MULTI-SPACECRAFT MEASUREMENT OF TURBULENCE WITHIN A MAGNETIC RECONNECTION JET

K. T. Osman1, K. H. Kiyani1,2, W. H. Matthaeus3, B. Hnat1, S. C. Chapman1,4, and Yu. V. Khotyaintsev5

1 Centre for Fusion, Space and Astrophysics; University of Warwick, Coventry, CV4 7AL, UK; k.t.osman@warwick.ac.uk
2 Laboratoire de Physique des Plasmas, École Polytechnique, Route de Saclay, F-91128 Palaiseau, France
3 Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, DE 19716, USA
4 Department of Mathematics and Statistics, University of Tromsø, NO-9037 Tromsø, Norway
5 Swedish Institute of Space Physics, Uppsala, Sweden

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ABSTRACT

The relationship between magnetic reconnection and plasma turbulence is investigated using multipoint in situ measurements from the Cluster spacecraft within a high-speed reconnection jet in the terrestrial magnetotail. We show explicitly that work done by electromagnetic fields on the particles, \( \mathbf{J} \cdot \mathbf{E} \), has a non-Gaussian distribution and is concentrated in regions of high electric current density. Hence, magnetic energy is converted to kinetic energy in an intermittent manner. Furthermore, we find that the higher-order statistics of magnetic field fluctuations generated by reconnection are characterized by multifractal scaling on magnetofluid scales and non-Gaussian local scale invariance on kinetic scales. These observations suggest that \( \mathbf{J} \cdot \mathbf{E} \) within the reconnection jet has an analog in fluid-like turbulence theory that it proceeds via coherent structures generated by an intermittent cascade. This supports the hypothesis that turbulent dissipation is highly nonuniform, and thus these results could have far reaching implications for space and astrophysical plasmas.

Key words: magnetic reconnection – plasmas – turbulence

1. INTRODUCTION

Turbulence is a universal nonlinear phenomenon that is ubiquitous in space plasmas (Zimbardo et al. 2010). It produces a cascade of coherent structures in neutral fluids (Anselmet et al. 1984) and plasmas (Matthaeus & Montgomery 1980; Karimabadi et al. 2013). These are concentrated structures that are phase correlated over their spatial extent and have relatively long lifetimes, such as current or vorticity sheets. Indeed, current sheets have been observed extensively in turbulent plasmas, and are associated with magnetic reconnection in the solar wind (Osman et al. 2014), at the magnetopause (Mozer et al. 2002), and in the magnetosheath (Retinó et al. 2007; Sundkvist et al. 2007). In the terrestrial plasma sheet, in situ coherent structures display signatures of intermittent turbulence (Vörös et al. 2004; Weygand et al. 2005) in the form of rare large amplitude fluctuations that are highly non-Gaussian. These spatially inhomogeneous turbulent flows have been proposed as central to plasma sheet dynamics (Borovsky et al. 1997; Chang 1999; Borovsky & Funsten 2003), and are thus critical to understanding how stored electromagnetic energy in the magnetotail is converted into plasma energy. Here we consider whether coherent structures generated by magnetic reconnection reflect the nonlinear dynamics of intermittent turbulence and might be sites of nonuniform dissipation. These longstanding questions (Matthaeus & Lamkin 1986) are the subject of this Letter.

The nature of turbulent dissipation within collisionless plasmas remains an open problem. A strong turbulent cascade is far from equilibrium and smaller scale behavior is driven by larger scale dynamics, with faster response times for decreasing scales (Matthaeus et al. 2014). In addition, intermittent turbulence generates small-scale coherent structures that are responsible for nonuniform dissipation (Frisch 1995; Biskamp 2003). For neutral fluids, the Kolmogorov refined similarity hypothesis (hereafter KRSH; Kolmogorov 1962; Obukhov 1962) relates the statistics of increments of the velocity field on a given spatial scale to local averages of the dissipation rate on the same scale. Hence, large intermittent fluctuations on small scales are concomitant with high local concentrations of dissipation.

