Physics of fusion power

Lecture 5: particle motion

Gyro motion

- The Lorentz force leads to a gyration of the particles around the magnetic field

$$x - x_0 = \rho sin \omega_c t$$

$$y - y_0 = \rho cos \omega_c t$$

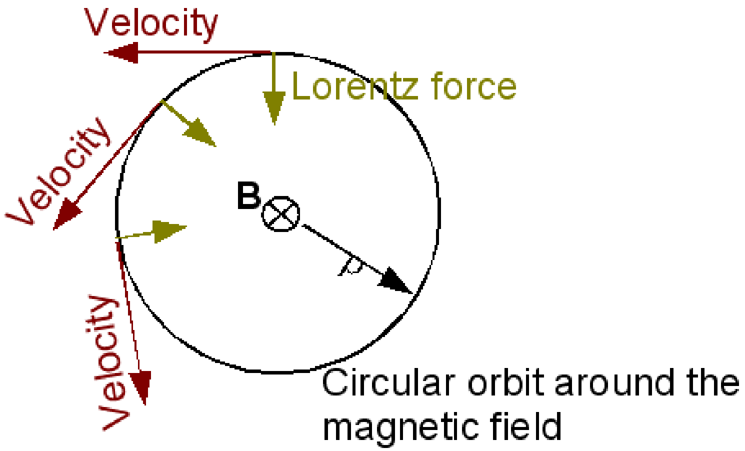
$$\rho = \frac{m v_\perp}{|q| B} \quad \omega_c = \frac{|q| B}{m}$$

- We will write the motion as

$$\mathbf{v} = v_{||} \mathbf{b} + \mathbf{v}_g$$

Parallel and rapid gyro-motion

$$m \frac{d\mathbf{v}}{dt} = q \mathbf{v} \times \mathbf{B}$$



The Lorentz force leads to a gyration of the charged particles around the field line

Typical values

- For 10 keV and B = 5T. The Larmor radius of the Deuterium ions is around 4 mm for the electrons around 0.07 mm
- Note that the alpha particles have an energy of 3.5 MeV and consequently a Larmor radius of 5.4 cm
- Typical values of the cyclotron frequency are 80 MHz for Hydrogen and 130 GHz for the electrons
- Often the frequency is much larger than that of the physics processes of interest. One can average over time
- One can not however neglect the finite Larmor radius since it lead to specific effects (although it is small)

Additional Force F

- Consider now a finite additional force F

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = q\mathbf{v} \times \mathbf{B} + \mathbf{F}$$

- For the parallel motion this leads to a trivial acceleration

$$m\frac{\mathrm{d}v_{\|}}{\mathrm{d}t} = F_{\|}$$

- Perpendicular motion: The equation above is a linear ordinary differential equation for the velocity. The gyro-motion is the homogeneous solution. The inhomogeneous solution

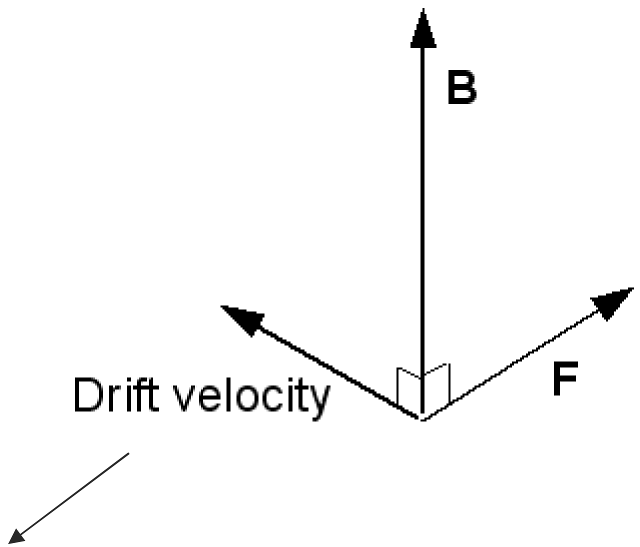
$$q\mathbf{v}_1 \times \mathbf{B} + \mathbf{F} = 0$$

