutions. Stationary waves are waves that travel at a constant velocity and without change of form.

We change to a frame of reference which moves with the stationary wave, by the substitution $\chi = \xi - v\tau$, where v is the speed of the stationary wave relative to the magnetoacoustic wave speed. Equation 17 becomes

$$\frac{d^2V_z}{d\chi^2} + \left(\frac{\epsilon}{\nu}V_z - \frac{v}{\nu}\right)\frac{dV_z}{d\chi} + \frac{\mu}{\nu}V_z = 0. \quad (21)$$

In any stationary solution, V_z must remain finite as χ goes to infinity. This is ensured due to a balance between $\frac{\epsilon V_z}{\nu}V_z - \frac{v}{\nu}$ and $\frac{\mu}{\nu}$ terms. These can only be balanced when $v = 0$, so there can only be stationary solutions which propagate at a magnetoacoustic wave speed. [16] The stationary solutions are then solutions of the ODE

$$\frac{d^2V_z}{d\chi^2} + \frac{\epsilon}{\nu}V_z\frac{dV_z}{d\chi} + \frac{\mu}{\nu}V_z = 0. \quad (22)$$

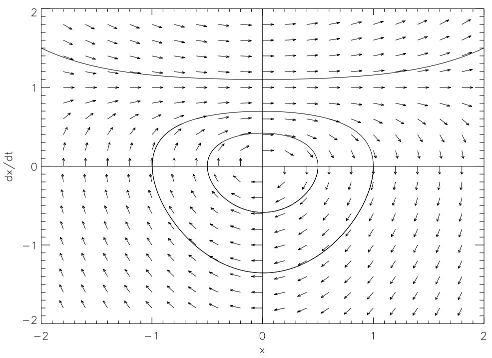

Figure 3. Phase diagram ($\frac{dx}{dt}$ vs x) for the thermally unstable case, $\mu < 0$. The closed loops represent stationary solutions.

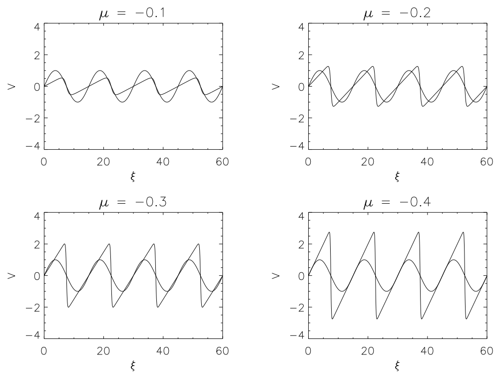

Figure 4. The same sine wave evolves to four different stationary solutions under different values of the parameter μ.

This equation is equivalent to Eq. 17 with $\frac{\partial}{\partial\tau} = 0$, ie., no wave evolution. We can now proceed to rewrite this equation in a dimensionless form. There are two forms, the first corresponds to activity, negative μ, and the second to dissipation, positive μ. Clearly, we cannot expect stationary solutions in the latter case. The variables x and t defined here are not related to their previous uses.

$$t = \sqrt{\frac{|\mu|}{-\nu}}\chi \tag{23}$$

$$x = \frac{\epsilon}{\sqrt{-|\mu|\nu}} V_z \tag{24}$$

$$\frac{d^2x}{dt^2} - x\frac{dx}{dt} + x = 0 \quad \text{(Activity)} \tag{25}$$

$$\frac{d^2x}{dt^2} - x\frac{dx}{dt} - x = 0 \quad \text{(Dissipation)} \tag{26}$$

Figure 3 shows the phase diagram for the case with the thermal instability. The closed loops around the origin are stationary solutions. These stationary solutions propagate at the magnetoacoustic wave speed, there are an infinite number of them and they symmetrical around the y-axis.

Even though we know that such solutions exist, we cannot say from this information whether or not these solutions are stable. Without a stability analysis we cannot say whether these waves will exist in nature. It is difficult to perform a stability analysis on these waves analytically as there is no analytical expression for them. The next step is, therefore, to investigate the stability of these solutions numerically.

4. NUMERICAL SIMULATION

We proceed to examine the solutions to Eq. 17 numerically. This has been done using the MacCormack finite difference scheme, as described by [17].

