Characterising the Scaling Properties of Incompressible Isotropic Three-Dimensional MHD Turbulence

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Abstract: Recent improvements in the scale and accuracy of direct numerical simulations (DNS) of three-dimensional MHD turbulence enable many of its fundamental properties to be investigated anew. Here we report progress on several questions, achieved using the DNS of Biskamp and Muller [Phys. Plasmas 7, 4889 (2000)]. This employs the incompressible resistive MHD equations to simulate decaying isotropic turbulence, with finite magnetic helicity and initially equal magnetic and kinetic energy densities. It has a spatial resolution of 512^3 Fourier modes.

Although a formal statistical description of turbulence remains elusive, much progress has been made by the heuristic treatment of universal scaling laws such as those proposed by A. N Kolmogorov.

A central question is the nature (including dimensionality) of the localised dissipative structures that develop in turbulent flows and their role in these universal scaling laws.

Biskamp and Muller interpreted the scaling laws obtained from this DNS (via extended self-similarity) in a generalised frame work of the theory of She and Leveque which, explicitly links the nature of the most intensely dissipating structures to the universal scaling laws. This relies on the existence of extended self-similarity in the local rate of dissipation which conforms with Kolmogorov's refined similarity hypothesis. This has not been shown for MHD

Evidence is presented here which suggests that this assumption may be true.

1. Universal Scaling Laws:

Branching Process (*Direct Cascade shown here*)

Large Scale eddies - pumping or driving scale (l_0)

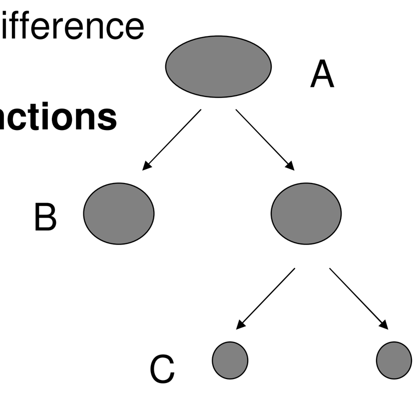
Cascade to smaller scales unaffected by dissipation - *inertial range* ($l_D \ll l \ll l_0$)

Dissipate at small scales - dissipative scale (l_D)

- Cascade of energy from eddy on scale A to B can be viewed as a scaled version of the same process on scales B to C
- Eddies of scale l associated with an expected distribution of velocity difference
- Cascade process then captured by statistics of velocity differences
- These statistics are described by set of moments called **structure functions**

$$\langle \partial v_l^p \rangle = \langle |v(x+l) - v(x)|^p \rangle \sim l^{\phi_p}$$

Cascade process characterised by set of scaling exponents ϕ_p that are thought to be universal



2. MHD Structure Functions & Elsasser Field Variables

•Elsasser fields ($z^{\pm} = v \pm \frac{B}{\sqrt{\rho\mu_0}}$) represent oppositely travelling Alfvenic disturbances

•Structure function are constructed in terms of these variables

$$S_l^p = \langle \partial z_l^{(\pm)p} \rangle = \left\langle \left| z^{(\pm)}(x+l) \cdot \frac{l}{|l|} - z^{(\pm)}(x) \cdot \frac{l}{|l|} \right|^p \right\rangle$$

3. Extended Self-Similarity

•Direct Numeric Simulation (DNS) must fully resolve the dissipation range to prevent the 'pile up' of energy at small scales

•Most resolution is used for this purpose thus inertial ranges are small

•Use Extended Self-Similarity (ESS) which extends the universal scaling laws into the dissipation range such that:

$$S_l^{(\pm)p} \sim l^{\xi_p} \quad \Longrightarrow \quad S_l^{(\pm)p} \sim S_l^{(\pm)q\left(\xi_p/\xi_q\right)}$$

