Characterising the Scaling Properties of MHD Turbulence

James A Merrifield[1] Sandra C Chapman[1,3] Richard O Dendy[4] Wolf-Christian Muller[2]

(1)University of Warwick, England, (2) IPP, Garching, Germany, (3) Radcliff Institute, Harvard, MA, USA, (4) UKAEA Fusion Division, Culham, Oxfordshire, England

1. Universal Scaling Laws:

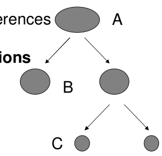
Branching Process (*Direct Cascade shown here*)

Large Scale eddies - pumping or driving scale (l_0)

Cascade to smaller scales unaffected by dissipation - *inertial range* ($l_D << l << l_0$)

Dissipate at small scales - dissipative scale (l_D)

- Cascade of energy from eddy on scale A to B can be viewed as a scaled version of the same process on scales B to C
- Eddies of scale l associated with an expected distribution of velocity differences
- Cascade process then captured by statistics of velocity differences
- These statistics are described by set of moments called **structure functions**



Cascade process characterised by set of scaling exponents ϕ_p that are thought to be universal

$$\left\langle \partial v_l^p \right\rangle = \left\langle \left| v(x+l) - v(x) \right|^p \right\rangle \sim l^{\phi_p}$$

2. MHD Structure Functions & Elsasser Field Variables

•Elsasser fields ($z^{\pm} = v \pm \frac{B}{\sqrt{\rho\mu_0}}$) represent oppositely travelling Alfvenic disturbances

•Structure function are constructed in terms of these variables

$$S_l^p = \left\langle \partial z_l^{(\pm)p} \right\rangle = \left\langle \left| z^{(\pm)}(x+l)\cdot\frac{l}{|l|} - z^{(\pm)}(x)\cdot\frac{l}{|l|} \right|^p \right\rangle \sim l^{\zeta_p(\pm)}$$

•Find that ζ_p is nonlinear in **p** → ***intermittency***

3. Intermittency & the Model of She-Leveque (SL)

•SL model belongs to a class that describes intermittency in local rate of dissipation to infer ζ_p by:

$$S_l^p \sim \chi_l^{p/3} l^{p/3} \text{; here } \chi_l^{(\pm)p} = \left\langle \left[\int_{x_i}^{x_i+l_i} \frac{\left(\partial_i z_i^{(\pm)}\right)^2}{l} dx \right]^p \right\rangle \sim l^{\tau_p(\pm)} \text{; to give } \zeta_p(\pm) = \tau_{p/3} + \frac{p}{3}$$

Where χ acts as a proxy to the local rate of dissipation

•SL is the **favoured model** [1] of MHD turbulence though this relationship was **previously untested**

•If true, a specific type of scaling should exist in χ

•Evidence to support this scaling was extracted from the simulation [1] of Biskamp and Muller

4. New Results: Extended Self-Similarity in Dissipation

•Figure A shows the extended self-similarity process for dissipation calculated from the simulation of Biskamp and Muller [1] . Blue dots show points used to calculate scaling exponents.

•Power law scaling found, see A.

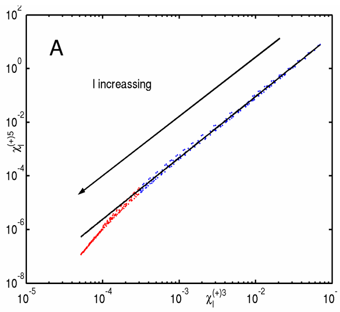
•Break in scaling at large l (possible finite size effect)

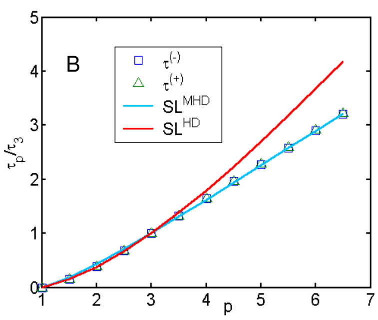
•Scaling exponents consistent with relationship shown above (section 3), see B

• The turquoise line on B represents the She-Leveque theory as in section 3.

The redline is that of hydrodynamics turbulence

•Results in Phys. Plasmas: Merrifield, J. A, & Muller, W.-C, & Chapman, S C, & Dendy, R. O. "*The Scaling Properties of Incompressible Isotropic MHD Turbulence*" (Accepted November 2004) [4]





5. Current and Further Research

Use **H**igh **O**rder Finite Difference **C**ode (HOC) authored by T. D. Arber to study **decaying 2D compressible** MHD turbulence.