2. ANALYSIS

We use 450 Hz burst mode magnetic field \( \mathbf{B} \) and electric field \( \mathbf{E} \) measurements from the FGM (Balogh et al. 2001), STAFF (Cornilleau-Wehrlin et al. 2003), and EFW (Gustafsson et al. 2001) instruments on board the Cluster spacecraft. While components of the dc electric field in the spacecraft spin plane are measured directly, the third component is reconstructed assuming \( \mathbf{E} \cdot \mathbf{B} = 0 \). We also use 4 s resolution proton moments from the CIS CODIF experiment (Rème et al. 2001).

The curlometer technique (Dunlop et al. 1988, 2002) is used to estimate the current density \( \mathbf{J} \) through a tetrahedron formed by four spacecraft, where the Maxwell–Ampere law is written as:

\[
\mu_0 \mathbf{J}_{jk} \cdot (\Delta r_{ik} \times \Delta r_{jk}) = \Delta B_{ik} \cdot \Delta r_{jk} - \Delta B_{jk} \cdot \Delta r_{ik},
\]

where \( i, j, \) and \( k \) are the spacecraft indices, \( \mathbf{J}_{jk} \) is the average current density normal to the surface made by spacecraft \( i \) and \( j \), and \( \Delta r_{ik} = r_i - r_k \) is the distance between spacecraft \( i \) and \( k \),
and $\Delta B_{ik} = B_i - B_k$ is the magnetic field difference between spacecraft $i$ and $k$. The total average current density is then determined by projecting the current normal to three faces of the tetrahedron into suitable Cartesian coordinates. However, this technique is not without its limitations (Vallat et al. 2005). The main assumptions are that the spatial variation of the magnetic field is a linear function of the spacecraft separation, such that $\mathbf{J}$ is constant over the tetrahedron, and that the medium is stationary. Since non-stationarity leads to the generation of nonlinear gradients, the only source of error to consider is the nonlinear variation of the magnetic field. This is determined by computing $\nabla \cdot \mathbf{B}$ from:

$$
\nabla \cdot \mathbf{B} |\Delta r_{ik} \cdot \Delta r_{jk} \times \Delta r_{jl}| = |\sum_{\text{cyclic}} \Delta B_{ik} \cdot \Delta r_{jk} \times r_{jl}|.
$$

(2)

While $\mathbf{B}$ is solenoidal, the expression above can produce non-zero values that result from nonlinear gradients that are neglected in the estimate. Hence, $\nabla \cdot \mathbf{B} / |\nabla \times \mathbf{B}|$ provides an indicator of the error on curlometer estimates of $\mathbf{J}$. We require this error to be less than 10%, and remove all data that does not satisfy this condition. Note that certain spacecraft configurations and separations can reduce the accuracy of the curlometer technique, but these effects are likely insignificant in this study since the Cluster quartet is in a regular tetrahedral configuration.

3. RESULTS

Figure 1 shows selected plasma and magnetic field data from Cluster 4 for a 12 minute interval encompassing an earthward magnetic reconnection jet observed in situ on 2003 August 17 (e.g., Henderson et al. 2006; Asano et al. 2008; Huang et al. 2012; Wang et al. 2014). The Cluster quartet is in the magnetotail lobe prior to 16:55 UT, and then enters the plasma sheet while located around $[−16.8, 5.6, 3.2]$ Earth radii in geocentric solar magnetospheric (GSM) coordinates with spacecraft separations near 220 km. A high-speed earthward flow ($V_x$ exceeds 1200 km s$^{-1}$) is detected in the plasma sheet, where the number density $n \approx 0.25$ cm$^{-3}$ and the plasma beta $\beta = n k_B T / (B^2/2 \mu_0) \approx 1$ have typical values. However, the proton temperature is significantly elevated within the earthward flow, which is suggestive of proton heating by magnetic reconnection. The spacecraft exits the reconnection jet and enters the central plasma sheet at 17:03 UT.

A tailward flow was detected by Cluster 4 ahead of the earthward flow from 16:33 to 16:52 UT, and the associated $B_y$ was mostly negative (positive) for the tailward (earthward) flow. These correlated changes in $V_x$ and $B_y$ are consistent with a tailward retreating X-line being swept past the spacecraft, and suggest that the earthward flow is close to this X-line. The negative sign of $B_y \approx -15$ nT (roughly 60% of the asymptotic magnetic field) agrees with the expected Hall magnetic field polarity in the southern hemisphere, eastward of the X-line. Indeed, the reversal in $J_y$ observed around 16:55:12 UT could be associated with a reversal in the nearly field-aligned currents that close the Hall currents across the reconnection separatrices. Hence, the spacecraft may have observed an ion diffusion region with a moderate guide field.

