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Scaling, asymmetry and a Fokker–Planck model of the fast and slow solar wind as seen by WIND

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The solar wind plasma is a natural laboratory for studies of plasma turbulence. Long, evenly sampled satellite data sets are natural candidates for statistical studies and these are often performed in the context of magnetohydrodynamic (MHD) turbulence. In this paper, scaling properties of solar wind bulk plasma parameters are discussed. Low order probability density function (PDF) asymmetry analysis is applied to data that combines slow and fast solar wind. Results are compared to those obtained using a PDF rescaling technique. A break in scaling, identified by the rescaling method, is confirmed to occur at a temporal scale of $\sim$26 hours. Low asymmetry levels of the fluctuations PDF are identified for the quantities that also exhibit self-similar statistics. A generalized structure function analysis is then applied to the kinetic energy density obtained from slow and fast solar wind streams. The applicability of a Fokker–Planck model for slow and fast wind ρv^2 fluctuations is investigated. © *2004 American Institute of Physics.*
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I. INTRODUCTION

Astrophysical plasmas, dominated by high Reynolds number flows can provide insights into the nonlinear dynamics of magnetized fluids. Conditions found in those systems can be difficult to recreate in the laboratory experiments or by direct numerical simulation where the range of system sizes and observed Reynolds number are limited.[1,2] In this context, *in situ* solar wind measurements are particularly relevant as they provide long, evenly sampled data sets with reasonable temporal resolution. Such data sets are conducive to statistical studies and these have recently revealed scaling as a unifying feature found in fluctuations of plasma parameters.

Due to the large Reynolds number and the $f^{-5/3}$ power laws observed for the solar wind, scaling is often discussed in the context of the Kolmogorov's 1941 (hereinafter K41) hypothesis[3] that energy is transferred from one spatial scale to the next at a constant rate ϵ through local interaction within the inertial range. The lack of any characteristic spatial scale within the inertial range itself leads to self-similar scaling for the moments of velocity structure functions:[4] $S_\ell^n = \langle |v(r+\ell)-v(r)|^n\rangle \propto (\epsilon\ell)^{n/3}$, where n is the nth moment, ℓ is a spatial scale, and ϵ represents the energy transfer rate. This self-similar scaling is obtained for nonmagnetized and incompressible fluids. Experimental results from hydrodynamical studies do not confirm such scaling in real flows, however. It has been suggested that intermittency, induced by random variations in the energy transfer rate ϵ, produces the observed deviation from self-similar scaling.[5,6] Intermittency requires a multi-fractal phenomenology to be invoked so that the self-similarity of the cascade can be broken. Multifractal, or intermittent behavior has also been found in solar wind plasma flows.[7] In this sense, statistical features of fluctuations found in hydrodynamic flows and those observed in the solar wind plasma appear to be very similar.[8,9] In recent years, models developed for hydrodynamic turbulence, such as the Castaing model,[10] has been successfully applied to magnetized plasmas.[11,12]

An obvious disadvantage of using spacecraft data in studies of turbulent plasmas comes, in general, from the lack of multi-point measurements. Available data sets are generated in the form of time series collected from a single-point location. The typical solar wind speed of about 500 km/s largely exceeds the speed of the spacecraft as well as the local Alfvén speed and the Taylor's hypothesis[13] is often used to relate scaling in the temporal and spatial domain. One assumes that the frequency of the fluctuations f is directly related to a spatial scale, or wave number k, through the formula $k=2\pi f/v_{sw}$. However, the solar wind velocity v_{sw} fluctuates between 200 and 900 km/s in time and exhibits complex multi-fractal scaling.[7]

Most statistical studies of the turbulent solar wind focus on the properties of the velocity field or on quantities such as magnetic field magnitude. Such an approach is often chosen because some statistical properties can be inferred from existing theoretical results[5,14] for these parameters. A number of models have also been proposed to incorporate the observed intermittency in these quantities.[10,15,16] These models predict a functional form of the fluctuation probability density function (PDF) or an exact formula for the dependence of the structure function on its order n. Here, we refer to a few selected studies that exemplify this approach.[8,11,17,18] These models, however, are based on a number of assumptions about the energy transfer rate that cannot be easily verified experimentally. An alternative way of modeling turbulence is to look for the set of variables that are mono-scaling and as such can offer a simplified description of the plasma

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flows. In this context it is important to identify these quantities and verify that their behavior is distinct from that of multi-fractal parameters.

