# Investigating Magnetic Holes
## From Hall-MHD to Plasma Particle Physics

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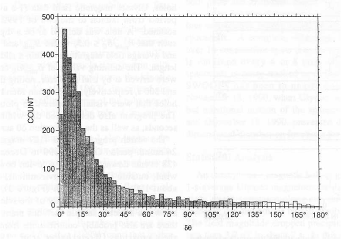
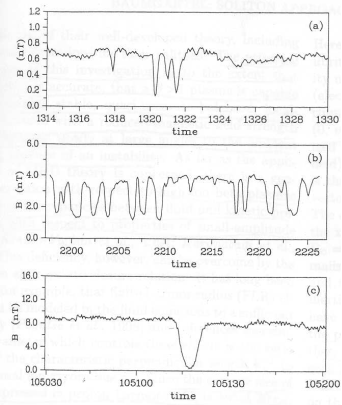
# Satellite Data

MHs are localised dropouts of the magnetic field accompanied by simultaneous increases in plasma density and pressure.




From Winterhalter et al (1994)




From Baumgärtel (1999)

# Possible Creation Mechanisms I

- Mirror Mode Structures
  - Instability found in pressure anisotropic plasma
  - However, the SW tends to be mirror stable
  - Suggestion is that MHs are remnants of mirror-mode structures
- DNLS Soliton
  - DNLS is an evolutionary equation derived from Hall-MHD
  - MHs signatures of DNLS soliton
  - DNLS not applicable in $\beta \sim 1$ plasmas
  - Cannot tell us anything about stability

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# Possible Creation Mechanisms II

- RHP Alfvénic Wavepackets (AWP) and the Ponderomotive Force (PF)
  - Simulations by Buti et al. (2000) suggest that RHP AWPs collapse into MHs
  - Dynamic process supported by Hybrid Simulations
  - PF identified as important for strong pulses
  - PF accelerates particles perpendicular to wave propagation, so MHs could be caused by diamagnetic effects (Tsurutani et al, 2002)
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# Model Premise

- Consider an 1.5D Hall-MHD model
- Move into time independent frame moving with a wave with speed $M_A = v_x/v_A$ at an angle $\theta$ to the magnetic field.
- Assume a polytropic equation of state

$$p_\perp = p_{\perp 0} n^\gamma$$

- Introduce an anisotropy parameter $a_p$

$$a_p = \frac{p_\parallel}{p_\perp} - 1$$

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# Model Equations

$$
\begin{aligned}
0 &= 2M_A^2\left(n^{-1}-1\right)+\beta\left(n^{\gamma}-1\right)+b^2-1+b_{x0}^2 a_p\beta\left(n^{\gamma}b^{-2}-1\right)\\
\frac{\partial}{\partial s}b_y &= f(b)-b_z g(b)\\
\frac{\partial}{\partial s}b_z &= b_y g(b).
\end{aligned}
$$

with

$$
\begin{aligned}
f\left(b\right) &= b_{z0}n(b)\left[1-\frac{1}{M_x^2}\left(1-\frac{a_p\beta}{2}\right)\right]\\
g\left(b\right) &= 1-\frac{n(b)}{M_x^2}\left(1-\frac{a_p\beta}{2}n^{\gamma}(b)b^{-2}\right)
\end{aligned}
$$

System of 3 variables $n, b_y.b_z$ and 5 parameters $a_p, \beta, \gamma, \theta, M_A$

# Model Assumptions

- $M_A \ll 1$, i.e. we are dealing slow moving structures and can write $M_A = \varepsilon \bar{M}_A$. This is supported by satellite data (e.g. Stasiewicz 2004)
- $\beta \sim 2$. The spatial extent of MHs suggests that FLR effects are not important (Pogutse et al 1998)
- Oblique propagation angle but $\theta \lesssim 85^\circ$
- $a_p$ stays constant over the structure.

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We consider an $O(1)$ expansion and search for fixed points.

