

Robustness and scaling: key observables in the complex dynamic magnetosphere

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Abstract

Plasma transport and energy dissipation in the driven dynamic magnetosphere are intermittent (bursty), and occur on a range of spatiotemporal scales. System observables such as geomagnetic indices, and auroral images, show evidence of scaling in the statistics of these events. Taken together these are hallmarks of a complex system. Here we underline the importance of robustness in these statistical signatures with respect to variability in the drive, and of bursty transport as opposed to intermittent structures, as key signatures of nonlinear complex avalanching systems. Power law statistics of bursty events do not necessarily require an underlying nonlinearity.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A complex system is understood here in its most general sense [1], as a system in which the emergent observed phenomenology is dominated by interactions between many processes on a disparate range of scales. The challenge in real physical systems is to distinguish this *complexity* from a system that is *complicated*, that is, a collection of many weakly coupled processes occurring in a system that is nonuniform and finite, with subtle boundary conditions. Unlike a complex system, the latter may be understood more appropriately by being broken down into one or more constituent physical processes, occurring on distinct characteristic scales which are only weakly coupled. It is increasingly clear that, especially as regards their global phenomenology, both magnetospheric and confined fusion plasmas may be complex systems as distinct from ('simply') complicated. This provides opportunities, as well as challenges, for observational interpretation and prediction.

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Driven confined plasmas, both in nature (such as the earth's magnetosphere) and in the laboratory, include examples where plasma transport and energy dissipation occur via intermittent (bursty) events (see, e.g. [2–7]). This, when accompanied by evidence of scaling, is a characteristic signature of a complex system [1]. Here, scaling refers to self-similarity over a broad range of signal amplitudes, lengthscales and timescales. This is characteristically manifested in power law distributions of event statistics, obtained over many ensembles of a given system.

The earth's magnetosphere is a particularly attractive subject for complex systems approaches, in that observations over long time intervals exist for the driver (in the form of *in situ* plasma measurements of the solar wind) and for global energy dissipation (measured indirectly through geomagnetic indices, and arguably more directly through auroral images). In addition, multiple satellite passes of the magnetotail provide *in situ* observations of bursty plasma transport (bursty bulk flows [8]).

Scaling is well established in geomagnetic indices [9, 10], which recent work (e.g. [11]) suggests to be distinct from that of the turbulent solar wind driver. This is consistent with an interpretation in terms of self-organized criticality (SOC [12, 13]); see for example [4, 5, 14–19], for reviews. The indices are, however, derived from the maximal response of individual magnetometers from a chain at similar geomagnetic latitudes. An arguably more direct measure of bursty energy release across the system is provided by auroral images, in particular the space-based observations of POLAR UVI [20, 21].

Standard avalanche models of SOC incorporate an important idealization which is not necessarily reflected in real plasma confinement systems. Their fuelling, which would correspond in principle to magnetospheric loading, or to plasma heating in a confined fusion plasma, is assumed to be slow. In general, the conditions necessary for SOC to occur constrain the inflow rate (see [30] for some proposed criteria); to emphasize this, Jensen [13] has proposed a category of slowly driven, interaction-dominated, thresholded (SDIDT) systems. However in both natural and laboratory plasmas, the loading rate can be strong, and can fluctuate substantially. We discuss this here using a simple sandpile model [23] that has been applied successfully to the earth's magnetosphere [15, 22], and to a wide range of distinctive tokamak plasma phenomenology including enhanced confinement and ELMing [24], internal transport barriers [25], and off-axis ECH temperature profiles [26]. There are, of course, significant observational and *a priori* reasons for adopting the sandpile—a paradigm for critical gradient-driven nondiffusive transport in finite macroscopic systems—for plasma modelling.

Power law scaling as a key observable is also problematic, since both natural and laboratory plasmas are finite sized, thus limiting the dynamic range over which scaling can be established. Scaling *per se* is not unique to nonlinear complex systems, such as those in a state of SOC [1]. We will demonstrate this with a physically relevant example, and argue that robustness against system details (such as the drive, and internal thresholds for instability) is a key observable for complex plasmas.