Here we first consider the following bulk plasma parameters: Magnetic field magnitude B, velocity magnitude v, ion density ρ, kinetic and magnetic energy density (ρv^2 and B^2) and Poynting flux approximated by vB^2 (such an approximation of the Poynting flux assumes ideal MHD where $\mathbf{E}=\mathbf{v}\times\mathbf{B}$). It has been reported that PDFs of fluctuations in ρ, B^2, ρv^2, and vB^2 exhibit mono-scaling for up to 10 standard deviations,[20] while B and v are multi-fractal.[11,21] In this paper we will use two independent techniques to further investigate statistical and scaling features of these mono-scaling quantities. First, a low order analysis[22] will be used to compare the asymmetry in statistics of positive and negative fluctuations. We will examine if such a low order method can detect statistical differences between quantities that are self-similar and those that are multi-fractal. Low order measures are preferred in statistical studies due to their better convergence and accuracy. They produce very stable results as the contributions from large but very infrequent events are small. Results of this asymmetry analysis indicate that the mono-scaling character of the fluctuations coincides with low asymmetry levels, while the multi-fractal fluctuations, like those of the velocity, show elevated levels of asymmetry. We will also use the asymmetry analysis to verify the relevance of the ~26 hours time scale, above which the PDFs no longer show scaling.

While the fundamental differences between the slow and the fast solar wind are well documented, earlier results discussed here were obtained for the data set that combined both slow and fast streams.[20] We now explore the differences in scaling of the kinetic energy density ρv^2 derived from both slow and fast solar wind. The non-Gaussian character of the fluctuations, whilst also present in low moments of the PDF, is most visible in the higher moments. For this reason structure function analysis is often performed in studies of nonlinear systems. Here we will use generalized structure functions (GSF) to verify that the mono-scaling of the fluctuations persists for higher moments of the PDFs. These GSFs will also be conditioned at 10 standard deviations.[8,23] Such conditioning not only eliminates statistical noise in large but infrequent events and removes coherent structures (shocks, Coronal Mass Ejection events) that are not part of the turbulent cascade but also effectively quantifies the impact of intermittency on fluctuations of different sizes. The GSF analysis has the advantage of being easily incorporated into K41 theory.[4] We will later show that the scaling index for these functions can also be related to that found via the PDF rescaling technique. We note that the similar scaling properties of mono-scaling quantities and, in particular, the nearly identical functional form of their fluctuation PDFs[20] suggest that the differences between slow and fast wind found in $\delta(\rho v^2)$ could be representative for all mono-scaling parameters considered here.

Results presented here have implications for modeling solar wind fluctuations and our understanding of turbulence in magnetized plasmas. We confirm that the mono-scaling approximation of the PDFs for the investigated quantities holds up to a temporal scale of ~26 hours. Low levels of asymmetry found for these PDFs clearly distinguish them from the velocity fluctuations that are well known to be multi-fractal. We also demonstrate that the fluctuations used in the PDF rescaling analysis are statistically independent. The mono-scaling of a fluctuation PDF combined with the statistical independence of the increments allows a Fokker–Planck[24] model to be introduced not only to identify the functional form of the space dependent diffusion coefficient but also to develop a diffusion model for the shape of their PDFs.[20] We discuss the applicability of this model to slow and fast solar wind fluctuations in kinetic energy density.

II. THE DATASET

The single point time series investigated in this study was collected at the Earth–Sun L1 point using the MFI[25] and the SWE[26] instruments of the WIND spacecraft. This data set is identical to that used in our previous work.[20] It contains both slow and fast velocity streams; previously we did not differentiate between these. We will examine the differences in scaling properties of the kinetic energy density obtained from slow and fast solar wind streams. The slow streams are defined as these for which the speed on time scale τ is lower than 500 km/s. We will examine fluctuations in the investigated quantities $\delta x(t,\tau)=x(t+\tau)-x(t)$. For the slow (fast) solar wind stream we require both $x(t+\tau)$ and $x(t)$ to have speeds below (above) the threshold.