Drift velocity

- Inhomogeneous solution

$$q\mathbf{v}_1 \times \mathbf{B} + \mathbf{F} = 0$$

$$\mathbf{v}_{\perp 1} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$



- Solution of the equation

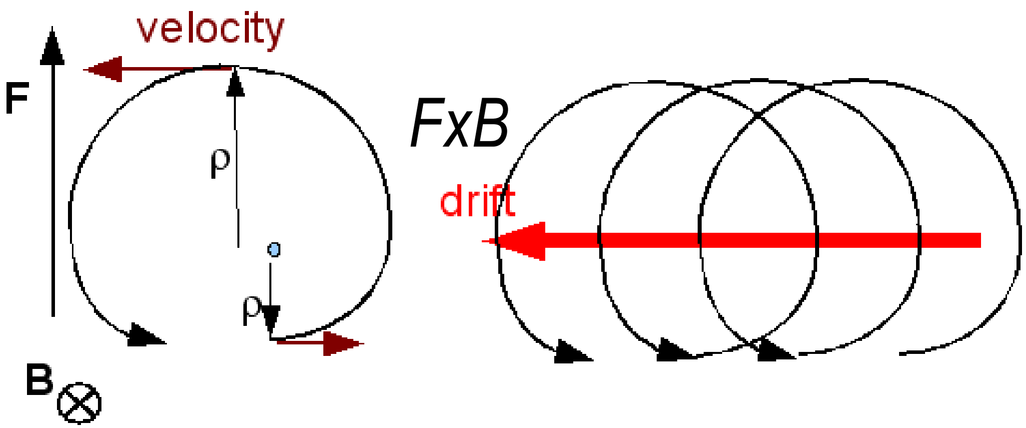
$$\mathbf{v} = v_{\|}\mathbf{b} + \mathbf{v}_g + \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Physical picture of the drift

- The force accelerates the particle leading to a higher velocity
- The higher velocity however means a larger Larmor radius
- The circular orbit no longer closes on itself
- A drift results.

Physics picture behind the drift velocity

$$\rho = \frac{mv_\perp}{|q|B}$$



Electric field

- Using the formula

$$\mathbf{v}_{\perp 1} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

- And the force due to the electric field

$$\mathbf{F} = q\mathbf{E}$$

- One directly obtains the so-called ExB velocity

$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

Note this drift is independent of the charge as well as the mass of the particles

Electric field that depends on time

- If the electric field depends on time, an additional drift appears

$$\mathbf{v} = \frac{m}{q^2 B^2} \frac{\mathrm{d}\mathbf{F}_\perp}{\mathrm{d}t}$$

$$\mathbf{F}_\perp = q\mathbf{E}_\perp$$

$$v_{\text{polarization}} = \frac{m}{qB^2} \frac{\mathrm{d}\mathbf{E}_\perp}{\mathrm{d}t}$$

Polarization drift. Note this drift is proportional to the mass and therefore much larger for the ions compared with the electrons