2. Robustness under slow and fast driving

2.1. Numerical sandpile experiments

Simple avalanche models are useful as a 'proof of concept' tool to demonstrate emergent phenomenology which is relevant to the system at hand. For example, in the magnetospheric context our simple model demonstrates [15] that avalanche models can yield systemwide avalanches whose statistics have a well defined mean (intrinsic scale), together with internal

avalanches whose statistics are scale-free. Thus a single avalanche model could in principle encompass both internal intermittent dynamics (bursty bulk flows, pseudobreakups) and global energy release (substorms).

This sandpile model was modified to examine its robustness against varying both the average value of the drive and its level of fluctuation [6, 20, 22], and we review briefly the main conclusions here. In this model, we take a one-dimensional, unit-spaced grid of N cells. The height of sand at the j th cell is h_j , defining a local gradient $z_j = h_j - h_{j+1}$. The pile is assumed stable at the angle of repose z_R , and both h_j and z_j are measured from this stable state. At each cell we assign a critical gradient $z_{c,j}$ drawn from the single distribution $P(z_c)$: thus in this simple model the average $\langle z_{c,j} \rangle$ does not vary across the pile. Redistribution of sand is instantaneous and conservative and only occurs when the critical gradient is exceeded, i.e. when $z_j > z_{c,j}$. A parameter L_f specifies the lengthscale of the region over which instantaneous redistribution occurs and the model exhibits a crossover in its behaviour as we vary this parameter [23], with implications for its confinement properties [24] *inter alia*. Each avalanche releases energy (but not necessarily sand) from the sandpile,

$$\Delta E = \sum_{j=1}^N h_j^2 |_{\text{before}} - \sum_{j=1}^N h_j^2 |_{\text{after}} . \quad (1)$$

Fuelling occurs by adding sand at the first cell. We normalize L and T to the average loading rate, g , so that unit volume sand is added in unit time (i.e. after $1/g$ iterations). The conservative redistribution rules [15] are such that avalanches, built up from instantaneous redistribution events at a sequence of neighbouring cells, occur on all scales up to the size of the pile. The extent to which the loading rate, g , is ‘fast’ or ‘slow’ can be measured by comparing g with the mean value of z_c [22].

A typical time-series of $E(t) = \sum_1^N h_j^2(t)$ is shown in figure 1 for the case $L_f = N$. The 4096-cell sandpile was loaded slowly ($\langle g \rangle / \langle z_c \rangle = 0.01$). Here the angle of repose is normalized to zero: the time evolution is then characterized by a systematic growth as sand is added, interspersed with occasional systemwide avalanches where the energy falls back to zero and more frequent internal avalanches where the energy is reduced to some nonzero value.

In the case of the earth’s magnetosphere, the driver (i.e. the solar wind) varies both in its mean level and in its variability. Numerical experiments that explore order of magnitude differences in both the average value of g and its variability have been performed [20, 22]. Essentially, for any loading rate there is an effective minimum avalanche length required to dissipate the energy associated with the sand added in each Δt . In the slow limit this corresponds to less than one cell, in the fast limit to many. If the mean fuelling rate, $\langle g \rangle$, is increased towards and beyond $\langle z_c \rangle$, the smallest avalanches will be increasingly eliminated. This will be reflected in the lower bound of any range of power law event statistics for energy release in the system. We illustrate this in figure 2, which shows the probability distributions for energy released by all avalanches (internal and systemwide) in an $L_f = N$ system for the above driving rate, and for an average driving rate increased by a factor of 10.

From these experiments we find that the statistics of the larger events remain unaffected by the changes in loading, illustrating the fact that the largest avalanches are robust against the fuelling process. The smallest events in the internal avalanche distribution are modified by changes in the drive, but as we consider successively larger events which propagate further from the feeding site, we find that these follow the same power law slope independent of details of the drive. The crossover between events that are strongly affected by the drive, and those dominated by the dynamics internal to the sandpile, is determined by the inflow rate. Essentially, those avalanches that are power law distributed are still in the slowly driven limit, in that they involve an amount of sand that is introduced to the pile over many time-steps. This

