Physics of fusion power

Lecture 6: Conserved quantities / Mirror device / tokamak

Reminder

- Perpendicular forces lead to drifts of the particles

Electric field acceleration

Inertia connected with a change in the ExB velocity

Centrifugal force due to a particle moving along a curved magnetic field

Grad B force (Lorentz force depends on magnetic field strength)

$$\mathbf{v} = v_{||}\mathbf{b} + \mathbf{v}_g + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{qB^2}\frac{\mathrm{d}\mathbf{E}_\perp}{\mathrm{d}t} + \frac{mv_{||}^2 + mv_\perp^2/2}{qB}\frac{\mathbf{B}\times\nabla B}{B^2}$$

Parallel motion

Gyration

ExB drift

Pololarization drift

Grad-B and curvature drift

Dynamics perpendicular to the field

- Force in the x-direction

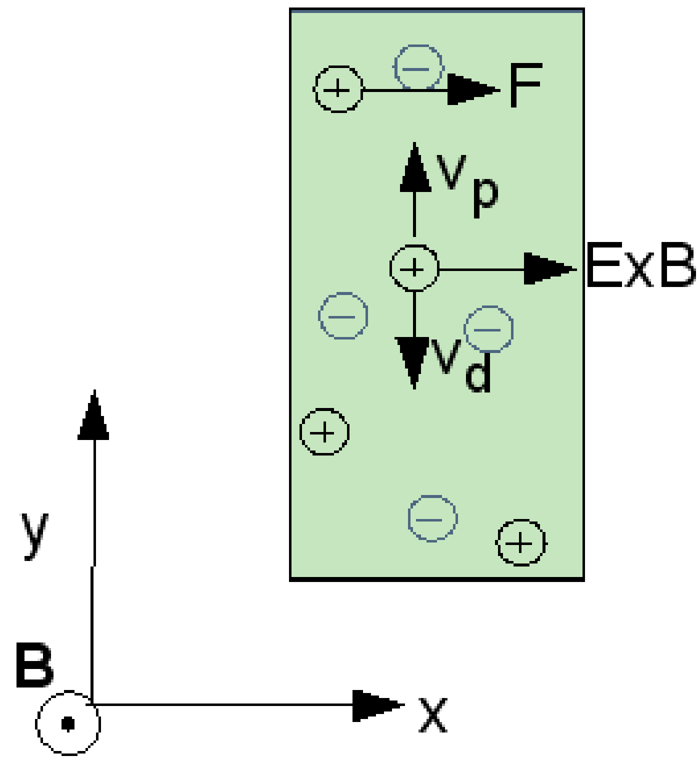
$$\mathbf{v}_d = -\frac{F}{eB}\mathbf{e}_y$$

- Balanced by polarization drift

$$\frac{m}{eB^2}\frac{\partial E_y}{\partial t} = \frac{F}{eB}$$

- Generating an electric field, and consequent ExB drift

$$\mathbf{v}_E = \frac{E_y}{B} = \frac{F}{m}t$$

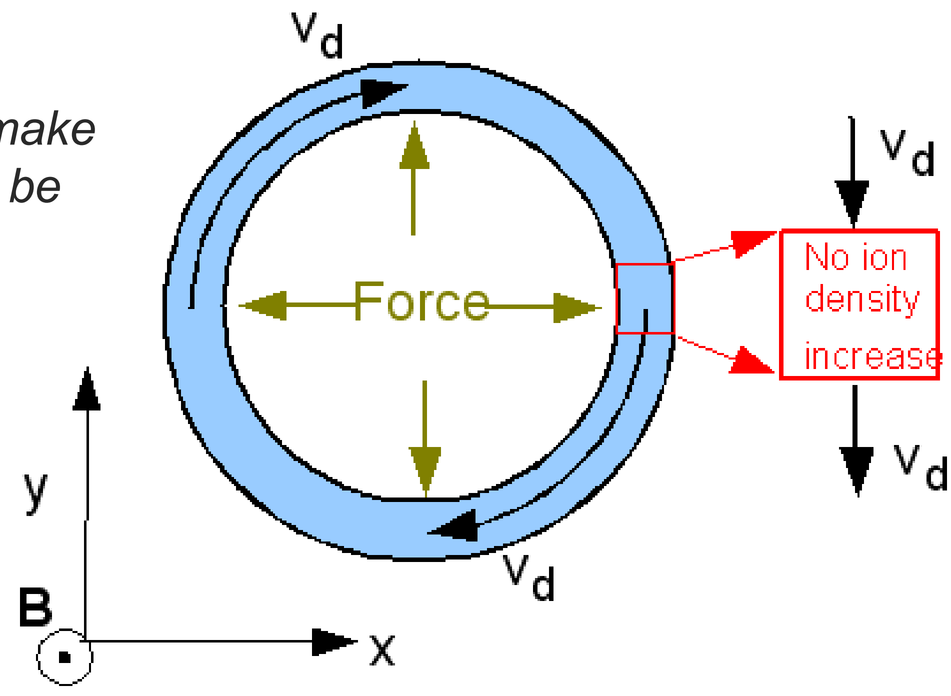


Dynamics of the particles perpendicular to the magnetic field

NOTE ......

- Does not apply to a uniform plasma.

Drawing that hopes to make clear that a plasma can be confined



Conserved quantities

- In the absence of an electric field

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B}$$

- Perpendicular energy is conserved

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}mv_\perp^2\right] = m\mathbf{v}_\perp \cdot \frac{\mathrm{d}\mathbf{v}_\perp}{\mathrm{d}t} = q\mathbf{v}_\perp \cdot (\mathbf{v}_\perp \times \mathbf{B}) = 0$$

- And consequently the total energy is conserved

$$E = \frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2$$

More tricky .....