HOC is fully parallel and possesses near spectral accuracy for less computation time [3]

•**Decaying Turbulence**: a convenient framework to study small scale dynamics

•Reliable statistics can be collected over a few eddy turnover times → **self-similar decay** (Fig.C)

•Results do not depend on a specific choice of forcing

6. Output from HOC: Self-Similar Decay and Structure Function Analyses for Low Mach Number (nearly incompressible) 1024^2 Run

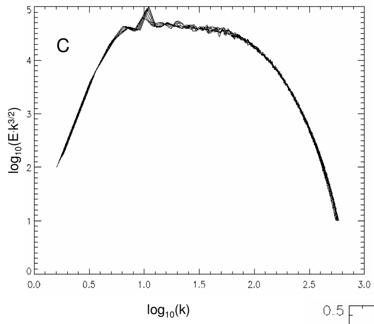
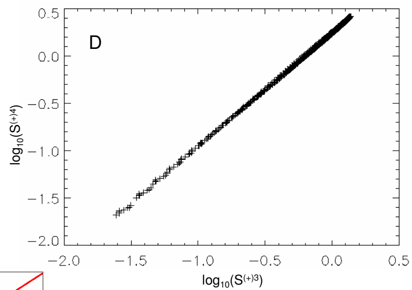


Figure C. shows the self-similar decay of the angle integrated total power (kinetic plus magnetic) spectrum Eight spectra obtained from HOC at intervals of 0.5 initial eddy turn over times are overlaid after self-similar normalisation, c.f. [2] The normalisation procedure is based on the cascade process proposed by Iroshnikov and Kraichnan (IK) in which energy transfer is governed by isotropic Alfvenic collisions. The small scatter goes someway to validating this picture for 2D MHD, c.f. [2] Any inertial range would appear as a plateaux on this plot.

Figure D. shows Extended self-similarity (ESS) in Elsasser field structure functions (see section 2) for order 4 against order 3. Points are calculated from the same eight snapshots as in Figure C. Normalisation by total energy shows the points lie on the same slope despite the decay process.



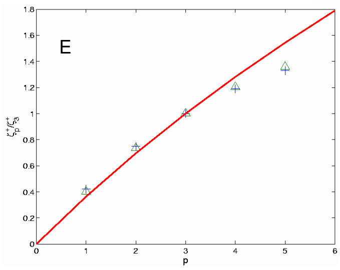
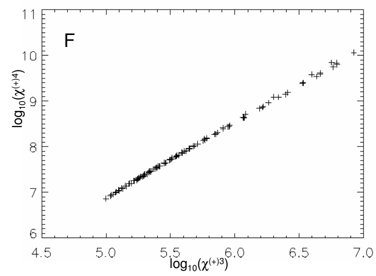


Figure E. shows the ratio of scaling exponents recovered by ESS (triangles). Those recovered for incompressible MHD decaying turbulence by Biskamp and Schwarz [2] are shown as crosses. The bold line shows exponents predicted by an IK based She-Leveque model. Neither set of exponents can be said to fit this model. Further investigation is needed.

Figure F. shows an example of ESS for the shear in the Elsasser field variables (χ in section 3) order 4 against order 3. Normalisation by total dissipation (Ohmic plus viscous) shows the points lie on the same slope despite the decay process. The plot is again constructed from the same eight snap shots.



7. References

[1] Biskamp, D, & Muller, W.-C.:2000,"*Scaling properties of three-dimensional isotropic magnetohydrodynamic turbulence*", Phys. Plasmas **7**(12) 4889-4900

[2] Biskamp, D, & Schwarz, E.: 2001, "On two-dimensional magnetohydrodynamic turbulence", Phys. Plasmas **8**(7) 3282-3292

[3] Brandenburg, A., & Dobler, W.: 2002, "*Hydromagnetic turbulence in computer simulations,*" Comp. Phys. Comm. **147,** 471-475

[4] Merrifield, J. A, & Muller, W.-C., & Chapman, S. C., & Dendy, R. O. *"The Scaling Properties of Incompressible Isotropic MHD Turbulence"* arXiv:physics/0412014

This work was supported in part by the United Kingdom Engineering and Physical Research Council

WARWICK