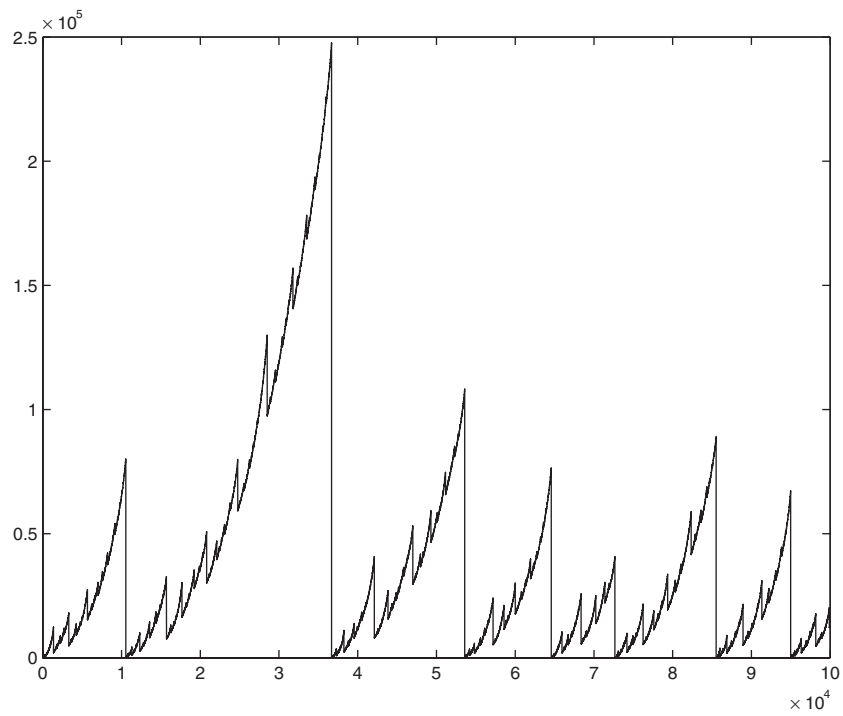


Figure 1. Energy, $E(t)$, as a function of time in the sandpile model, after [16].

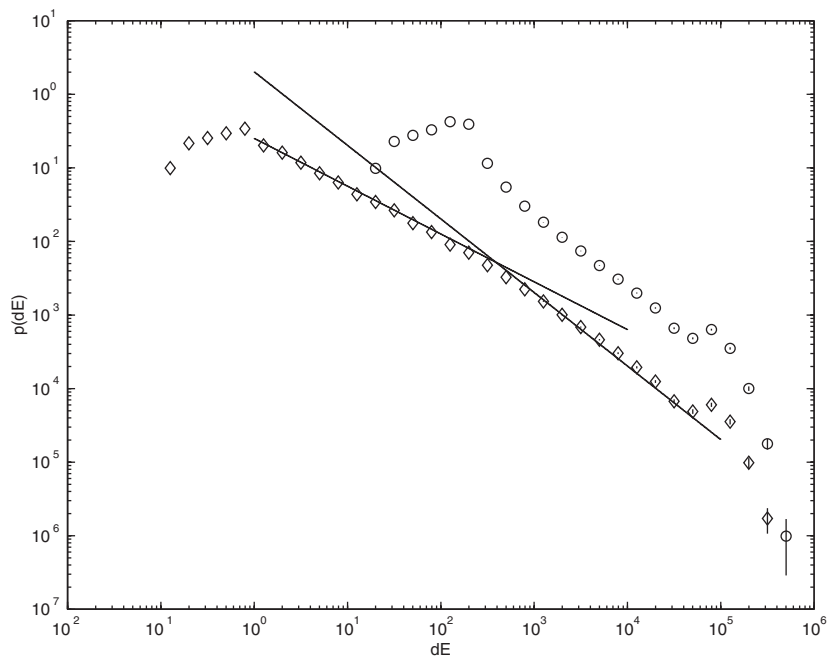


Figure 2. Logarithmically binned histogram of energy dissipated by avalanches under slow (\circ) and fast (\diamond) driving, after [6].