- Consider a changing magnetic field. An electric field is generated

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}$$

- Integrating over the area of the Larmor orbit

$$-\pi\rho^2 \frac{\partial B}{\partial t} = \int \mathrm{d}^2\mathbf{A}\, \nabla \times \mathbf{E} = \oint \mathrm{d}\mathbf{s} \cdot \mathbf{E}$$

$$\rho = \frac{m v_\perp}{qB} \longrightarrow -\frac{\pi m^2 v_\perp^2}{q^2 B^2}\frac{\partial B}{\partial t} = \oint \mathrm{d}\mathbf{s} \cdot \mathbf{E}$$

Acceleration

- Derive a second equation for the integral of the electric field from

$$m\frac{\mathrm{d}\mathbf{v}_\perp}{\mathrm{d}t} = q\mathbf{E}_\perp$$

- Solve through the inner product with the velocity

$$m\mathbf{v}_\perp \cdot \frac{\mathrm{d}\mathbf{v}_\perp}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}mv_\perp^2\right] = q\mathbf{v}_\perp \cdot \mathbf{E}_\perp = q\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} \cdot \mathbf{E}_\perp$$

- Integrate towards time

$$\Delta\left[\frac{1}{2}mv_\perp^2\right] = q\oint \mathrm{d}\mathbf{s} \cdot \mathbf{E} \approx \frac{\partial}{\partial t}\left[\frac{1}{2}mv_\perp^2\right]\tau$$

Acceleration

- Integrate in time

$$\Delta\left[\frac{1}{2}mv_\perp^2\right] = q\oint \mathrm{d}\mathbf{s}\cdot\mathbf{E} \approx \frac{\partial}{\partial t}\left[\frac{1}{2}mv_\perp^2\right]\tau$$

$$\oint \mathrm{d}\mathbf{s}\cdot\mathbf{E} = \frac{2\pi m}{q^2 B}\frac{\partial}{\partial t}\left[\frac{1}{2}mv_\perp^2\right]$$

$$\tau = \frac{2\pi}{\omega_c} = \frac{2\pi m}{qB}$$

- Note the integration has the opposite orientation compared with the one from Maxwell equation. One is minus the other

$$\frac{\pi m^2 v_\perp^2}{q^2 B^2}\frac{\partial B}{\partial t} = \frac{2\pi m}{q^2 B}\frac{\partial}{\partial t}\left[\frac{1}{2}mv_\perp^2\right]$$

Magnetic moment is conserved

- The equation

$$\frac{\pi m^2 v_\perp^2}{q^2 B^2}\frac{\partial B}{\partial t} = \frac{2\pi m}{q^2 B}\frac{\partial}{\partial t}\left[\frac{1}{2} m v_\perp^2\right]$$

$$\frac{1}{B}\frac{\partial}{\partial t}\left[\frac{1}{2} m v_\perp^2\right] - \frac{m v_\perp^2}{2B^2}\frac{\partial B}{\partial t} = 0$$

$$\boxed{\frac{\partial \mu}{\partial t} = 0 \qquad \mu = \frac{m v_\perp^2}{2B}}$$

- The magnetic moment is a conserved quantity

Flux conservation

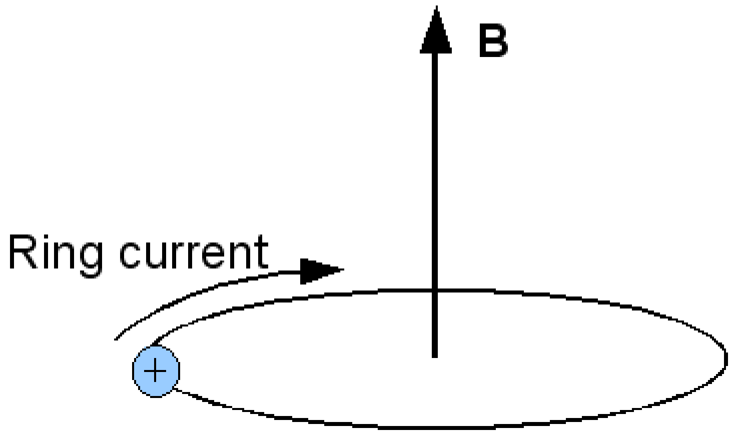
- The magnetic moment is conserved

$$\frac{\partial \mu}{\partial t} = 0 \qquad \mu = \frac{m v_\perp^2}{2B}$$

- Calculate the flux through the gyro-orbit

$$F = \pi \rho^2 B = \frac{\pi m^2 v_\perp^2}{q^2 B}$$

$$= \frac{2\pi m}{q^2} \mu$$



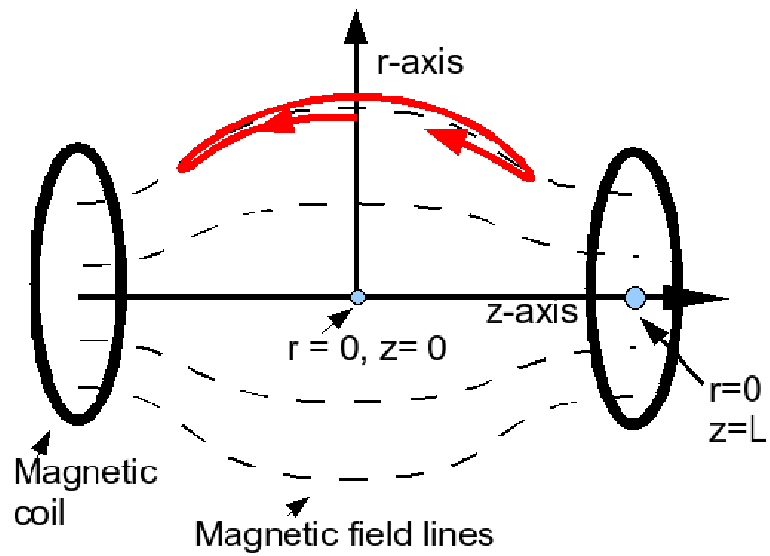
Drawing of the ring current of a particle in a magnetic field. The ring will conserve the flux which is related to the magnetic moment