III. RESULTS AND DISCUSSION

A. Asymmetry measure

Asymmetry of the fluctuation PDF is one of the characteristic, but not unique, features of a turbulent fluid. If considered in the context of K41, it is an immediate consequence of the directional cascade which imposes a nonzero third moment on spatially averaged fluctuations of velocity, $\langle(\delta v_\ell)^3\rangle\propto\ell$. However, the PDF asymmetry is not uniquely associated with turbulent systems and could be explained by models other than an energy cascade. Using low order asymmetry measures as an indicator of the turbulent flow was first proposed by Vainshtein.[22] The analysis is performed separately for the positive and negative fluctuations, δx^+ and δx^-. Vainshtein computed these differences for the smallest temporal scale τ_0 but, in principle, the asymmetry level should be the same over the entire inertial range.[22] A zeroth order correlation function for these difference is defined as follows:[22]

$$C^{\pm}(\tau)=\langle\{[\delta x^{\pm}(t+\tau_1)-\mu][\delta x^{\pm}(t)-\mu]\}^0\rangle, \qquad (1)$$

where μ is an average of the δx time series and $\langle\,\rangle$ denotes averaging over time. It is often assumed that the mean value of fluctuations μ is 0 for all temporal scales. Such an assumption can be made when the higher moments are considered. For the real data a slight departures from this assumed zero mean do occur. The low order asymmetry measure has functional dependence on the position of this mean and thus we subtract it in (1). We compute $C^{\pm}(\tau)$ for different values

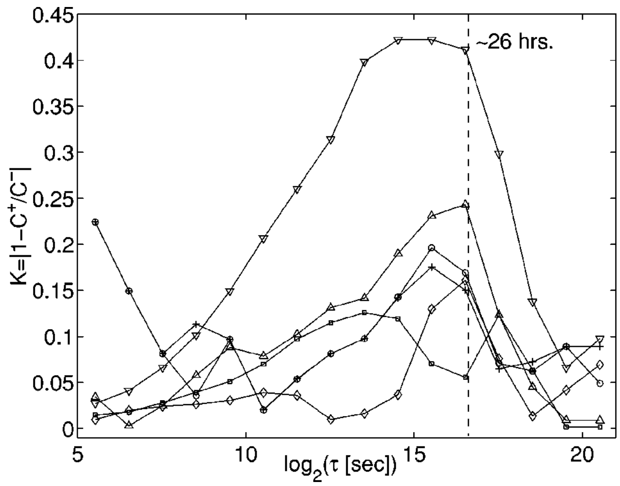

FIG. 1. Levels of the PDF asymmetry K for all quantities under investigation: ○ corresponds to δB^2, □ ion density $\delta\rho$, ◇ kinetic energy density $\delta(\rho v^2)$ and △ Poynting flux component $\delta(vB^2)$, ▽ velocity magnitude δv and + magnetic field magnitude δB.

of τ to determine if the change in the asymmetry level found in the fluctuations PDF is consistent with the PDF rescaling results. Namely, the time scale where the PDF's self-similarity is no longer present should coincide with the time scale where a sharp drop in a measured asymmetry occurs. In order to visualize changing asymmetry levels on different temporal scales τ, we consider the ratio $K=|1-C^+/C^-|$ which is close to 1 for a completely asymmetric PDF and tends to 0 as the asymmetry level decreases.

Figure 1 shows the PDF asymmetry K as measured by the zeroth order correlation function (1). There is a common behavior for all curves in this figure; all quantities exhibit increasing asymmetry levels with increasing temporal scale τ. The temporal scale where the maximum asymmetry is reached clearly coincides with that found in the PDF rescaling method.[20] There are, however, important differences in the level of asymmetry between mono-scaling and the velocity PDFs. We see that the fluctuation PDFs for all mono-scaling quantities investigated here are nearly symmetric. A low level of the measured asymmetry, on average smaller than 10%, persists through the region between ~ 2 minutes and ~ 10 hours. The asymmetry rises to about 15% when we approach the temporal scale of ~ 26 hours for which the scale break occurs.[20] This is in sharp contrast with the strong asymmetry of the PDF of velocity fluctuations. A monotonic increase of the asymmetry is observed, and, interestingly, the maximum of this curve also coincides exactly with the temporal scale where the self-similarity of the PDF is no longer valid. Such strong asymmetry of the velocity fluctuations has been also found in hydrodynamic turbulence.[10,22,27] This suggests that the high asymmetry of the statistics for the velocity increments is a good indicator of the highly intermittent character of the flow and multi-fractal scaling of these fluctuations. We also find a very different behavior of the the asymmetry curve of the magnetic field magnitude fluctuations. Statistically these fluctuations are known to be multi-fractal when treated on longer timescales[21] but here their PDF asymmetry level is quite low except for the first two temporal scales. In fact, the curve nearly coincides with that of the magnetic energy density B^2, for which a mono-scaling PDF appears to be a good approximation.[20] Such behavior suggests that the mechanisms leading to a lack of mono-scaling in the fluctuations of velocity and magnetic field magnitudes could be different.