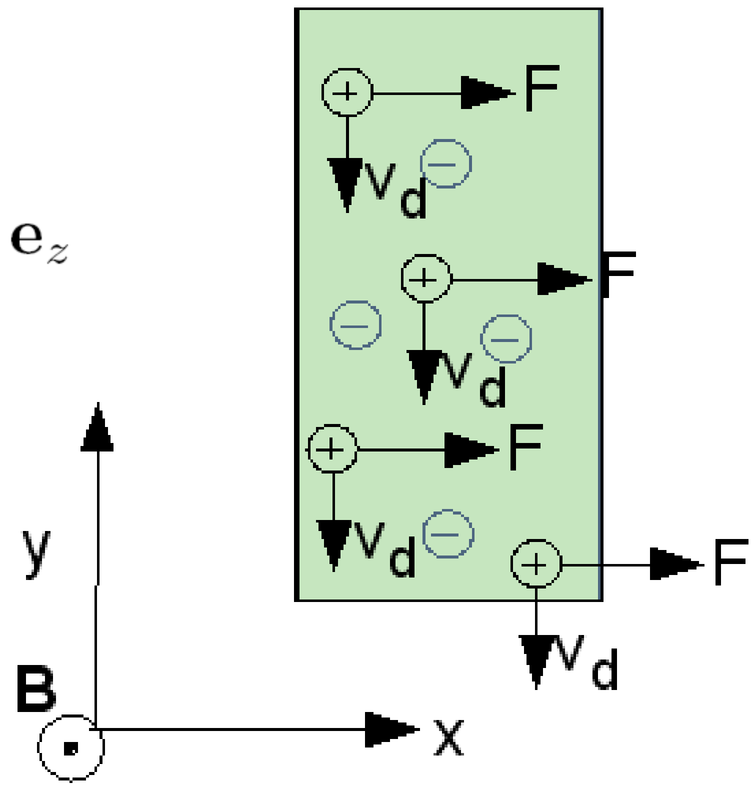
Meaning of the drifts

- Assume a Force F on each ion in the x-direction

$$\mathbf{v}_d = \frac{\mathbf{F}\times\mathbf{B}}{eB^2} = \frac{F}{eB}\mathbf{e}_x \times \mathbf{e}_z$$

$$\mathbf{v}_d = -\frac{F}{eB}\mathbf{e}_y$$

- Electrons are stationary

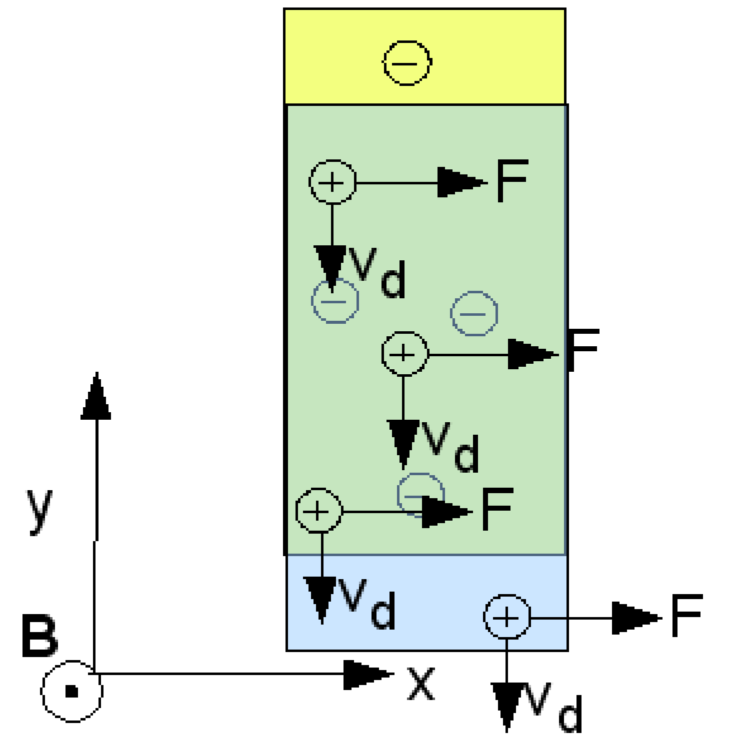


Drawing of the slab of plasma with a force F on the ions in the x-direction

Drift leads to charge separation

- The drift of the ions leads to charge separation.
- A small charge separation will lead to a large electric field, i.e. a build up of an electric field can be expected
- This would lead to a polarization drift
- Quasi-neutrality