The mirror

- Theta pinch has end losses
- But one could use the mirror force to confine particles

$$\mathbf{F} = -\mu \nabla B$$

- The mirror has a low B field in the centre and a high field near the coils
- Particles moving from the centre outward experience a force in the opposite direction



Drawing the mirror concept and the motion of a particle in the field (in red)

Mirror configuration

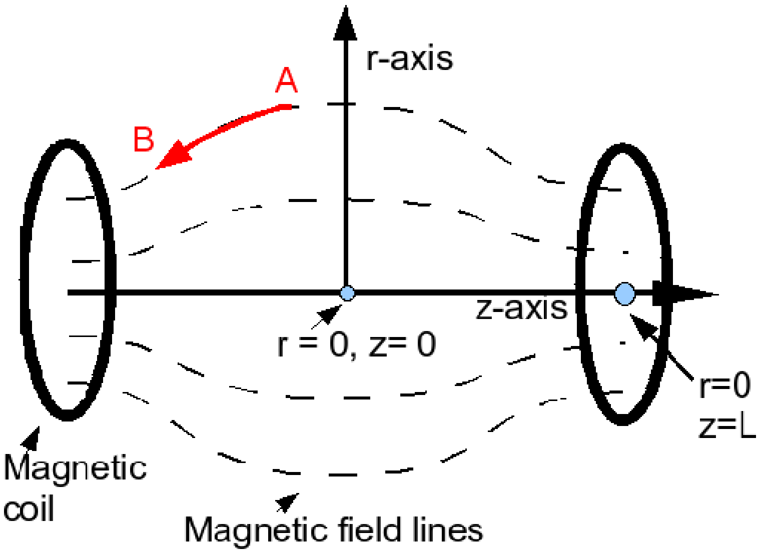
- From magnetic moment conservation follows the perpendicular energy

$$\mu = \frac{mv_{\perp A}^2}{2B_A} = \frac{mv_{\perp B}^2}{2B_B}$$

$$\frac{1}{2}mv_{\perp B}^2 = \frac{B_B}{B_A}\frac{1}{2}mv_{\perp A}^2$$

- Energy conservation then dictates that the parallel velocity must decrease

$$\frac{1}{2}mv_{\parallel A}^2 + \frac{1}{2}mv_{\perp A}^2 = \frac{1}{2}mv_{\parallel B}^2 + \frac{1}{2}mv_{\perp B}^2$$



Particle moving from A to B

Bouncing condition

- Assume the particle moving from A to B is reflected in the point B

$$\frac{1}{2}mv^2_{\parallel A} + \frac{1}{2}mv^2_{\perp A} = \frac{1}{2}mv^2_{\parallel B} + \frac{1}{2}mv^2_{\perp B}$$

Zero because the particle is reflected

$$\frac{B_B}{B_A}\frac{1}{2}mv^2_{\perp A}$$

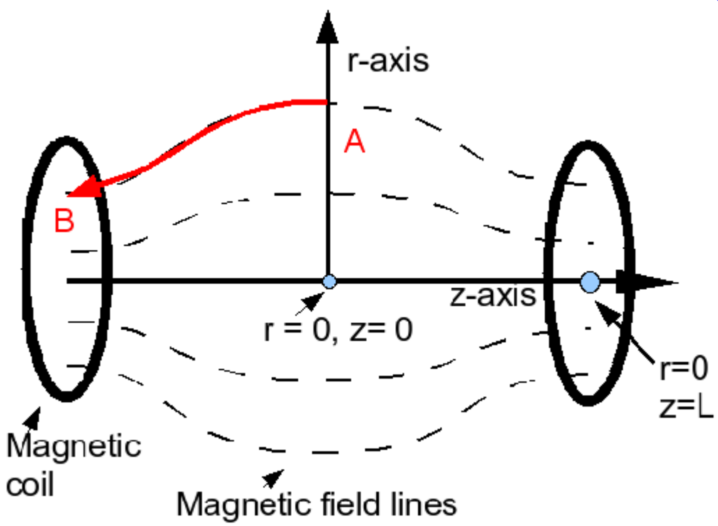
$$v^2_{\parallel A} = v^2_{\perp A}\left[\frac{B_B}{B_A} - 1\right]$$

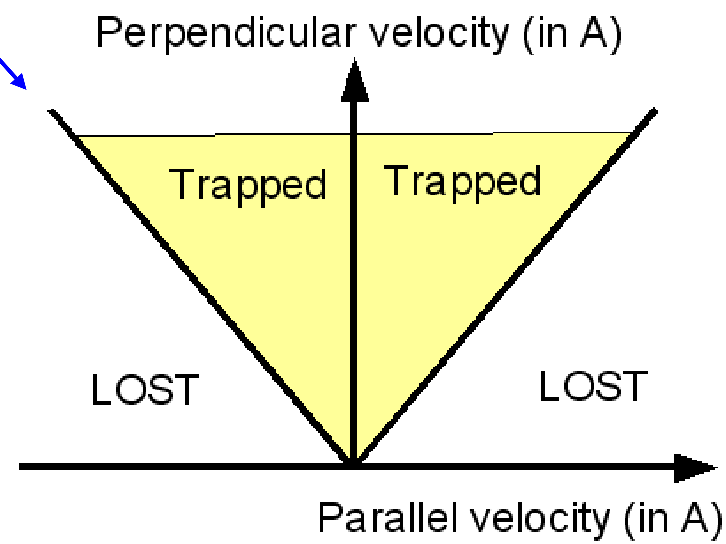
$$v_{\parallel A} = \pm v_{\perp A}\sqrt{\frac{B_B}{B_A} - 1}$$

The first key problem of the mirror

- Only part of the particles are confined (Collisional scattering in the loss region will lead to a rapid loss of the particles from the device)