Figure 1 indicates that fluctuations in the kinetic energy density ρv^2 exhibit the lowest level of asymmetry between positive and negative branches of the PDF. The nature and statistical properties of fluctuations in density, velocity and magnetic field are different for slow and fast solar wind streams. Recently Pagel and Balogh[19] quantified these differences by applying P-model[16] to the magnetic field magnitude fluctuations derived from Ulysses data sets collected during the solar minimum and maximum. They found an appreciable difference in the intermittency levels between slow and fast solar wind with the fast wind showing higher intermittency levels. These differences could affect scaling properties of bulk plasma parameters fluctuations examined here and in the next section we investigate if this difference can be detected for $\delta(\rho v^2)$ when slow and fast streams are treated separately.

B. Generalized structure functions

Generalized structure functions (GSF) S_m are widely used to characterize non-Gaussian processes where departure of a PDF from the normal distribution is the most pronounced in the higher moments.[4] These functions can be defined for fluctuations $\delta x(t,\tau)=x(t+\tau)-x(t)$ as $S_m(\tau) \equiv \langle|\delta x|^m\rangle$, where m can be any real number, not necessarily positive. If these moments of the differenced data exhibit scaling with respect to the time lag τ we have $S_m \propto \tau^{\zeta(m)}$. A log–log plot of S_m versus τ should then reveal a straight line for each m and the gradients $\zeta(m)$. If $\zeta(m)=\alpha m$ (α constant) then the time series is self-similar with single scaling exponent α. Recently, conditioned GSFs have been used to quantify the impact of intermittency on fluctuations of different sizes.[8,23] Practically, the method relies on eliminating the largest fluctuations, where the statistical errors are the greatest. In our case, this threshold will be based on the standard deviation of the fluctuation time series for a given τ, $A(\tau) = 10\sigma(\tau)$. We will use symbol S_m^c to indicate that the structure function has been conditioned.

Generalized structure functions can be easily related to the generic and model independent PDF rescaling technique described previously in Refs. 20 and 28. Here we only recall that this method is based on the rescaling of the PDFs, obtained for fluctuations δx, on different time scales τ. If the process under investigation exhibits statistical self-similarity, a single argument representation of the PDF can be found such that:

$$P(\delta x,\tau)=\tau^{-\alpha}P_s(\delta x\,\tau^{-\alpha}). \quad (2)$$

We can also express S_m using the fluctuations' PDF, $P(\delta x,\tau)$ as follows:

$$S_m(\tau)=\int_{-\infty}^{\infty}|\delta x|^m P(\delta x,\tau)\,d(\delta x), \quad (3)$$

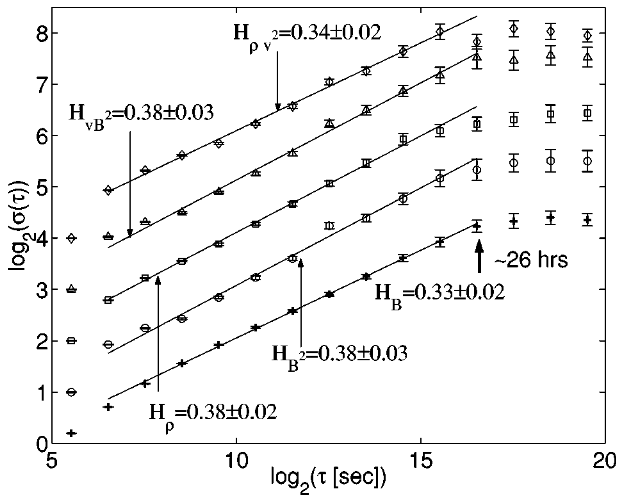

FIG. 2. Scaling of the standard deviation $\sigma(\tau)$ of the PDFs for all quantities under investigation: ○ corresponds to δB^2, □ ion density $\delta\rho$, ◇ kinetic energy density $\delta(\rho v^2)$, △ Poynting flux component $\delta(vB^2)$ and $+\,\delta B$. The plots have been offset vertically for clarity. The error bars on each bin within the PDF are estimated assuming Gaussian statistics for the data within each bin.