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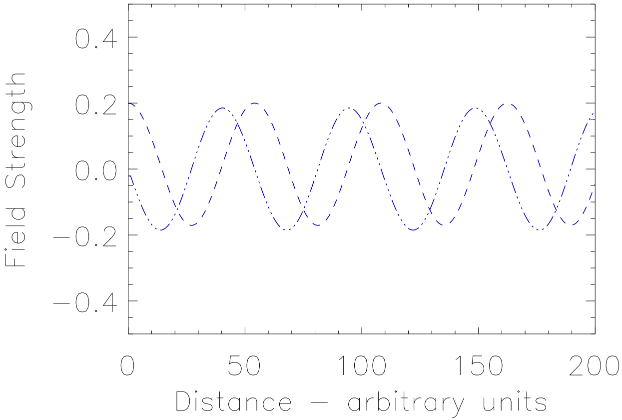
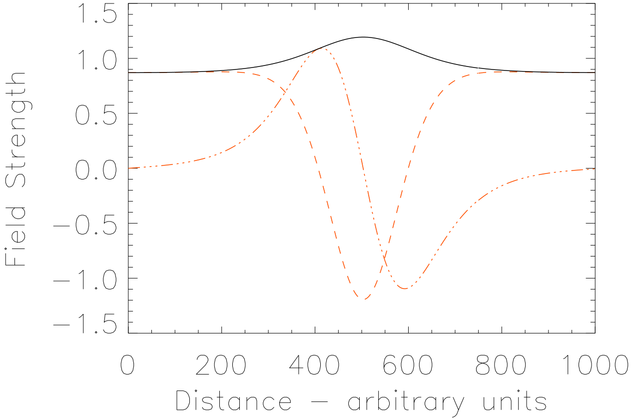
We can show $b_y = 0$ at fixed points and label them as either centre or saddle

# Possible Solutions



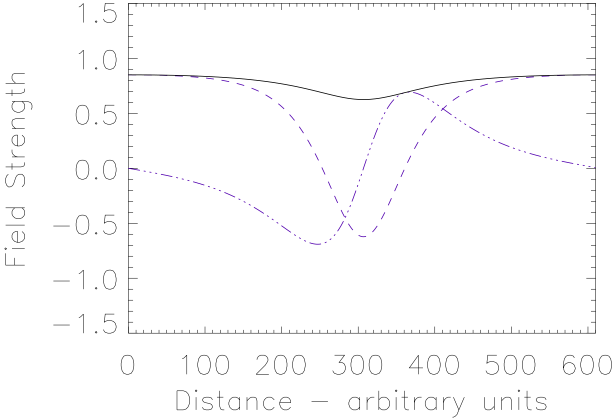

- Periodic wave solutions

# Possible Solutions



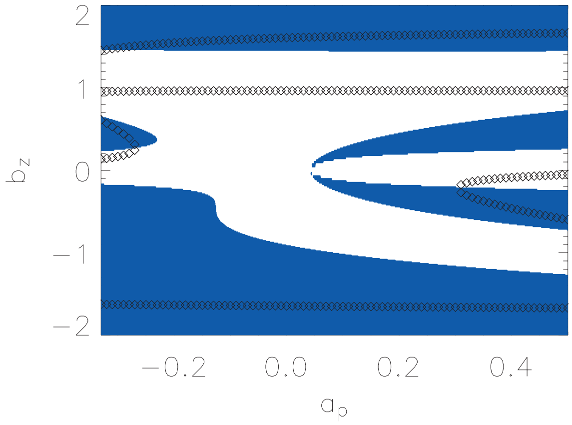

- Bright solitary wave solutions

# Possible Solutions




- Dark solitary wave solutions

# Pressure Anisotropy Dependence




From Simon and Rowlands (2007)

# Developing a Potential Formalism I

- The equations obtained from the curl of Ohm's law can be combined to derive an evolutionary equation for $b$
- Introduce $H(b) = \int bg(b)/f(b)\mathrm{d}b = b_z - b_{z0}$ and $\tau = \int f(b)/b\mathrm{d}s$
- These substitutions yield

$$\begin{aligned} \frac{\partial^2 b}{\partial \tau^2} &= \left[ b - \frac{bg(b)H(b)}{f(b)} \right] - \frac{bb_{z0}g(b)}{f(b)} \\ &= F(b) \end{aligned}$$

- Mathematically equivalent to Newton II, so we can construct a pseudo-potential [Simon et al. (submitted)]

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$$V(b) = -\int F(b)\mathrm{d}b$$

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$$V(b) = \frac{H(b)^2 - b^2}{2} + b_{z0}H(b)$$

# The $O(1)$ Approximation to the Potential

- In the case of slowly moving structures $M_A = \varepsilon \bar{M}_A$, one can show that

$$f(b) \approx \frac{b_{z0} b_{x0}^2 n}{\varepsilon \bar{M}_A} \left( \frac{a_p \beta}{2} - 1 \right)$$

$$g(b) \approx \frac{b_{x0}^2 n}{\varepsilon \bar{M}_A} \left( \frac{a_p \beta}{2} \frac{n^\gamma}{b^2} - 1 \right)$$

$$n^\gamma \approx \frac{b^2 \left[ 1 + \beta \left( 1 + a_p b_{x0}^2 \right) - b^2 \right] )}{\beta \left( b^2 + a_p b_{x0}^2 \right)}$$