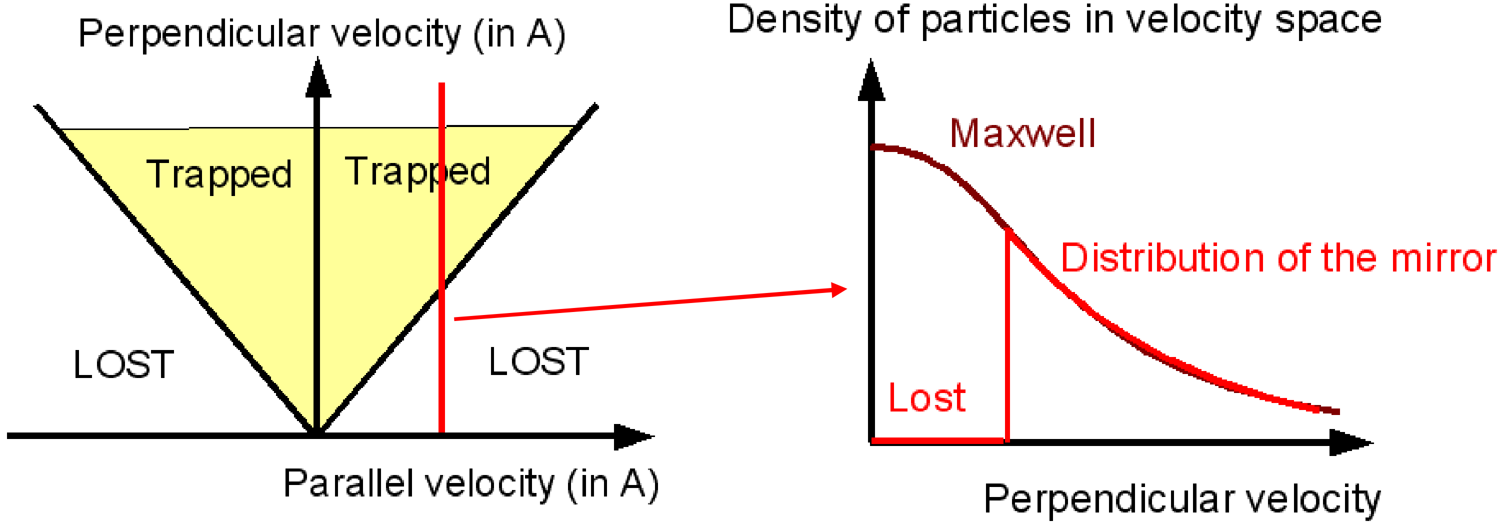
$$v_{\parallel A} = \pm v_{\perp A}\sqrt{\frac{B_B}{B_A} - 1}$$





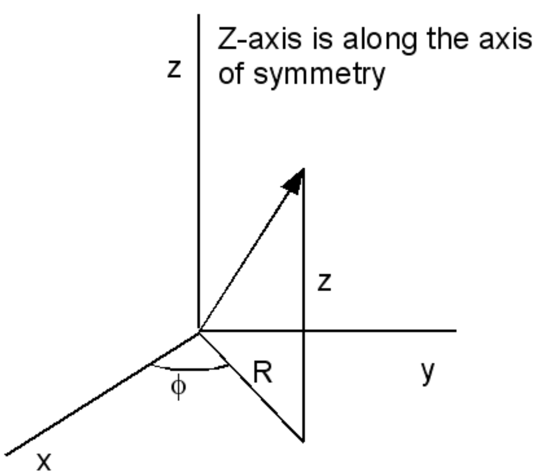
Second key problem of the mirror

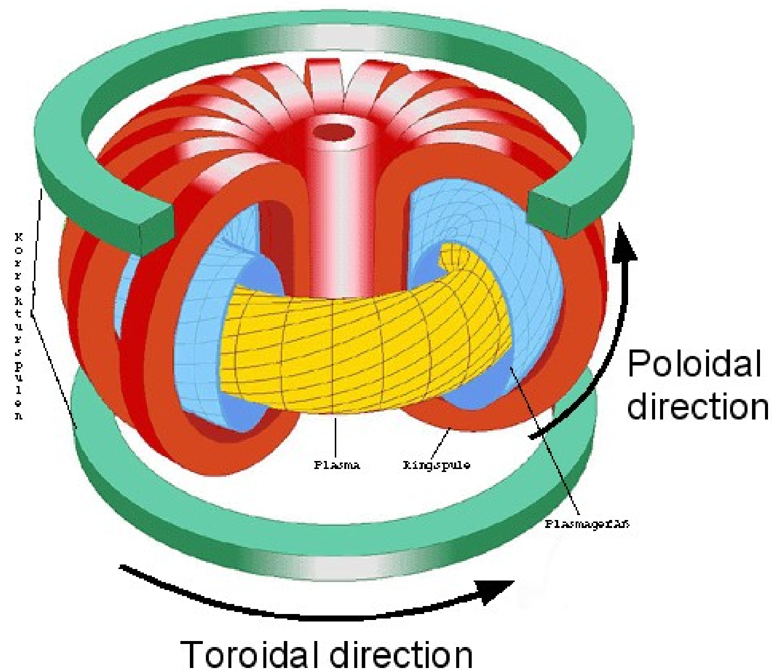
- The rapid loss of particles makes that the distribution of particles in velocity space is far from the Maxwell of thermodynamic equilibrium
- The ‘population inversion’ can drive all kinds of kinetic instabilities



Tokamak

- Bend the theta pinch into a donut shape
- No end losses because the field lines go around and close on themselves