In terms of rescaled variables, P_s given by (2) and $\delta x_s = \delta x \tau^{-\alpha}$, the integral in (3) can be expressed as:

$$S_m(\tau) = \tau^{m\alpha} \int_{-\infty}^{\infty} |\delta x_s|^m P_s(\delta x_s) d(\delta x_s), \tag{4}$$

which allows us to identify scaling exponent $\zeta(m)$ in (4) to be $\zeta(m) = m\alpha$ for a statistically self-similar process.

Structure functions for high values of m are difficult to obtain due to the large statistical errors and a slow convergence of the method. We first examine the second moment structure function by investigating scaling of the standard deviation, $\sigma(\tau) = [S_2^c(\tau)]^{1/2}$, with τ. In this case, the scaling exponent $\zeta(2)/2$ provides an estimate of the Hurst exponent. Figure 2 shows the standard deviation $\sigma(\tau)$ plotted versus τ on log–log axes for $\delta x = \delta(\rho)$, $\delta(\rho v^2)$, $\delta(B^2)$, $\delta(vB^2)$ and δB. The region of scaling, within which all lines were fitted in Fig. 2, was established by the R^2 goodness of fit analysis. The optimal interval was found to extend between 1.5 minutes and 26 hours and the power laws suggest self-similarity persists up to this temporal scale. The slopes of these lines yield the rescaling exponents $\alpha = H$ and these are summarized in Table I. We then compute the Hurst exponent for the fluctuations of the kinetic energy density ρv^2 in the slow and fast solar wind. Figure 3 shows the scaling of their standard deviations and we see that the slow solar wind fluctuations

TABLE I. Scaling indices derived from $\sigma(\tau)$ versus τ power law fit.

Quantity	H from $\sigma(\tau)$	τ_{max}	PDF scales
δB	0.33±0.02	~26 hrs	No
δv	No scaling		No
$\delta(B^2)$	0.38±0.03	~26 hrs	Yes
$\delta(\rho)$	0.38±0.02	~20 hrs	Yes
$\delta(\rho v^2)$	0.34±0.02	~20 hrs	Yes
$\delta(vB^2)$	0.38±0.03	~26 hrs	Yes

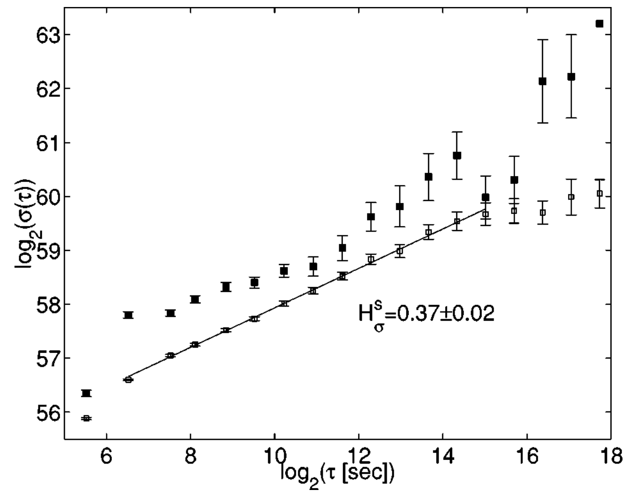

FIG. 3. Standard deviation scaling of the ρv^2 for slow (□) and fast (■) solar wind.

exhibit scaling up to ~20 hours while the fast stream data do not show a clear region of scaling. The Hurst exponent of the slow wind fluctuations deduced from this plot is $H = 0.37 \pm 0.02$.