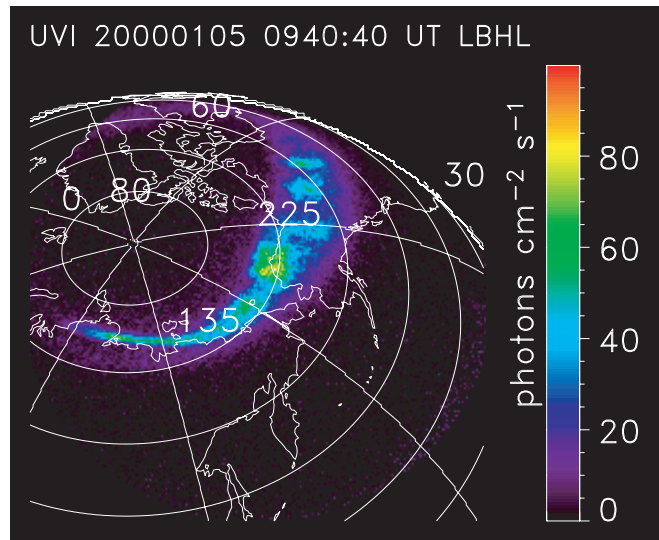


Figure 3. A snapshot of the aurora from POLAR UVI. (Courtesy G Parks.)

is a modification of the idealized concept of SOC in which the entire system is in a slowly driven limit (SDIDT [13]).

The simple sandpile numerical experiments thus suggest a key criterion for complex dynamics: robustness of the statistical distribution of internal events (whether power law or otherwise) against variation in the drive.

2.2. Auroral 'blob' statistics

A representative example of the global images of the aurora obtained by the ultraviolet imager on the NASA Polar spacecraft is shown in figure 3, with the auroral intensity represented by a false colour scale. Typically there are several spatially distinct patches (or blobs) of auroral activity. To perform analysis on the auroral images, they are transformed onto a Mercator projection of magnetic latitude and local time. To determine the distribution of power and size of these auroral activity regions, the background intensity in the polar region outside the luminous auroral belt is subtracted from the intensity in the auroral belt. We then identify individual auroral blobs by applying an intensity level corresponding to a threshold well above (ten times) the noise level; the statistics remain unchanged when other thresholds are used. The power in the auroral intensity and the area of each blob are then calculated. Each individual image is sorted to select intervals of active times (when a substorm is occurring) and quiet times, excluding intermediate conditions (see also [27]).

This analysis was performed [20] for the entire data set of 1–31 January 1997, consisting of 9033 frames of auroral images. Normalized, logarithmically binned histograms of auroral blob size and power are shown in figure 4. The total number of auroral blobs is 81 794 for quiet time (left panels), and 155 347 for substorm intervals (right panels). The upper range of the area is limited by the total viewing scope of the instrument, and the lower range by the instrument's effective spatial resolution and sensitivity, which vary along the POLAR orbit.

These plots suggest that there is an ever-present component of auroral activity seen in the small scale events, which can be fitted by a finite range power law. To within the errors, the slope obtained by fitting a power law is insensitive to the presence or absence of substorms,