- In this case, one can approximate $H(b)$ by

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# General Potential Considerations

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- Periodic wave solutions would correspond to oscillations about a potential minimum
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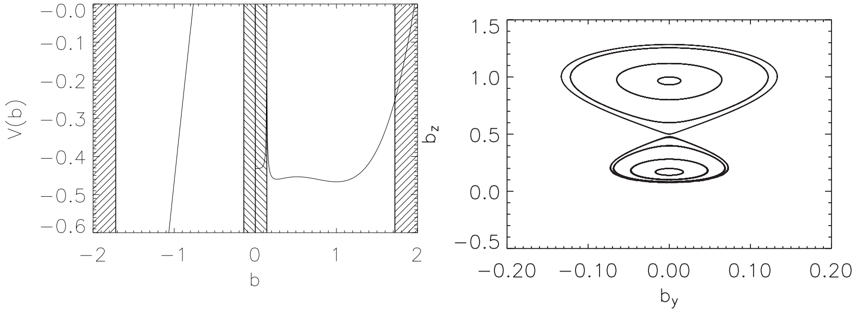

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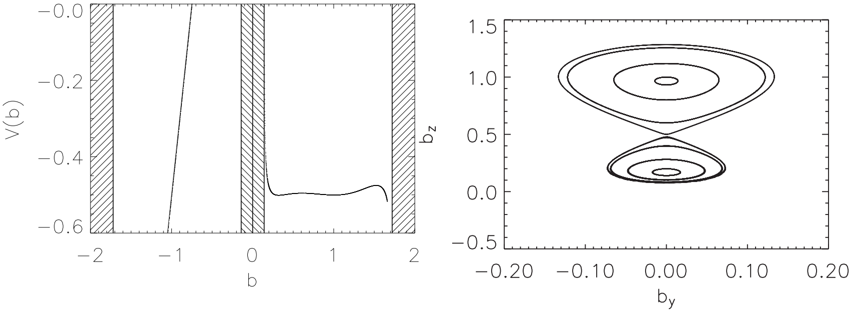

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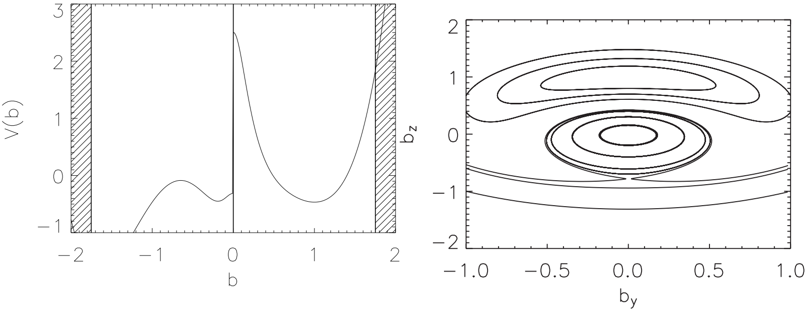

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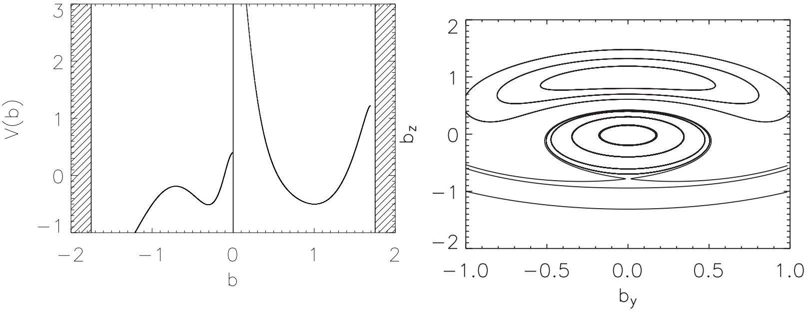