We now extend the analysis to higher orders of structure functions. Figure 4 shows exponents $\zeta(m)$ derived from the conditioned GSFs of orders from 0 to 8 for quantity ρv^2 obtained from the slow (ρv_s^2) solar wind. We see that, when conditioned to $10\sigma(\tau)$, structure functions of the slow stream kinetic energy density are scaling and the exponent can be estimated to be $\alpha_{\rho v^2} = 0.31 \pm 0.02$. Although one could, in principle, fit a very weak multi-fractal model to those points, the straight line fit is within the errors and thus captures the essential behavior of the data. The fast solar wind kinetic energy density does not scale and we conclude that the statistical features of the slow and fast solar wind kinetic energy density are different, however, the deviation of fluctuations from self-similarity is not strong since the combined data set

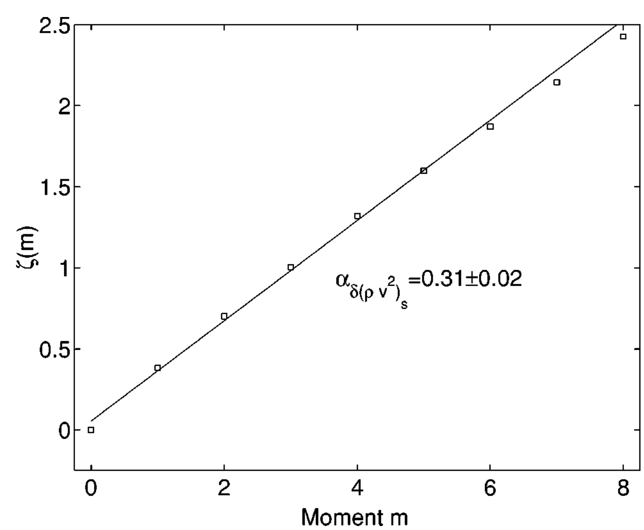

FIG. 4. Conditioned structure functions of the ρv^2 for the slow solar wind.

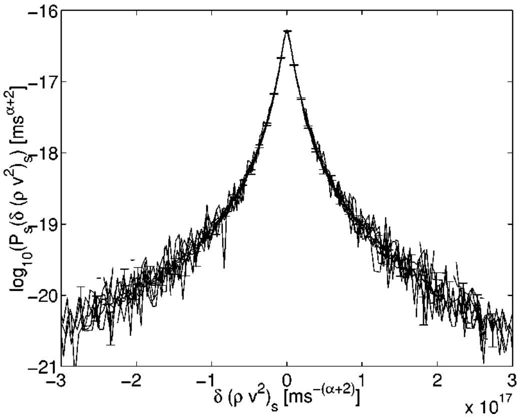

FIG. 5. One parameter rescaling of the fluctuation PDFs for the slow solar wind ρv^2. Scaling exponent is equal to the Hurst exponent. Curves correspond to temporal scales between ~ 2 minutes and ~ 26 hours.

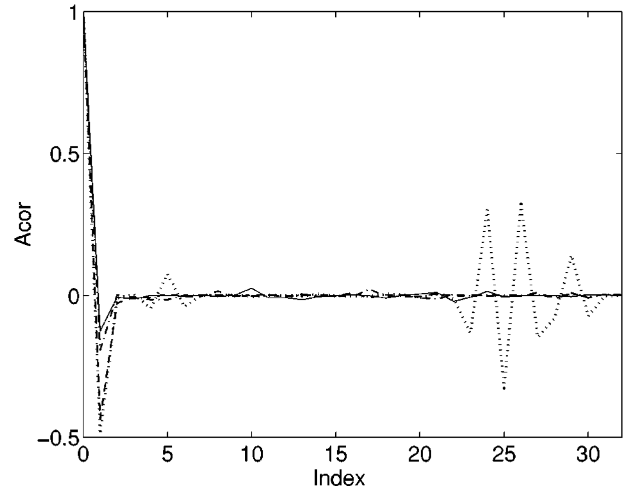

FIG. 6. Autocorrelation function for the time series of the ρv^2 differenced with $\tau = 4$ (solid line), $\tau = 64$ (dashed line), $\tau = 256$ (dash–dot line) and $\tau = 1024$ (dotted line).

still shows mono-scaling of the PDF.[20] The mono-scaling of the slow solar wind fluctuation in ρv^2 was then verified by applying the PDF rescaling, with the rescaling exponent $\alpha = 0.37$ (the Hurst exponent shown in Fig. 2), to the fluctuations $\delta(\rho v^2)_s$. Figure 5 shows the result of this one parameter rescaling. The χ^2 test was applied to these collapsed PDFs by comparing the PDF for $\tau = 2$ minutes with all other curves. All collapsed curves lie within 3%–5% error band for the slow solar wind ρv_s^2 fluctuations.