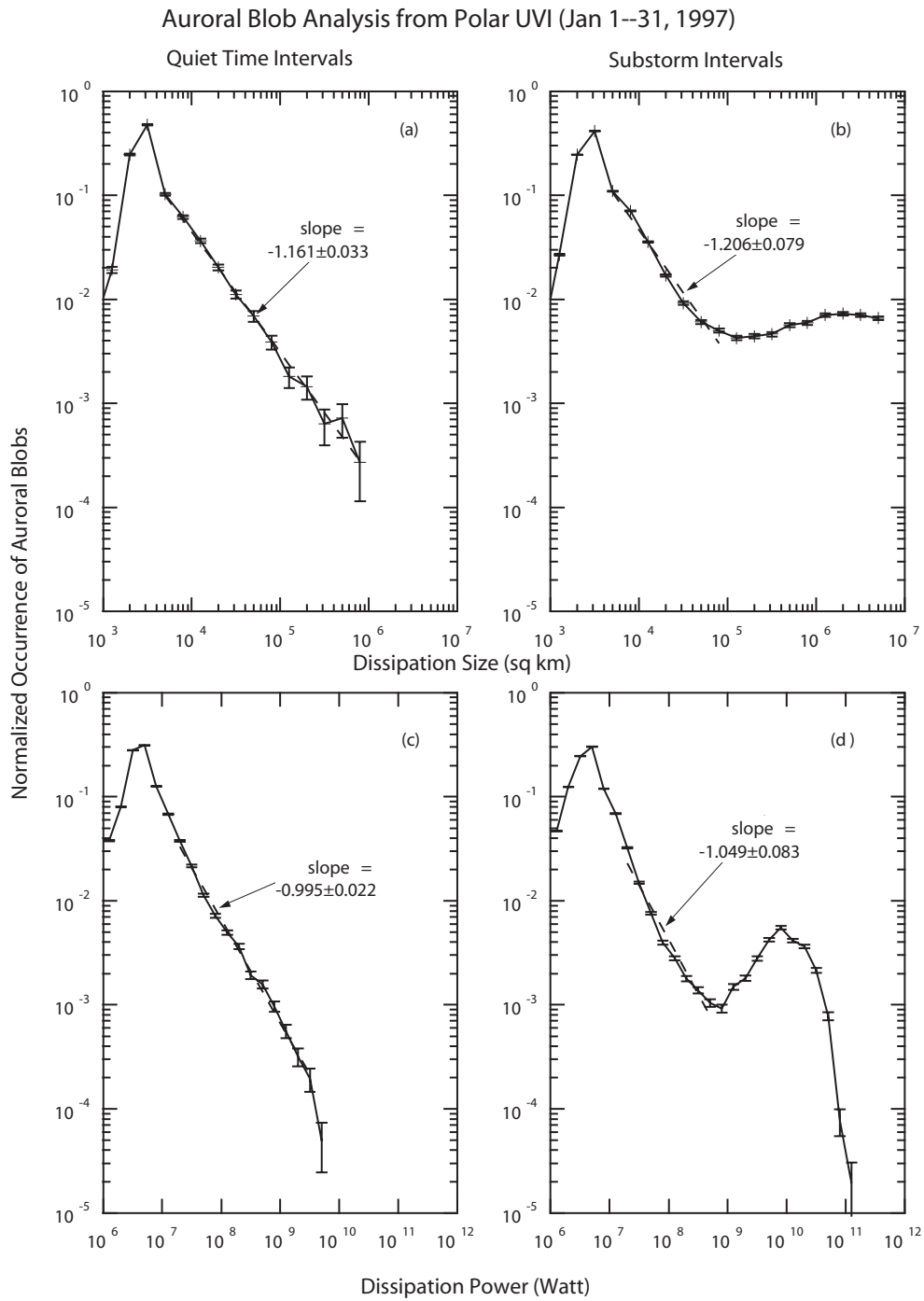


Figure 4. Histograms of patches of activity seen in POLAR UVI snapshots of the aurora. Data are ordered by geomagnetic activity. After [20].

and hence independent of the level of activity in the system. We interpret these power law-distributed events as bursty, internal reconfigurations of the system. The other events, which show well defined distribution peaks in terms of power dissipated and size, are interpreted as global reconfigurations. The points of similarity between figures 4 and 2 were noted in [20].

3. Edwards–Wilkinson: a linear model for power law blob distributions

Power law blob distributions need not, however, imply an underlying *nonlinear* critical phenomenon. There are a variety of mechanisms that generate power law scaling but are not related to nonlinear phenomena and/or collective effects (see, e.g. [1]). Here we consider an example which could arguably arise from some of the simple physical considerations that we have also discussed.

Consider a system where there is constant random precipitation of a scalar quantity onto a plane. This quantity can then spread across the plane by linear diffusion. At each point on the plane, the magnitude of the scalar quantity defines a growing surface (or interface) which evolves in both space and time.

The Edwards–Wilkinson (EW) model (see [28] for a comprehensive review, and [29] for its relevance to fusion materials science issues) for interface growth advances the height of an interface, $h(\mathbf{r}, t)$, by random addition of grains over the plane, with stochastic source term η , and by diffusion, with constant coefficient D . If the source term is simply uniformly distributed noise, then the average height over the plane, $\langle h \rangle$, grows linearly with time. We can express the evolution of the surface in terms of the height with this average subtracted, $\tilde{h} = h - \langle h \rangle$, giving

$$\frac{\partial \tilde{h}}{\partial t} = D \nabla^2 \tilde{h} + \eta. \quad (2)$$

This linear system grows an interface with scaling properties. An important nonlinear generalization of the EW model is the Kardar–Parisi–Zhang equation, obtained by adding a term $+|\nabla \tilde{h}|^2$ to the rhs of equation (2); this in turn maps onto Burgers' equation under a simple transformation [31]. Equation (2) also generalizes to an SOC model if we introduce a nonlinear form for the diffusion, $D(\tilde{h})$, which has a hysteresis, such as that discussed in [30].