There are large fluctuations in the magnetic field and current density associated with the high-speed reconnection jet. Here we investigate the statistical properties of these fluctuations to identify signatures of intermittent turbulence. In order to determine the higher-order scaling of magnetic field fluctuations, the absolute moments of the increments $\delta B(t, \tau) = B(t + \tau) - B(t)$ are computed for each vector component $B \rightarrow B_x, B_y$ or $B_z$. The $n$th order structure function
is given by:

\[ S^m(\tau) = \frac{1}{N} \sum_{i=1}^{N} |S(t_i, \tau)|^m. \]  

(3)

where \( \tau \) is the time lag and \( N \) is the signal sample size. The higher-order structure functions progressively capture the more intermittent fluctuations. These represent the spatial gradients responsible for dissipating energy in fluid-like turbulence. We examine the power-law scaling behavior of structure functions such that

\[ S^m(\tau) \propto \tau^{-\zeta(m)} \]  

(4)

and \( \zeta(m) \) are the scaling exponents. Figure 2 shows the Cluster 4 GSM x-component magnetic field structure functions and scaling exponents for the earthward flow data interval indicated in Figure (1). Note that the three magnetic field components on all four Cluster spacecraft exhibit essentially identical statistical scaling. The inertial and dissipation ranges are well-defined, with a break around 3 s, in agreement with the temporal signature of the proton gyroradius (~260 km) where the mean solar wind flow is 656 ± 28 km s\(^{-1}\). This suggests that Taylor’s hypothesis (Taylor 1938) is valid within the magnetic reconnection jet, and thus each spacecraft time series can be considered a spatial snapshot of the plasma. However, this cannot be confirmed without knowing the characteristic timescale on which the observed fluctuations vary. Note that hereafter we assume time lags and frequency spectra are equivalent to spatial lags and wavenumber spectra. In effect we rely on some form of random sweeping of small scales (Borovsky et al. 1997).

The scaling exponent \( \zeta(2) \) is directly related to the power spectrum spectral index \( \alpha \) when Equation (4) is satisfied:

\[ \zeta(2) = \alpha - 1 \]  

(Monin & Yaglom 1975). This relationship only applies to \( \alpha < 3 \) when using two-point structure functions (Kiyani et al. 2013). Here the inertial range follows a power law with \( \alpha = 1.65 \pm 0.03 \), which then steepens to \( \alpha = 2.52 \pm 0.02 \) in the dissipation range. These are similar to other reported observations in magnetic reconnection diffusion regions (Eastwood et al. 2009; Huang et al. 2010) and in the turbulent solar wind (Smith et al. 2006).

Figure 2 shows results from the higher-order scaling analysis of magnetic field fluctuations in the reconnection outflow. The errors on \( \zeta(m) \) are estimated as the sum of the regression error from Figure 2(a) and the variation in \( \zeta(m) \) found by repeating the regression over a subinterval of the scaling range (Kiyani et al. 2006). For inertial range fluctuations, \( \zeta(m) \) is nonlinear in \( m \), which is typical of hydrodynamic (Frisch 1995) and magnetofluid (Biskamp 2003) turbulence. This behavior is associated with a statistical distribution of energy dissipation that is highly nonuniform, and distributed on a spatial multifractal. In contrast, dissipation range fluctuations are characterized by a linear \( \zeta(m) \). This indicates global scale invariance and is associated with a distribution of energy dissipation that is also nonuniform, but in this case is distributed on a monofractal. Hence, the probability density functions (PDFs) of dissipation range magnetic field increments should collapse onto a unique scaling function. Figure 2(d) shows PDFs corresponding to \( \tau = [0.01, 0.22, 0.44, 0.67, 0.89, 1.11, 1.44] \) s that are rescaled by their standard deviations and overlaid, where the smallest \( \tau \) shows the associated errors. It is apparent that there is a very good collapse onto a single curve even up to several