It is informative to discuss these results in the context of the work by Pagel and Balogh.[19] In both cases a clear difference between statistics of the slow and fast solar wind is apparent. Similarly to their work we also find that, at least in the slow wind, the third order structure function's scaling exponent, $\zeta(3) \approx 1$, which seems to excludes predictions of Kraichnan's turbulence model.[14] Intriguingly, Pagel and Balogh found multi-fractal scaling for δB (in the sense of a P-model) in both slow and fast solar wind, whereas we find no evidence of scaling for $\delta(\rho v^2)$ in the fast solar wind at all. This may suggest that one characteristic of the fast wind is the presence of coherent structures, to which the scaling in fluctuations of ρv^2 are sensitive, thus breaking the scaling, whereas the corresponding signature in magnetic field fluctuations is a change in scaling.

IV. THE FOKKER–PLANCK MODEL

The mono-scaling of the fluctuation PDF allows for the direct comparison of the functional form of the PDF as well as the development of a Fokker–Planck model for the PDF dynamics.[20] This approach is correct only in the case when the fluctuations can be treated as statistically independent.[24] The degree of correlation in a data set can be implied by obtaining a scaling exponent of the power spectrum. In terms of the integrated data set exponent $\beta = -2$ indicates a lack of any correlation. A random Brownian walk is an example of such system. In the case of the solar wind a weak correlation is present in the integrated time series. The power spectra for the solar wind velocity and magnetic field magnitude are known to exhibit scaling[7,9] close to Kolmogorov's $-5/3$. This is also true for the for all quantities discussed here. The PDF rescaling method and the asymmetry analysis (but not structure functions) were performed with nonoverlapping intervals of τ in order not to introduce correlation in the differenced time series. Figure 6 shows the autocorrelation function for the quantity $\delta(\rho v^2)$ and $\tau \approx 50$ minutes. We see that the assumption of the independent increments is a correct one for the differenced time series.

The Fokker–Planck diffusion model predicts the following functional form of the PDF for mono-scaling quantities:[20]

$$P_s(\delta x_s) = \frac{a_0}{b_0} \frac{C}{|\delta x_s|^{a_0/b_0}} \exp\left(-\frac{\alpha^2}{b_0}(\delta x_s)^{1/\alpha}\right) \times \int_0^{\delta x_s} \frac{\exp\left(\frac{\alpha^2}{b_0}(\delta x_s')^{1/\alpha}\right)}{(\delta x_s')^{1-a_0/b_0}} d(\delta x_s') + k_0 H(\delta x_s), \quad (5)$$

where k_0 is a constant and $H(\delta x_s)$ is the homogeneous solution:

$$H(\delta x_s) = \frac{1}{(\delta x_s)^{a_0/b_0}} \exp\left(-\frac{\alpha^2}{b_0}(\delta x_s)^{1/\alpha}\right). \quad (6)$$

Such a one parameter (α) description of the fluctuation PDF captures both the mono-scaling character of fluctuations and statistical intermittency revealed through the "heavy tails" of the distribution. Recently, a similar approach, based on the Lévy distribution,[29] has been proposed to model statistical properties of financial data.[28]

Figure 7 shows a fit of the Fokker–Planck model, with its solution given by (5), to fluctuation PDFs of slow and fast wind ρv^2. This fit is obtained with the following parameters $a_0/b_0 = 2.0$, $b_0 = 10$, $C = 0.00152$, $k_0 = 0.0625$, and $\alpha = 0.37$ as derived from the rescaling procedure. We emphasize that the divergence of the formula for the near-zero fluctuations is not surprising considering the form of the diffusion coefficient we have found, namely $D(\delta x) \propto (\delta x)^{2-1/\alpha}$