In the inertial range — In the inertial and dissipation range

4. Numerical Scheme

•DNS of Biskamp and Muller, Phys. Plasmas, 2000

•Pseudo-spectral incompressible MHD solver

•512^3 Fourier modes – **is 3D**

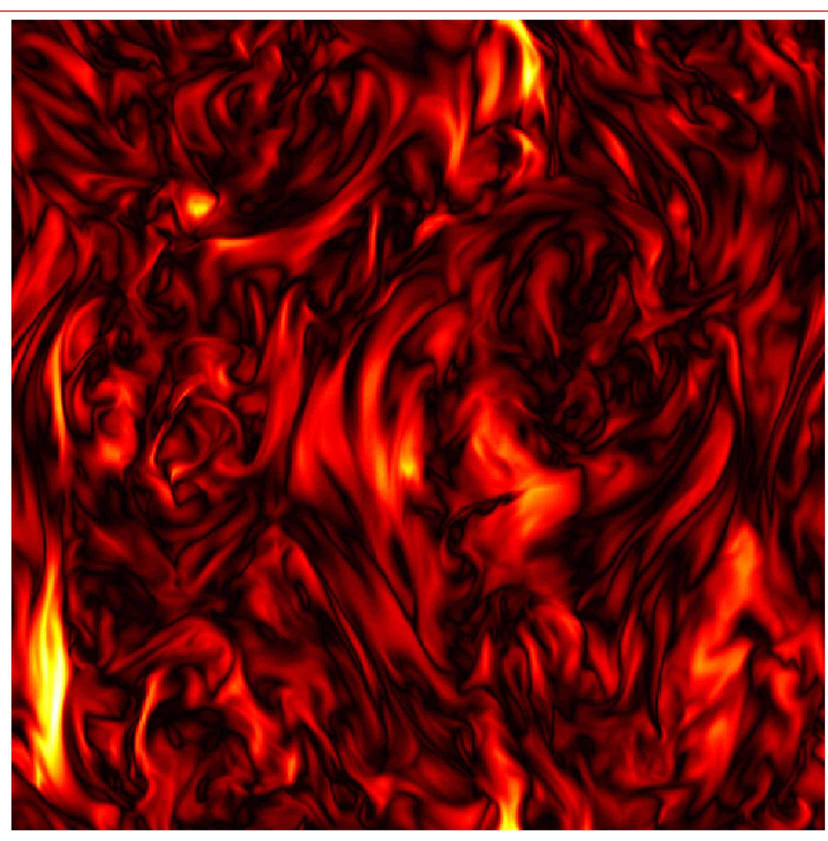
•Turbulence is decaying which allows:

- Reliable statistics from short runs since smooth time evolution
- Results independent of driving regime

•Micro-scale Reynolds number = 94

•Kinematic Viscosity = Magnetic Diffusivity

2D Slice of Velocity magnitudes from 3D simulation (right)



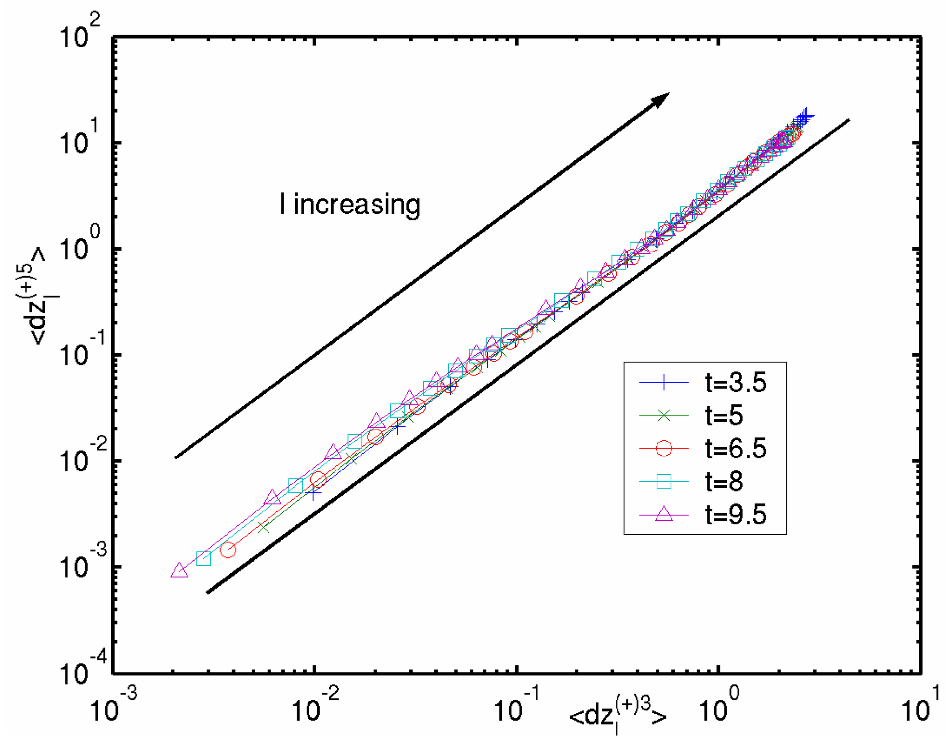
5.Known Result: Extended Self-Similarity in Elsasser Fields

•Turbulence decaying so structure functions from different times normalised to total energy

•Effect of normalisation and ESS procedure shown on plot

•Values of ξ_p/ξ_3 recovered are consistent with a type of She-Leveque theory that implies:

- Non linear process in cascade is same as in Hydrodynamic case
- The most intensely dissipating structures are 2D (perhaps current sheets)



6. New Results: Extended Self-Similarity in Dissipation

A She-Leveque theory predicts scaling in the local rate of dissipation (χ_l^p) so that $\chi_l^p \sim l^{\tau_p}$. Kolmogorov postulated that: $S_l^p \sim \chi_l^{p/3} l^{p/3} \sim l^{\xi_p} \rightarrow \xi_p = \tau_{p/3} + \frac{p}{3}$

(this is Kolmogorov's refined similarity hypothesis) which allows ξ_p/ξ_3 to be calculated from a She-Leveque theory. For the results of **5** to be validated, scaling must be found in the local rate of dissipation which conforms with Kolmogorov's refined similarity hypothesis.