Here we consider a numerical realization of equation (2) constructed in the spirit of avalanche models. We mimic diffusion by updating heights on a two-dimensional grid in the following way. For each cell at location (p, q) we remove a fraction of the height $\Delta_{pq} = Dh_{pq}$ ($D < 1$) and divide it evenly between neighbouring cells $i = p \pm 1, j = q \pm 1$, so that the updated heights $h'_{pq} = h_{pq} - \Delta_{pq}$ and $h'_{ij} = h_{ij} + \Delta_{pq}$. This operation mimics the diffusion term of equation (2) in that it is symmetric in space, is conservative in h and redistributes h at a rate that is proportional to the local gradient and to the diffusion coefficient, D . Except that height is treated as a continuous rather than integer variable, this model can be mapped to the original sandpile algorithm of Bak *et al* [12]. Importantly, net redistribution of sand occurs whenever there is a nonzero local gradient: in sandpile terms, the critical gradient is zero, so that there is no local threshold for this process.

If, for a two-dimensional surface, we threshold \tilde{h} at some arbitrary constant value h_0 , we obtain patches (or blobs) where $\tilde{h} > h_0$. A snapshot of these patches is shown in figure 5, which we have obtained by evolving our 'sandpile' implementation of equation (2) numerically on a 100×100 grid. One can then consider this as a naive model for a system, such as the aurora, perhaps, where there is constant random precipitation of energy which diffuses locally and is uniformly damped. Such a system when thresholded naturally evolves patches of different sizes. We have evolved this and a larger system (500×500) to obtain an adequate dynamic range of patch sizes, and have obtained the histogram of patch sizes shown in figure 6, binned

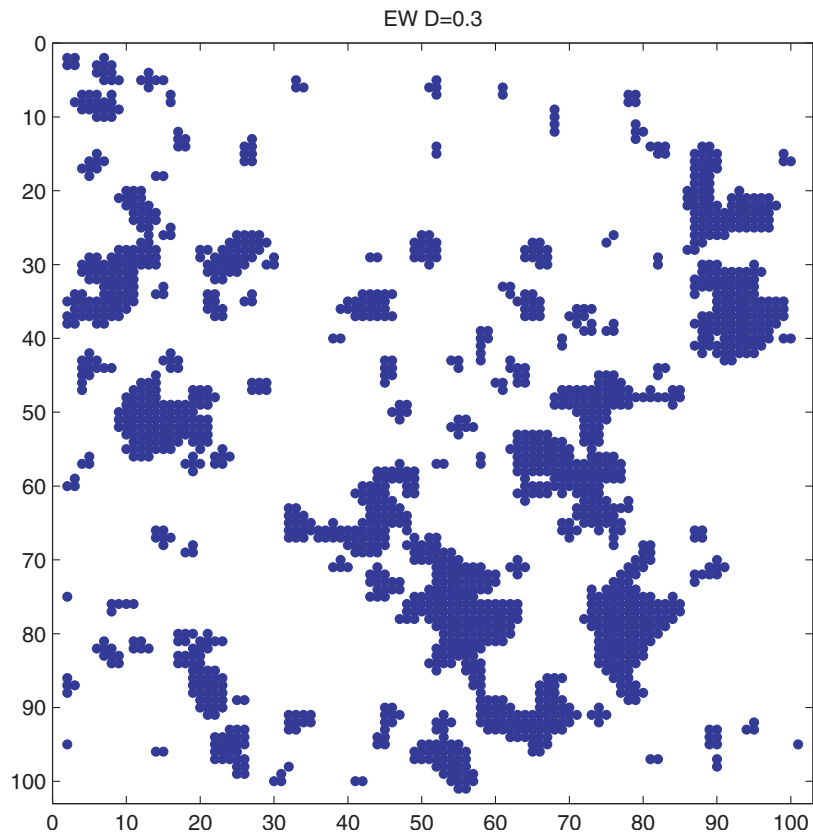


Figure 5. A snapshot of patches (grid points where the height exceeds a fixed threshold) generated in a 100×100 grid cells EW model.

logarithmically as before (cf [15, 20, 22, 23]). From the figure we see that the patch histogram shows power law scaling over much of its range.