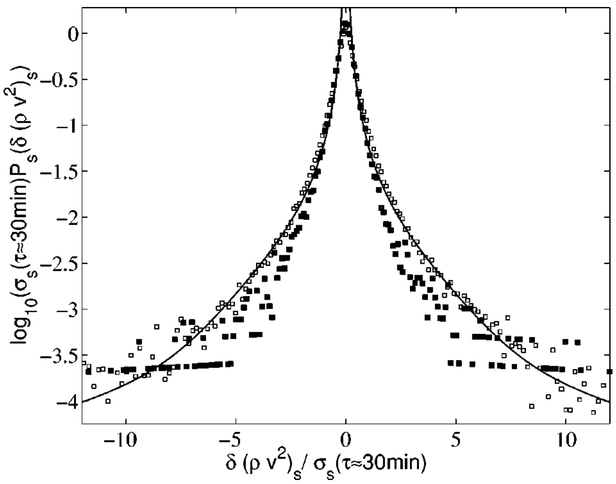

FIG. 7. Example of the fit of the PDF functional form predicted by a Fokker–Planck description (5) to the fluctuation PDF of the $\delta(\rho v^2)$ bulk parameter derived from slow (□) solar wind. Fast (■) solar wind is also shown.

with $\alpha<0.5$. This behavior can be tamed by assuming that the smallest fluctuations are Gaussian and they experience a constant diffusion. We note, however, that the Fokker–Planck equation can not be solved analytically for the diffusion coefficient given by $D_0+D(\delta x)$ where D_0 is a constant. We see that the functional form (5) fits slow wind fluctuations quite well while for the fast wind the $\delta(\rho v^2)_f$ PDF diverges appreciably from this predicted form for fluctuation sizes between ~ 2 and ~ 6 standard deviation. The Fokker–Planck approach appears to provide a good approximation of both PDFs in the limit of large fluctuations.

V. SUMMARY

In this paper we have discussed scaling properties of the bulk plasma parameters observed in the solar wind by the WIND spacecraft. We used the low order based PDF asymmetry analysis to gain additional information about the statistics of the processes. This analysis confirmed that the time scale of ~ 26 hours is a relevant scale for all quantities. We observe a maximum in the asymmetry levels around this time and then a significant drop in the asymmetry as the PDF of fluctuations converge toward a normal distribution. We find that all quantities that exhibit mono-scaling PDFs also show relatively low levels of asymmetry, on average about 7%. However, the velocity fluctuations PDF is clearly very asymmetric with asymmetry varying between 5% and about 35%, depending on the temporal scale. Interestingly, the asymmetry level for the PDF of δB are low despite the fact that the fluctuations are known to be multi-fractal on certain temporal scales. We have also obtained the Hurst exponents for all quantities. We find that the scale break occurs at ~ 26 hours.

We have examined differences in scaling properties of the fluctuations in kinetic energy density ρv^2 for slow and fast solar wind. The threshold for the solar wind velocity was set at 500 km/s. We found scaling in the standard deviation of the slow solar wind that extended to about 20 hours. Fast stream fluctuations do not exhibit such scaling. We have used the Hurst exponent, derived from the scaling of the standard deviations of the slow solar wind, to perform PDF rescaling analysis.[20] Consequently a PDF rescaling procedure applied to the fluctuation PDFs of the slow solar wind ρv_s^2 gives a good collapse of the curves within the temporal scales of ~ 20 hours, while the fast streams PDFs cannot be rescaled. The generalized structure functions, conditioned at 10σ, were also applied to examine the higher moments of the fluctuations in ρv^2 in slow and fast solar wind. Again, the slow wind fluctuations show self-similar scaling up to the 6th moment, while these of the fast wind show a lack of scaling.

We also considered a possible model for the self-similar slow solar wind fluctuations based on the Fokker–Planck equation. We have fitted functional forms for the PDF, predicted by this model, to an experimental PDF of the kinetic energy density fluctuation. It appears that the Fokker–Planck gives a good approximation of the slow solar wind fluctuations in ρv_s^2 while the fast wind fluctuations can not be treated with this approach. This approach does not depend on any particular model for turbulence but it is interesting and informative to compare our results with these based on turbulence models. We have used recent work of Pagel and Balogh to discuss differences emerging from these two approaches. These authors find evidence of a multi-fractal scaling in both fast and slow solar wind, but the character of the intermittency differs between the two. We find that the mono-scaling of kinetic energy density fluctuations only holds in the slow solar wind. This issue clearly requires further investigation, but does suggest both a fundamental difference, and a connection, between the physics of magnetic quantities and kinetic energy density. One possibility is that structures that are not part of the turbulent cascade can be seen in the fast wind in the kinetic energy density fluctuations, but not in fluctuations of the magnetic filed magnitude.

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