Schematic picture of the tokamak

Toroidal curvature

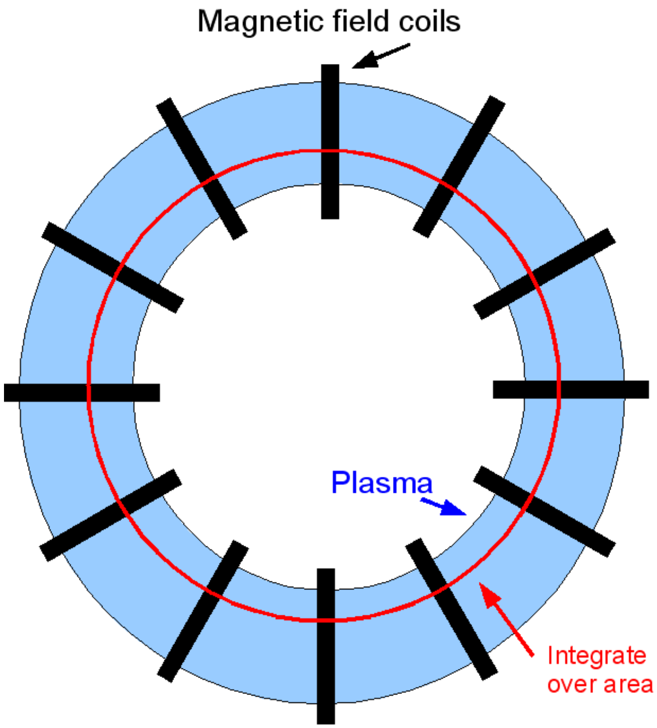
- The magnetic field follows form

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$2\pi R B_\phi = \mu_0 I$$

- And therefore varies with major radius R as

$$B_\phi = \frac{C}{R}$$



Top view of the tokamak

Toroidal curvature

- The toroidal magnetic field has a gradient

$$B_\phi = \frac{C}{R} \longrightarrow \nabla B = \nabla\left(\frac{C}{R}\right) = -\frac{C}{R^2}\mathbf{e}_R = -\frac{B}{R}\mathbf{e}_R$$

- Which leads to a drift in the vertical direction

$$\frac{\mathbf{B} \times \nabla B}{B^2} = \frac{1}{R}\mathbf{e}_z$$

$$\mathbf{v}_d = \frac{mv_\parallel^2 + mv_\perp^2/2}{qBR}\mathbf{e}_z$$

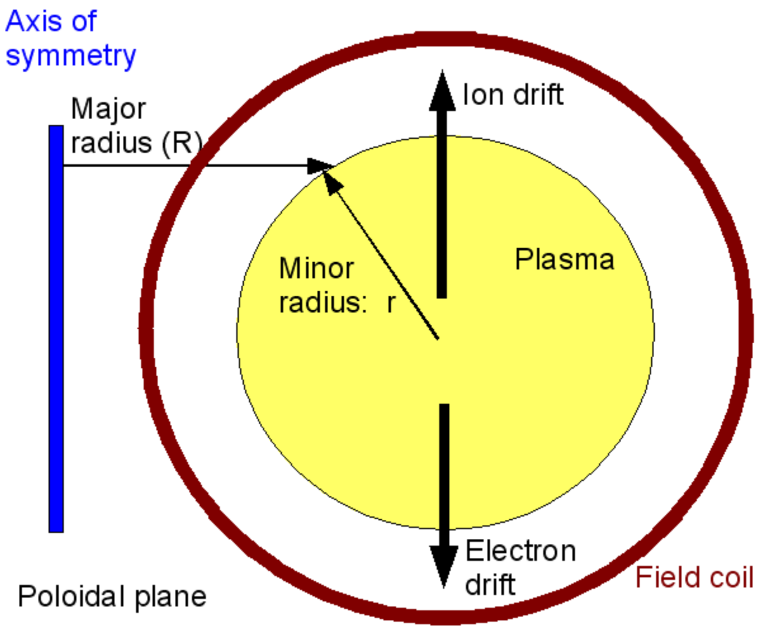
Note that the sign of the drift depends on the sign of the charge q

Toroidal curvature

- The drift

$$\mathbf{v}_d = \frac{mv_\parallel^2 + mv_\perp^2/2}{qBR}\mathbf{e}_z$$

- Leads to charge separation
- Build up of an electric field (calculate through the balance with polarization)
- And then to an ExB velocity



Poloidal cut of the tokamak.

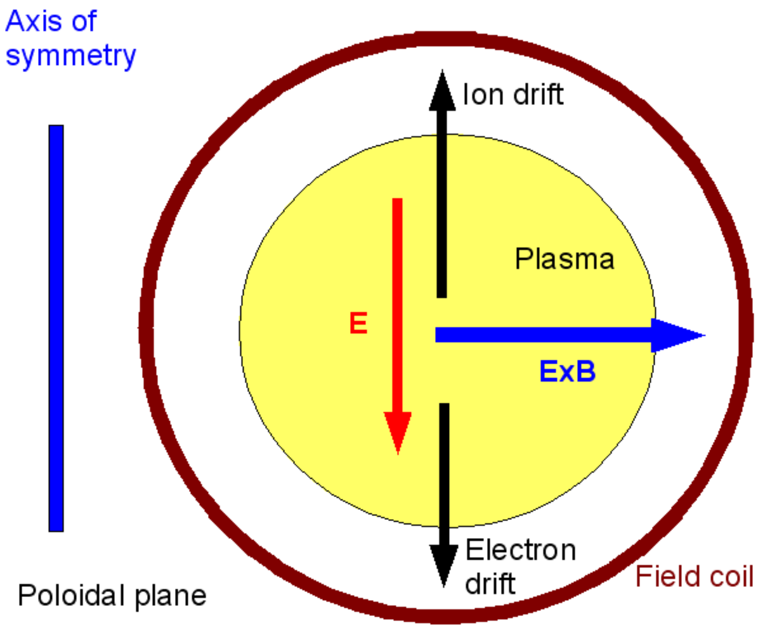
Toroidal curvature has its price

- The ExB velocity

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2} = -\frac{E_z}{B}\mathbf{e}_R$$

- Is directed outward and will move the plasma on the wall in a short timescale
- This effect is no surprise since