$$v_{\text{polarization}} = -v_d$$



Drawing of the slab of plasma with a force F on the ions in the x-direction

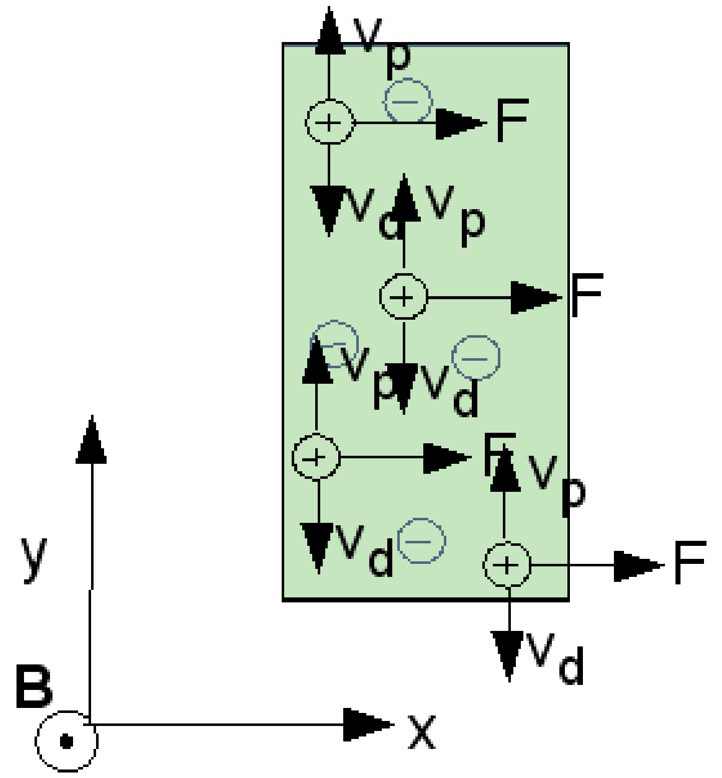
Electric field evolution

- The polarization drift balances the drift due to the force

$$v_{\text{polarization}} = -v_d$$

- The plasma remains quasi-neutral, and the electric field can be calculated from the polarization drift

$$\frac{m}{eB^2}\frac{\partial E_y}{\partial t} = \frac{F}{eB}$$



Drawing of the slab of plasma with a force F on the ions in the x-direction

The next drift : The ExB velocity

- The electric field evolution

$$\frac{m}{eB^2}\frac{\partial E_y}{\partial t} = \frac{F}{eB} \qquad E_y = \int_0^t \mathrm{d}t' \, \frac{FB}{m} = \frac{FB}{m} t$$

- leads to an ExB velocity

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2} = \frac{E_y}{B}\mathbf{e}_y \times \mathbf{e}_z = \frac{E_y}{B}\mathbf{e}_x$$

- Substituting the electric field

$$\mathbf{v}_E = \frac{E_y}{B} = \frac{F}{m} t$$

The ExB velocity

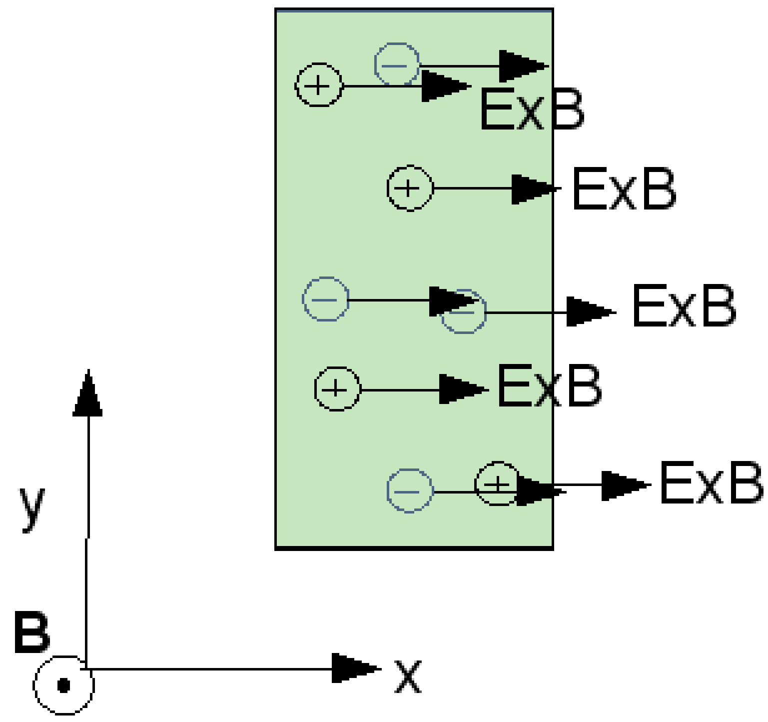
- The ExB velocity

$$\mathbf{v}_E = \frac{E_y}{B} = \frac{F}{m}t$$

- Satisfies the equation

$$m\frac{\mathrm{d}v_x}{\mathrm{d}t} = F$$

- Chain. Force leads to drift. Polarization drift balances the drift and leads to electric field, ExB velocity is in the direction of the force