Power law distributions of a single measurable do not constitute a unique indicator of an underlying nonlinear and critical process, such as SOC. To make this identification, one needs a relationship between several exponents that fully quantify the scaling properties of avalanches (see, e.g. [13, 32]); these are seldom readily available for physical systems of interest. Reconstructing the full evolution of avalanches in space and time from series of snapshots is problematic, but has been attempted for POLAR UVI [21]. There is a more readily observable feature that distinguishes the EW model from avalanching systems—the patches cannot propagate across the system, so that they do not transport information or matter. This can be verified numerically, but also follows directly from the symmetry in r of equation (2). This symmetry would be broken by introducing a threshold for instability, either directly in the redistribution rule, as in [12], or via a nonlinear form for the diffusion, as in [30], which would introduce avalanches which propagate in the direction where locally the critical gradient is smallest. Furthermore the scaling seen in the EW model is directly related to the randomness in the drive, which is needed to generate the ‘roughness’ of the surface. Thus a uniform drive (that is, uniformly rather than randomly added grains) generates a smooth surface, so that patches would not arise from the above thresholding procedure. Different scaling properties

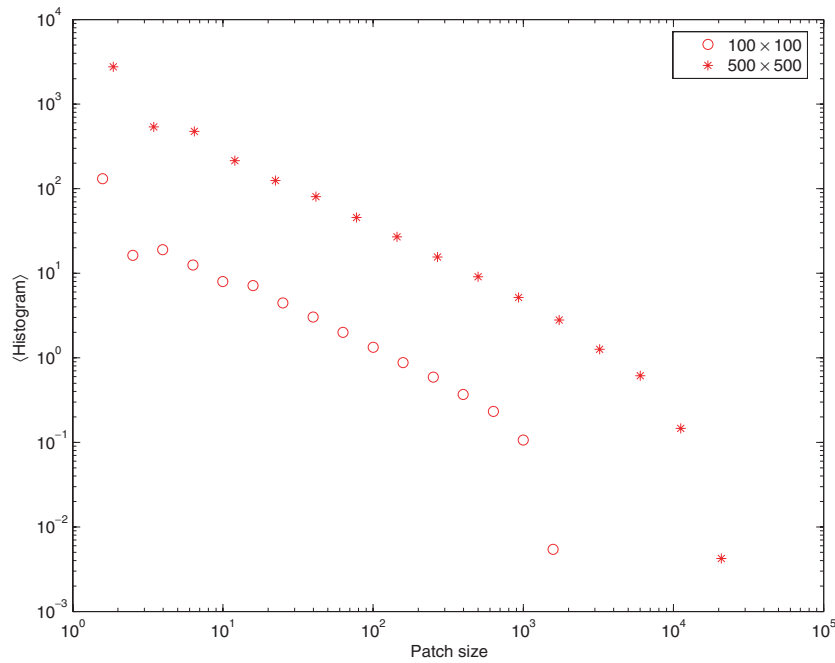


Figure 6. Logarithmically binned histograms of patches in snapshots of realizations of the EW model. Results shown for systems of 100×100 and 500×500 grid cells.

under a random drive are reflected in the scaling properties of the resulting surface [28]. This lack of robustness against details of the drive is in contrast to the case of both the auroral images and the simple avalanche model discussed previously.

4. Conclusions

Identifying observables that are unique indicators of complexity in real physical systems is a current challenge. Here we have discussed observable features and criteria relevant to a natural driven confined plasma system—the earth’s magnetosphere. We have contrasted these with the emergent phenomenology of a simple avalanche model, and of an interface model. Both these models generate bursty energy release with event statistics that are self-similar, displaying power law scaling. Importantly, the former is a nonlinear complex system whereas the latter is linear. The phenomenology of the avalanche model is insensitive to order of magnitude variation in both the average value of the drive and its variability, as well as to the random thresholds for internal instability. In the interface model, on the other hand, bursty activity displaying scaling arises as a direct consequence of the properties of the fluctuations in the random drive.

This example illustrates the importance of robustness in the statistics of events that lack a characteristic scale, rather than power law event statistics *per se*. It is a key indicator of a class of nonlinear complex systems relevant to driven natural and fusion plasmas. These physical systems can be in a domain which is far from the classic limit (SDIT) identified for idealized SOC.

As we have seen, the application of concepts and quantitative techniques from complex systems science to plasma physics can be very fruitful. The distinctive features of macroscopic

plasmas, and the constraints which these impose upon the types of measurement that can be taken, must be considered carefully, and give rise to questions of generic physical interest.

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