The gradient squared measure is used:

$$\chi_l^{(\pm)p} = \left\langle \left[\int_{x_i}^{x_i+l_i} \frac{\left(\partial_i z_i^{(\pm)}\right)^2}{l} dx \right]^p \right\rangle$$

7. New Results: Extended Self-Similarity in Dissipation II

•A and B show normalisation and ESS process for dissipation. Blue dots show points used to obtain scaling exponents. Those in red were excluded

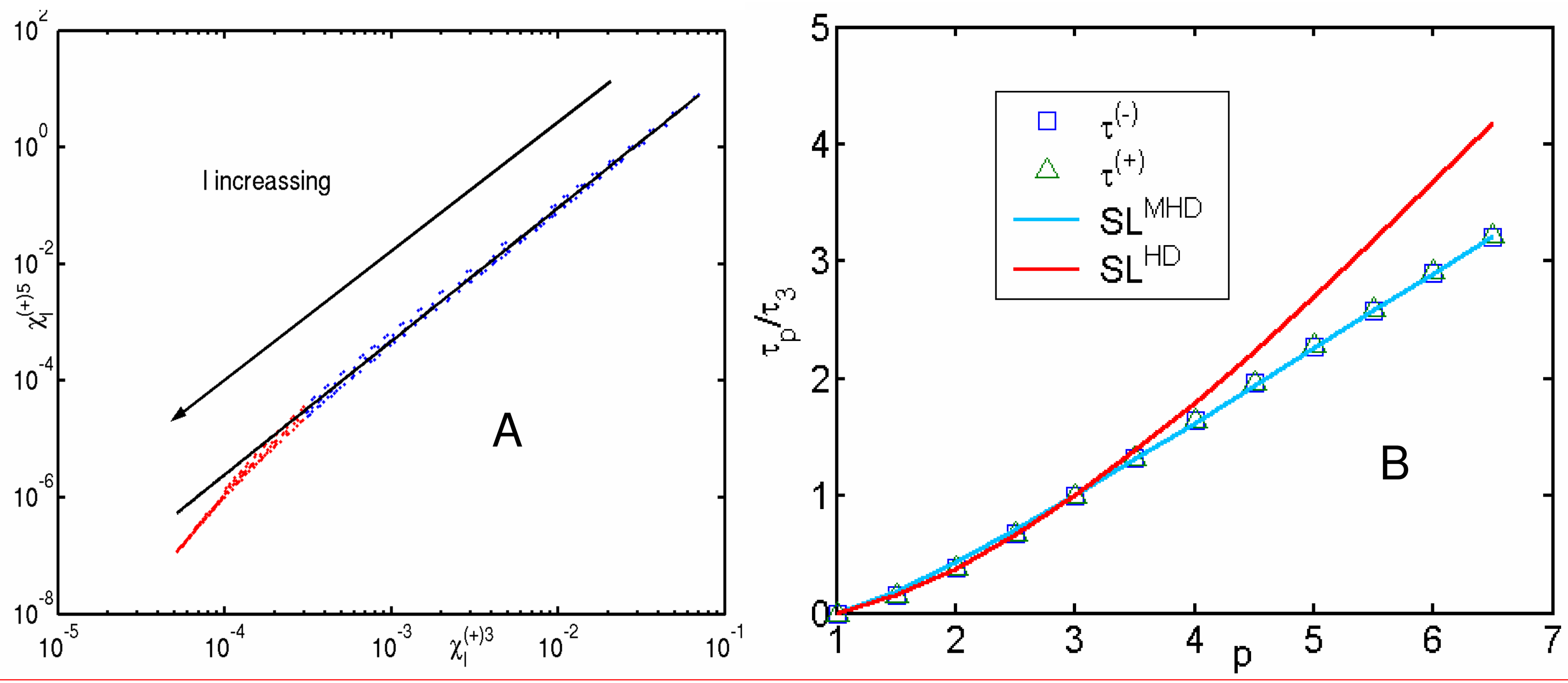
•Power law scaling found (see A)

•Break in scaling at large l (possible finite size effect)

•Scaling exponents consistent with Kolmogorov's refined similarity hypothesis for SL model

• The turquoise line on B represents the She-Leveque theory suggested by results in **5**.

The redline is that of hydrodynamics turbulence



8. Conclusion

•Extended Self-Similarity has been shown to exist in the local rate of dissipation

•Scaling exponents derived from $\chi_l^{(\pm)}$ agree with those obtained from the Elsasser field variables (see **5**) which is consistent with Kolmogorov's refined similarity hypothesis

•Further evidence that the cascade process is that of random eddy scrambling with sheet-like most intensely dissipating structures

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