Physics of Fusion

- **Fusion** here refers to the controlled process in which two light atoms are fused together generating a heavier atom with the aim of generating energy.
Binding energy is the energy that is released when a nucleus is created from protons and neutrons. It is released during the formation of a nucleus. The greater the binding energy per nucleon in the atom, the greater the atom’s stability.
Fusion and Fission

![Graph showing fusion and fission reactions with isotopes and energy changes.](graph.png)
Relevant fusion reactions

- Often considered fusion reactions (Note more than one reaction possible)

\[ \frac{2}{1}D + \frac{3}{1}T \rightarrow \frac{4}{2}He + \frac{1}{0}n + 17.6 \text{ MeV} \]

\[ \frac{2}{1}D + \frac{2}{1}D \rightarrow \frac{3}{2}He + \frac{1}{0}n + 3.7 \text{ MeV} \]

\[ \frac{2}{1}D + \frac{2}{1}D \rightarrow \frac{3}{1}T + \frac{1}{1}H + 4.03 \text{ MeV} \]

\[ \frac{2}{1}D + \frac{3}{2}He \rightarrow \frac{4}{2}He + \frac{1}{1}H + 18.3 \text{ MeV} \]
Calculation of energy released

- The released energy follows from the mass deficit. Consider the reaction

$$\frac{2}{1}D + \frac{3}{1}T \rightarrow \frac{4}{2}\text{He} + \frac{1}{0}\text{n}$$

- The masses of the different products are

$$m_D = (2 - 0.000994)m_H \quad m_T = (3 - 0.006284)m_H$$

$$m_{He} = (4 - 0.027404)m_H \quad m_n = (1 + 0.001378)m_H$$

- The mass deficit (Total mass before minus total mass after) is

$$\Delta m = 0.0187m_H \quad \quad m_H = 1.6727 \cdot 10^{-27} \text{ kg}$$
Calculation of the released energy

- The mass deficit is
  \[ \Delta m = 0.0187m_H \quad m_H = 1.6727 \cdot 10^{-27} \text{ kg} \]

- The energy then follows from Einstein’s formula
  \[ E = mc^2 = 0.0187m_Hc^2 = 2.8184 \cdot 10^{-12} \text{ J} \]

- Used unit of energy is the electron volt (eV), kilo-electron volt (1 keV = 1000 eV) or Mega-electron volt (1 MeV = 10^6 eV)
  \[ 1 \text{ eV} = 1.6022 \cdot 10^{-19} \text{ J} \]
  \[ E = \frac{2.8184 \cdot 10^{-12}}{1.6022 \cdot 10^{-19}} \text{ eV} = 17.56 \text{ MeV} \]
Energies in the MeV range are far in excess of usual chemical reactions

- 1 kg of a Deuterium/Tritium mixture would allow for a number of fusion reactions $N$

$$N = \frac{0.5}{2.5 \cdot 1.67 \cdot 10^{-27}} = 1.2 \cdot 10^{26}$$

- This amount of reactions would generate an energy

$$E = N \times 2.8184 \cdot 10^{-12} \text{ J} = 3.4 \cdot 10^{14} \text{ J}$$

- This is around 4 GW for 24 hours
eV is also used as the unit of Temperature

- Temperature is always used to express an averaged energy. The unit is again eV, i.e.

\[ T = \frac{kT_k}{e} \text{ (eV)} = 8.617 \cdot 10^{-5} T_k \text{ (eV)} \]

- Where \( T \) is the temperature and \( T_k \) is the temperature in Kelvin.

- Note

\[ 1 \text{eV} = 11605 \text{ K} \quad 17.56 \text{ MeV} = 2 \cdot 10^{11} \text{ K} \]
Distribution of energy over the products

- The energy is released in the form of kinetic energy
- The kinetic energy is not equally distributed over the products since both energy as well as momentum need to be conserved

\[
\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = E_{fus}
\]

\[
m_A v_A + m_B v_B = 0
\]

- These equations can be solved to give

\[
E_A = \frac{1}{2} m_A v_A^2 = \frac{m_B}{m_A + m_B} E_{fus}
\]

\[
E_B = \frac{1}{2} m_B v_B^2 = \frac{m_A}{m_A + m_B} E_{fus}
\]
Distribution of energy

- Momentum and energy conservation yield

\[ E_A = \frac{1}{2} m_A v_A^2 = \frac{m_B}{m_A + m_B} E_{\text{fus}} \quad E_B = \frac{1}{2} m_B v_B^2 = \frac{m_A}{m_A + m_B} E_{\text{fus}} \]

- Take the now famous reaction

\[ ^2_1 \text{D} + ^3_1 \text{T} \rightarrow ^4_2 \text{He} + ^1_0 \text{n} \]

- The Helium nuclei is roughly 4 times more heavy than the neutron and will thus acquire 20% of the energy (3.5 MeV) whereas the neutron obtains 80% (14.1 MeV)
Key problem of fusion

- Is the Coulomb barrier

![Graph showing potential energy as a function of distance between nuclei. The graph illustrates tunneling at higher energy and repulsion by Coulomb force.]
Cross section

- The cross section is the effective area connected with the occurrence of a reaction.
- For snooker balls the cross section is $\pi r^2$ (with $r$ the radius of the ball).