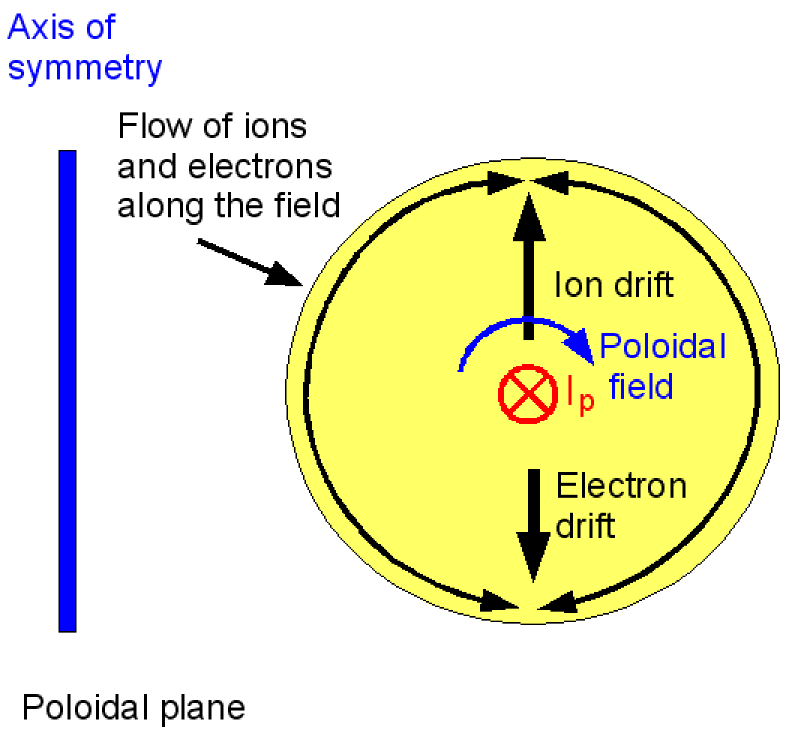
$$F = -\mu\nabla B \longleftarrow B_\phi = \frac{C}{R}$$



Poloidal cut of the tokamak.

Remedy : a plasma current

- A toroidal current in the plasma will generate a poloidal field
- Top and bottom are connected by the magnetic field line
- A vertical electric field would have a component along the field and leads to acceleration of the ions / electrons
- Drift will be balanced by a return flow along the field



Poloidal cut of the tokamak.

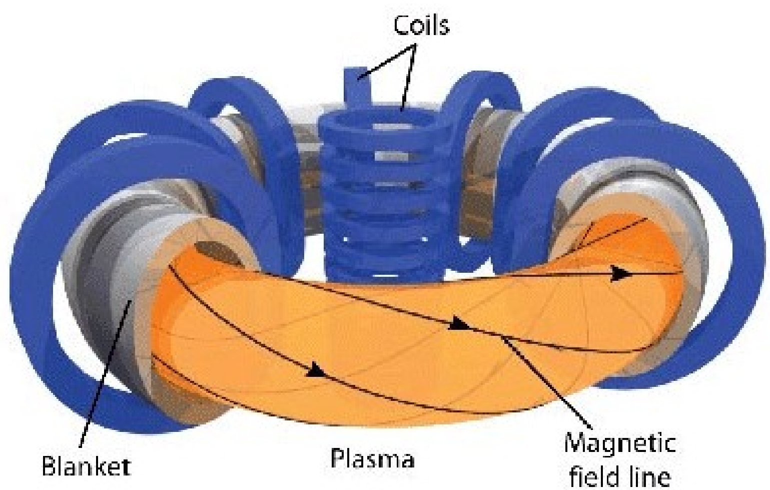
Plasma current

- Because of the plasma current the field lines wind around hellically
- Resistivity is small but finite

$$E_\phi = \eta J_\phi$$

- Note that the toroidal electric field that drives the current can not be electro-static

$$\nabla \Phi = \frac{1}{R}\frac{\partial \Phi}{\partial \phi} = 0$$



The field lines wind around helically .

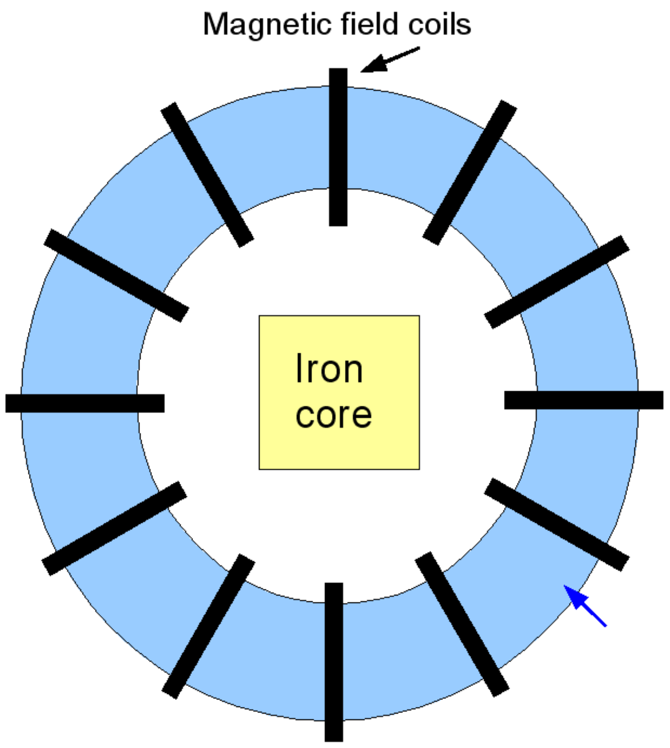
Electric field induced

- An iron core on the axis of symmetry is added
- The magnetic flux through the iron core is increase