# Stability of Solutions

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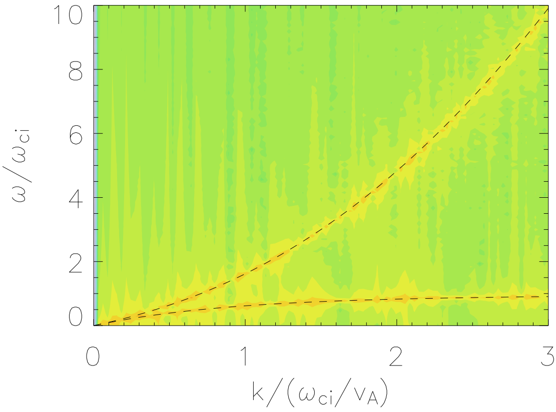
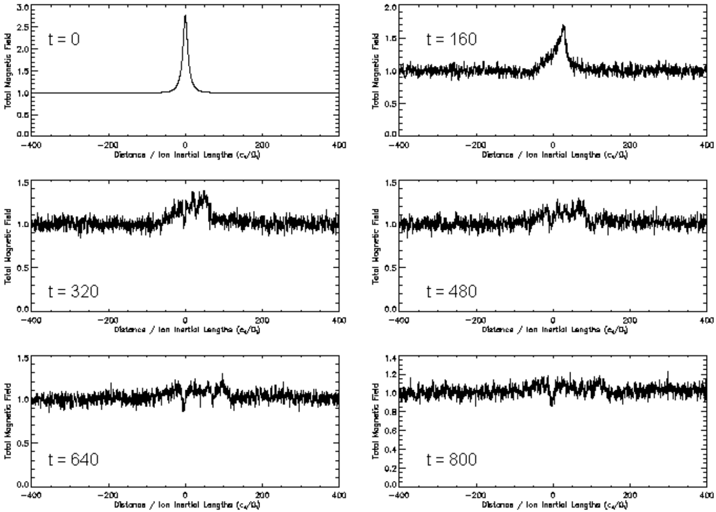


# Example: Dynamic Generation of an MH



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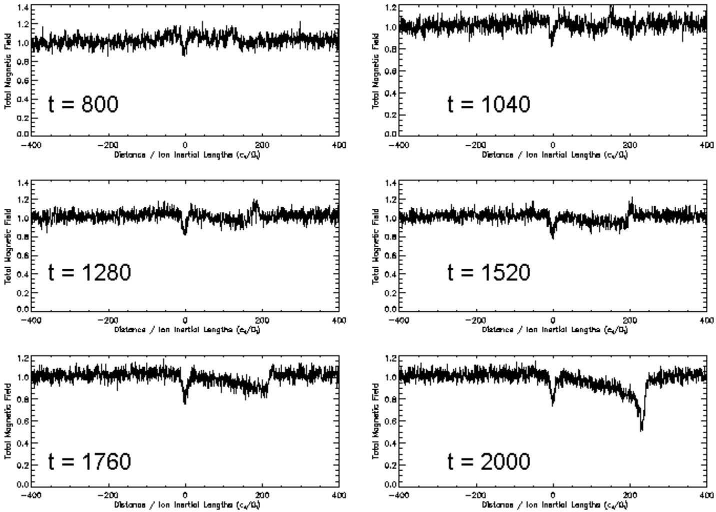

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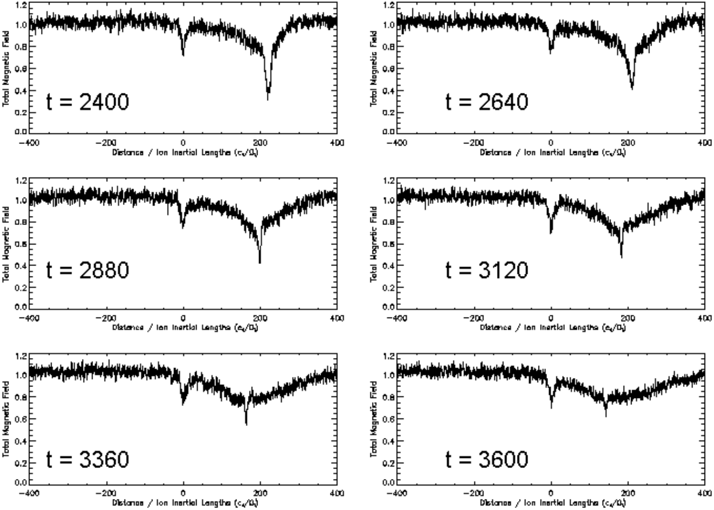

# The case for Full Kinetic Treatment

- Hybrid simulations can include some kinetic effects, but assume that the electron kinetic effects are unimportant.
- Lin et al (1995) found evidence of Langmuir wave creation in the holes. The suggestion is that these waves are created by electron beams.
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# Conclusions

- We present slow mode solitary wave solutions to Hall-MHD which are possible candidates for MHs
- We derive a pseudo potential description which explains solitary waves as local potential max/min combinations
- We are currently performing hybrid simulations to investigate the predicted stability of these structures
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