We start with a sine wave and examine how this sine wave evolves under the influence of the various evolutionary terms. Figure 4 shows the evolution of a sine wave for four different values of μ (-0.1, -0.2, -0.3 and -0.4) where $\epsilon = 1.0$ and $\nu = -0.3$. In each case the sine wave steepens into a sawtooth shape wave of a different amplitude which no longer evolves. An initial sine wave evolves to a stationary non-evolving solution. These sawtooth waves may look like shock waves but they are not, the presence of high-frequency dissipation prevents shock formation.

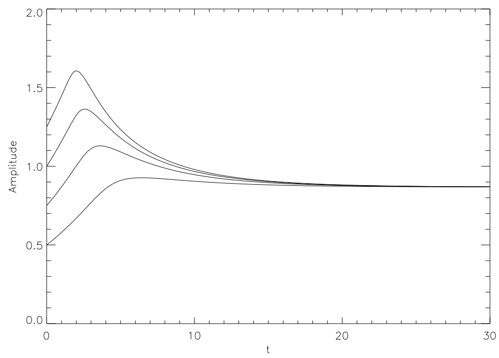

Figure 5. The amplitude of the final wave is independent of the initial amplitude, as shown in this graph of amplitude versus time for four waves with initial amplitudes of 0.5, 0.75, 1.0 and 1.25.

In order for these to be autowaves, the solution to which the wave evolves must not depend upon the initial conditions. To test this we ran the simulation with sine waves of various amplitudes and the same set of coefficients. The result is shown in Figure 5. The amplitude of the final wave is independent of the initial amplitude. This is the defining feature of an autowave.

5. EVIDENCE OF AUTOWAVES IN FLARING LOOPS

Recent simulations of oscillations in flaring loops has shown that these oscillations can be interpreted as standing acoustic waves. [18] However, in the conditions un-

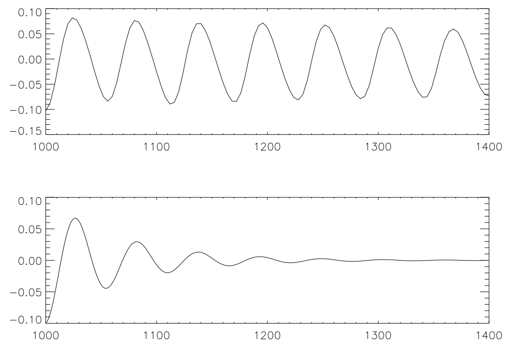

Figure 6. A comparison between the simulation results of [18] (top) and the level of thermal damping predicted by theory. (bottom)

der which these waves occur there is sufficient thermal conductivity to damp the waves on a timescale of less than one loop oscillation. A comparison between the simulation results of [18] and the damping time predicted by theory is shown in Figure 6.

One explanation for the discrepancy between these two results is that thermal instability or some other amplification mechanism such as gravitational stratification balances the dissipation due to thermal conductivity. These waves may then be autowaves. The extended Burgers Equation cannot be used to describe these waves as that equation describes travelling waves and these are standing waves.

6. CONCLUSIONS

In the solar corona slow magnetoacoustic waves propagating in loops and plumes and trapped between loop footpoints, are modified by thermal conductivity, radiative losses and non-linearity. Taking the MHD equations we have derived an extended Burgers equation to model the slow evolution of waves due to these factors. An equivalent equation can also be derived including effects such as viscosity and stratification without any change in the form of this equation.

We have shown that there exist stationary solutions to this equation and, using numerical simulations, that a sine wave of arbitrary amplitude evolves to a stationary solution of a specific amplitude. This is a propagating magnetoacoustic autowave. The amplitude of this autowave is determined only by the properties of the medium and not by the amplitude of the initial disturbance. However, the frequency of the wave is not determined, and determines the final amplitude.

The derivation of the extended Burgers equation is limited to weak non-adiabacity. Damping by thermal conductivity is often very strong and this limits the applicability of the equation. In addition, this technique is limited to the study of a single propagating mode and so is not useful in the study of standing waves. These problems can be solved by examining the results of hydrodynamic simulations.

The observable properties of autowaves (speed, amplitude and wavelength) could provide a tool for the study of heat deposition mechanisms in the solar corona.

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