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}$$

$$\frac{\partial \psi}{\partial t} = \oint \mathrm{ds} \cdot \mathbf{E}$$

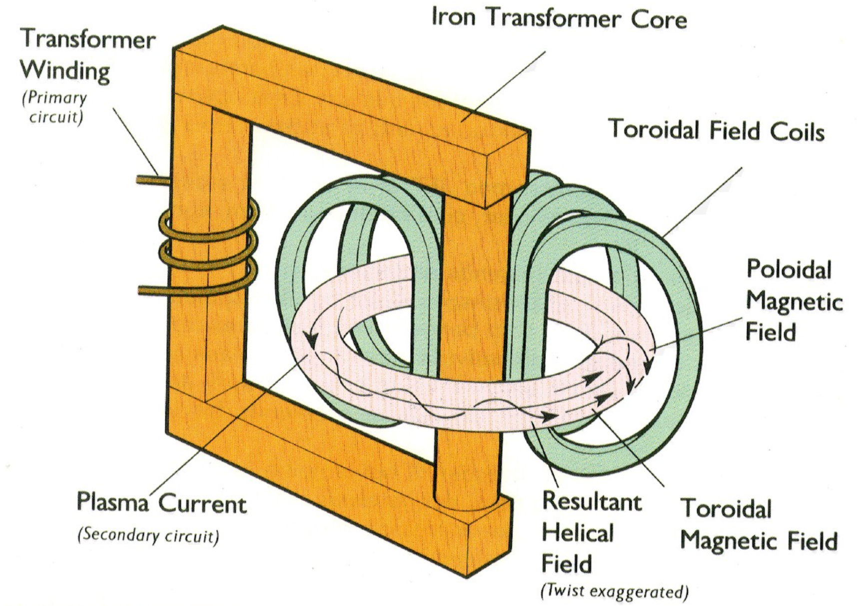
- This generates a toroidal electric field



Top view of the tokamak. An iron core is added through which the magnetic flux is increased

The toroidal electric field

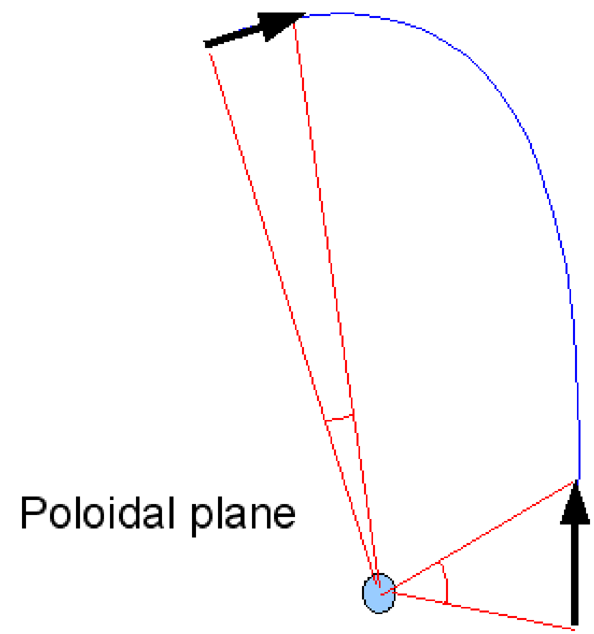
- Plasma is the second winding of a transformer
- Flux in the iron core cannot be increased forever. The tokamak is necessarily a pulsed machine
- That is not good for energy production
- Also thermal stresses are associated with the pulsed character
- One can either: live with it / drive current another way / use a different concept



The plasma is the second winding of a transformer.

Do we really need the plasma current?

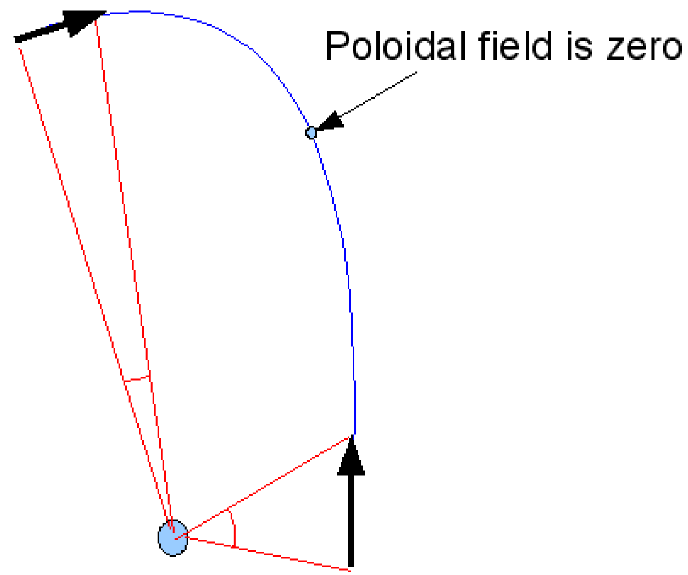
- It might at first appear obvious that the answer is yes since without current inside the plasma
- But a positive as well as negative poloidal field does not necessarily mean that the field line on average does not go around poloidally



On average the field line can go around even if the enclosed current is zero.