The cross section of various fusion reactions as a function of the energy. (Note logarithmic scale.)

$1 \text{ barn} = 10^{-28} \text{ m}^2$
Averaged reaction rate

- One particle (B) colliding with many particles (A)
- Number of reactions in $\Delta t$ is
  \[ \Delta N = n_A \sigma v \Delta t \quad \rightarrow \]
  \[ \frac{dN}{dt} = n_A \langle \sigma v \rangle \]
- Both $\sigma$ as well as $v$ depend on the energy which is not the same for all particles. One builds the average

Schematic picture of the number of reactions in a time interval $\Delta t$
Averaged reaction rate …..

- The cross section must be averaged over the energies of the particles. Assume a Maxwell-

\[ F_M(v) = \frac{n}{(2\pi T/m)^{3/2}} \exp\left[-\frac{mv^2}{2T}\right] \]

Averaged reaction rates for various fusion reactions as a function of the temperature (in keV)
Compare the two

The averaged reaction rate does not fall off as strongly when going to lower energies

Cross section as a function of energy

Averaged reaction rate as a function of Temperature
The reason.....

- Even for temperatures below the energy at which the cross section reaches its maximum, there is a sufficient amount of fusion reactions due to the number of particles in the tail of the Maxwell distribution.
Current fusion reactor concepts

- Are designed to operate at around 10 keV (note this is still 100 million Kelvin, matter is fully ionized or in the plasma state)
- Are based on a mixture of Deuterium and Tritium
- Both are related to the cross section

Averaged reaction rates for various fusion reactions as a function of the temperature (in keV)
Availability of the fuel

- The natural abundance of Deuterium is one in 6700. There is enough water in the ocean to provide energy for $3 \times 10^{11}$ years at the current rate of energy consumption (larger than the age of the universe).
- Deuterium is also very cheaply obtainable. Calculating the price of electricity solely on the basis of the cost of Deuterium, would lead to a drop of $10^3$ in your electricity bill.
- Tritium is unstable with a half age of 12.3 years. There is virtually no naturally resource of Tritium.
Availability of the fuel

- Tritium however can be bred from Lithium

\[
\frac{6}{3}\text{Li} + \frac{1}{0}n \rightarrow \frac{4}{2}\text{He} + \frac{3}{1}\text{T} + 4.8 \text{ MeV}
\]

\[
\frac{7}{3}\text{Li} + \frac{1}{0}n \rightarrow \frac{4}{2}\text{He} + \frac{3}{1}\text{T} + \frac{1}{0}n - 2.5 \text{ MeV}
\]

- Note that the neutron released in the fusion reaction can be used for this purpose

- The availability of Lithium on land is sufficient for at least 1000 if not 30000 years, and the cost per kWh would be even smaller than that of Deuterium.

- If the oceans is included it is estimated that there is enough fuel for \(3 \times 10^7\) years.
Why fusion ....

- There is a large amount of fuel available, at a very low price.
- Fusion is CO$_2$ neutral.
- It would yield only a small quantity of high level radio active waste.
- There is no risk of uncontrolled energy release.
- The fuel is available in all locations of the earth. Fusion is of interest especially for those regions that do not have access to other natural resources.
- There is only a small threat to non-proliferation of weapon material
But …

- A working concept is yet to be demonstrated. The operation of a fusion reactor is hindered by several, in itself rather interesting, physics phenomena.
- The cost argument isn’t all that clear, since the cost of the energy will be largely determined by the cost of the reactor.
Limitations due to the high temperature

- 10 keV is still 100 million Kelvin (matter is fully ionized, i.e. in the plasma state)
- Some time scales can be estimated using the thermal velocity
  \[ v_{th} = \sqrt{2T/m} \]
- This is \( 10^6 \) m/s for Deuterium and \( 6 \times 10^7 \) m/s for the electrons
- In a reactor of 10 m size the particles would be lost in 10 \( \mu \)s.
Two approaches to fusion

- One is based on the rapid compression, and heating of a solid fuel pellet through the use of laser or particle beams. In this approach one tries to obtain a sufficient amount of fusion reactions before the material flies apart, hence the name, inertial confinement fusion (ICF).
Magnetic confinement..

- The Lorentz force connected with a magnetic field makes it impossible for charged particles to move over large distances across the magnetic field.
- They gyrate around the field lines with a typical radius.

\[ \rho = \frac{mv_{th}}{ZeB} \]

At 10 keV and 5 Tesla, this radius is 4 mm for deuterium and 0.07 mm for electrons.