Motion due to the ExB velocity

Meaning of the drifts

- In a homogeneous plasma

$$\mathbf{v} = v_{||}\mathbf{b} + \mathbf{v}_g + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{eB^2}\frac{\partial \mathbf{E}_\perp}{\partial t}$$

Free motion along the field line

Fast gyration around the field lines

ExB drift velocity. Provides for a motion of the plasma as a whole (no difference between electrons and ions)

Polarization drift. Allows for the calculation of the electric field evolution under the quasi-neutrality assumption. Provides for momentum conservation.

Inhomogeneous magnetic fields

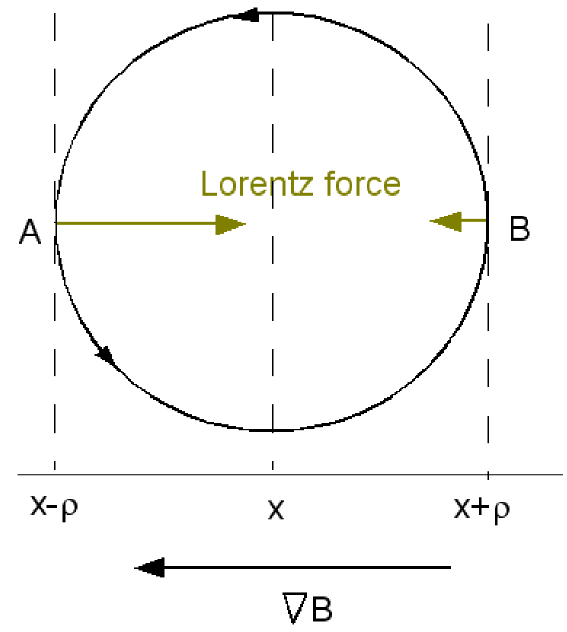
- When the magnetic field strength is a function of position the Lorentz force varies over the orbit
- Taking two points A and B

$$F = qv_\perp(B(x-\rho) - B(x+\rho))\mathbf{e}_x$$

$$= -2qv_\perp\rho\frac{\partial B}{\partial x}\mathbf{e}_x \longleftarrow \rho = \frac{mv_\perp}{|q|B}$$

$$F = -\frac{2mv_\perp^2}{qB}\frac{\partial B}{\partial x}$$

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$



Drawing of the Grad-B force

Inhomogeneous magnetic field

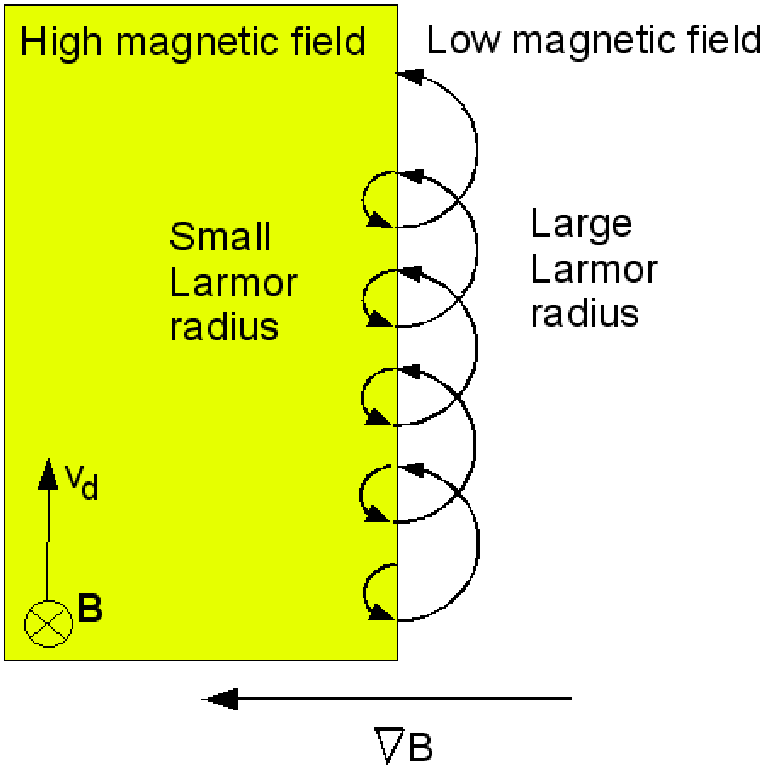
- Force due to magnetic field gradient is directed such that the particle tries to escape the magnetic field