Toroidal symmetry

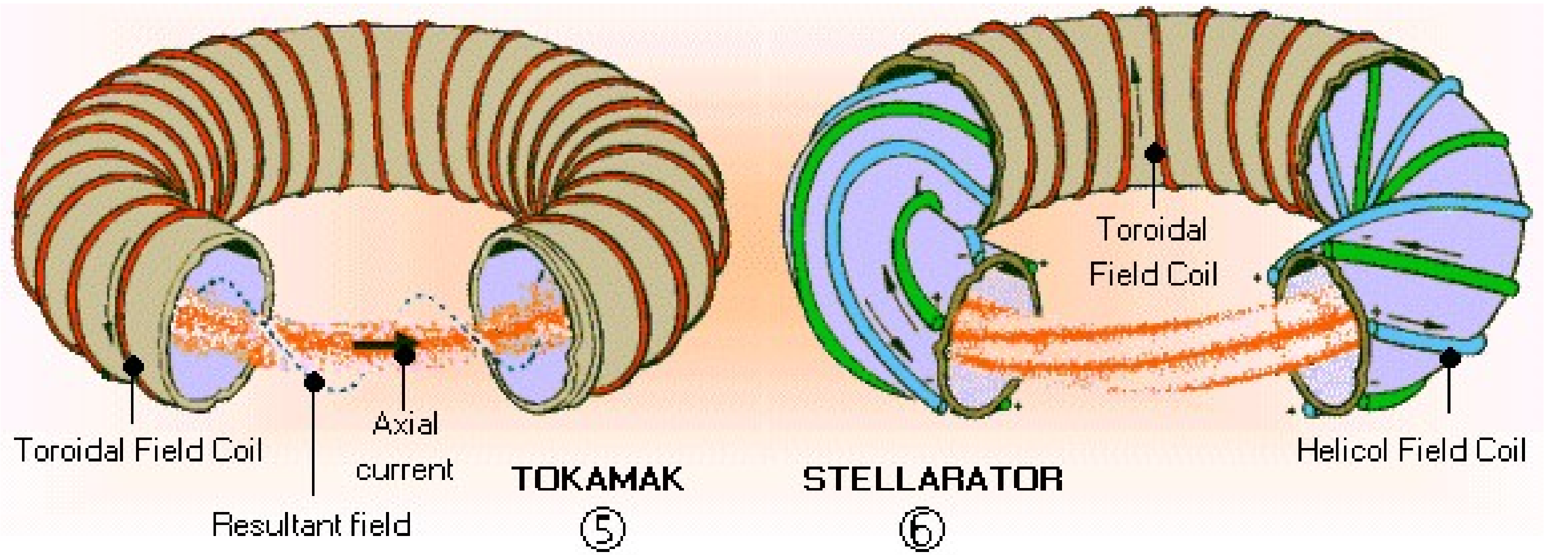
- At some point the poloidal field must be zero
- In the case of toroidal symmetry this field line closes upon itself
- Regions of positive and negative field are not connected
- A field line can not wind around poloidally
- Then top and bottom can not be connected



With toroidal symmetry one field line can not wind around poloidally

The stellarator

- If the field is not toroidally symmetric the motion in the toroidal direction will move the field line from regions of positive poloidal field into regions of negative field
- Then a net poloidal turn of the field line can be achieved
- Steady state operation is possible at the cost of greater complexity

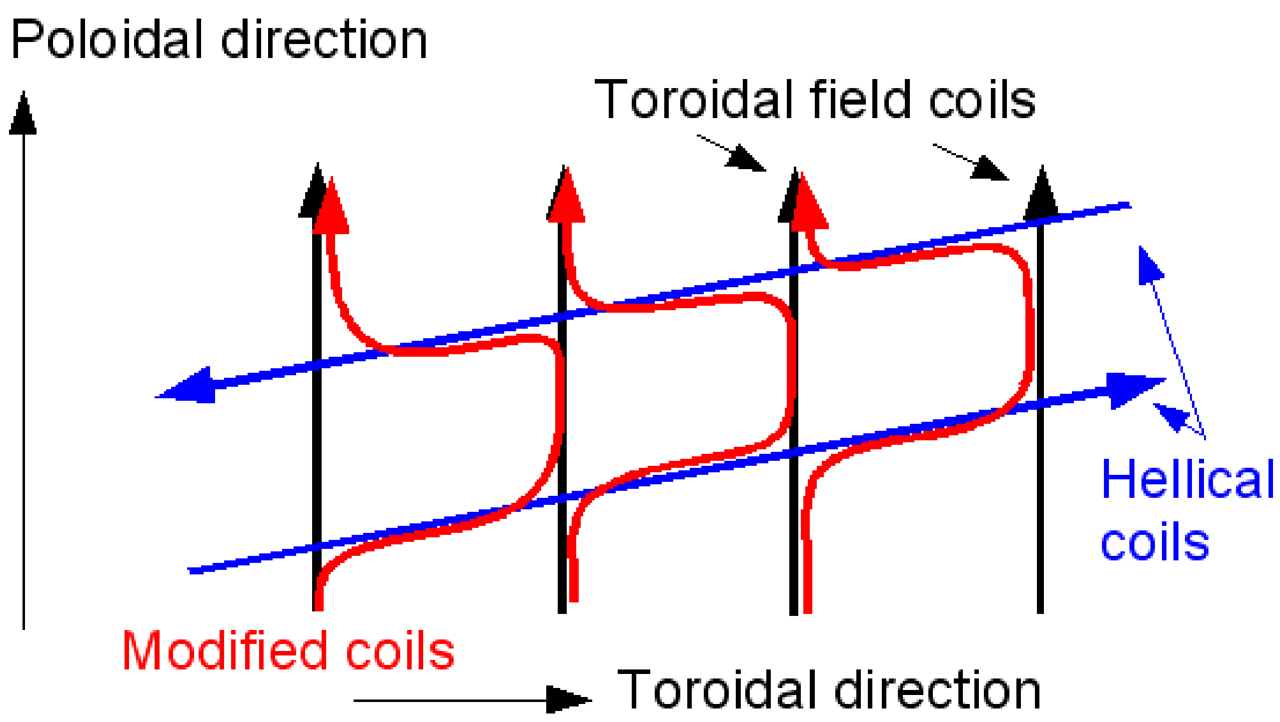


Stellarator

- Inside the device it looks something like this
- Picture from LHD in JAPAN

Hellical coils can be simplified

- The picture shows how the combination of helical coils and toroidal field coils can be changed to use modular coils



Applied in W7X

- Modular coils of W7x
- There is a large disadvantage in the use of the modular coils. They are highly bend and therefore there are large force on them
- Also difficult to generate tight aspect ratio machines because the poloidal field decays away from the coils