$$F = -\frac{2mv_\perp^2}{qB}\frac{\partial B}{\partial x}$$

$$F = -\frac{mv_\perp^2}{2qB}\nabla B$$

- Leads to the grad-B drift

$$\mathbf{v}_d = \frac{mv_\perp^2}{2qB}\frac{\mathbf{B}\times\nabla B}{B^2}$$



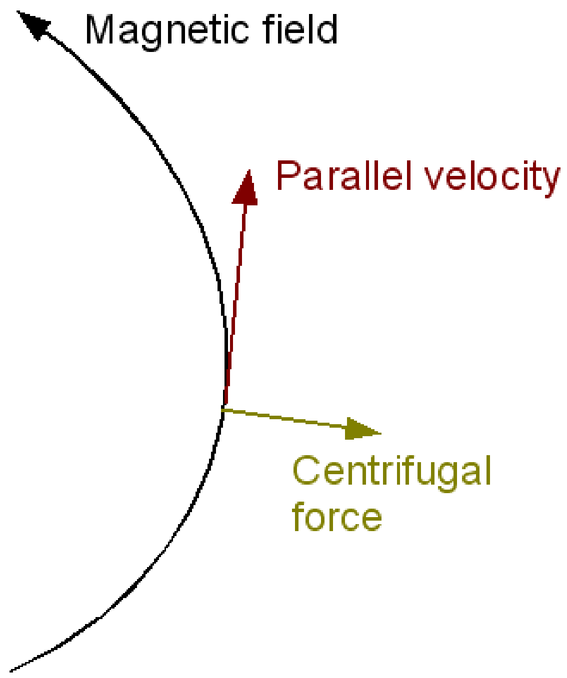
Curvature drift

- A particle moving along a curved field line experiences a centrifugal force

$$\mathbf{F} = \frac{mv_{\|}^2}{R_{\mathrm{curv}}}\mathbf{e}_{\mathrm{curv}}$$

- For a low beta plasma

$$\mathbf{v}_d \approx \frac{mv_{\|}^2}{qB}\frac{\mathbf{B}\times\nabla B}{B^2}$$



Centrifugal force due to the motion along a curved magnetic field

Drifts due to the inhomogeneous field

- The drifts due to the inhomogeneous field (curvature and grad-B)

$$\mathbf{v}_d = \frac{mv_{\|}^2 + v_{\perp}^2/2}{qB}\frac{\mathbf{B}\times\nabla B}{B^2}$$

Scales as ρv

Scales as 1/L where L is the scale length of the magnetic field

- The drift due to the magnetic field in homogeneity is in general much smaller than the thermal velocity

$$v_d \approx \frac{\rho}{L}v$$

All together ….

$$\mathbf{v} = v_{||}\mathbf{b} + \mathbf{v}_g + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{qB^2}\frac{\mathrm{d}\mathbf{E}_\perp}{\mathrm{d}t} + \frac{mv_{||}^2 + mv_\perp^2/2}{qB}\frac{\mathbf{B}\times\nabla B}{B^2}$$

Parallel motion

Gyration

ExB drift

Pololarization drift

Grad-B and curvature drift