![Figure 2](image_url)
Figure 3. PDFs of (a) spacecraft and proton frame work done, (b) current density, and (c) electric field fluctuations. Gaussian fits (dashed curves) are applied. The (d) mean proton frame work done conditioned on local current density thresholds, where $J/J_{\text{rms}}$ is larger than and equal to some value $n$. Also, the fraction of data in these averages is plotted.

standard deviations. The largest events are not well-sampled, as indicated by greater statistical spread at larger values of the increments. A fitted Gaussian distribution illustrates the non-Gaussian PDF tails, which reflects the presence of rare large amplitude fluctuations.

These results contradict earlier studies that found multifractal scaling at kinetic scales using PIC simulations (Leonardis et al. 2013) and from direct observation of the scale-dependent kurtosis (Huang et al. 2012). This inconsistency could be because the simulations were set within the complex topology of a reconnection region, whereas our observations sample the high-speed outflow jet, and kurtosis is sensitive to very large fluctuations that are not statistically well-sampled in heavy-tailed distributions. However, the higher-order scaling shown in Figure 2 is almost identical to that observed in the solar wind on both MHD and kinetic scales (Kiyani et al. 2009, 2013). This suggests that magnetic field fluctuations generated by reconnection exhibit a detailed correspondence with intermittent turbulence, including a cross-over from multifractal scaling to global scale invariance and distinct non-Gaussian statistics in the inertial and dissipation ranges.

Turbulence cascades energy from structures on larger to smaller spatial scales. The corresponding spatial field is nonuniform, with strong fluctuations that have non-Gaussian statistics. These fluctuations are small-scale coherent structures that support spatial gradients that can contribute to dissipation (Leonardis et al. 2013). In analogy with hydrodynamic turbulence this suggests the possibility that reconnection also related to activity on small scales, converts magnetic energy into kinetic and thermal energies. The connection between these dynamical processes is examined within the earthward flow interval by computing $D_t = J \cdot E$, the work done by electromagnetic fields on the particles in the spacecraft frame. Although this is not strictly a measure of irreversible dissipation it must necessarily include the work done to convert stored magnetic energy into heat. Indeed, the identification of $D_t$ as dissipation is complicated by contributions from particle acceleration, fluid motion, field line stretching, and compressions. In order to avoid some of the ambiguity associated with $D_t$ (e.g., Zenitani et al. 2011), the work done is evaluated in a frame moving with the bulk proton velocity measured by Cluster 4, $D_p = J \cdot (E + V \times B)$.

Figure 3(a) shows the broad and slightly asymmetric PDFs of the work done in the laboratory and proton frames rescaled by the standard deviation. These distributions are almost identical, which implies that the contribution to the work done from the convective electric field is minimal. There is an excess of positive values in the distribution cores and the average work done is $\langle D_t \rangle = 72 \pm 8 \text{ pWm}^{-3}$ and $\langle D_p \rangle = 62 \pm 8 \text{ pWm}^{-3}$. These may be interpreted as (imperfect) estimates of net magnetic energy dissipation into plasma internal energy, but will also contain other effects. Note, values beyond the dashed vertical lines ($\pm 2.5\sigma$) are not statistically well-sampled and are likely dominated by unphysical fluctuations on scales smaller than the spacecraft separations. Nonetheless, a fitted Gaussian distribution shows that these heavy-tailed PDFs are manifestly non-Gaussian. This is a direct indication of intermittency of dissipation, which in the context of the KRSH is associated with the presence of intermittent fluctuations of the fluid variables such as $B$. Figures 3 (b)–(c) show that the current density and electric field fluctuations, which together constitute $D_p$, are also independently non-Gaussian and intermittent.

If the work done in the proton frame is highly structured as implied by its leptokurtic PDF, then this inhomogeneity should be evident in suitable conditional statistics. Figure 3(d) shows averages of $D_p$ conditioned on thresholds of the local current density $\langle D_p | J \rangle / \langle D_p \rangle$. The fraction of data points used in each of
these conditioned averages is also plotted \( F(n) = \sum f / \sum j \), where \( \sum j \) only includes points that satisfy the threshold condition \( J / Je \, \mathrm{ms} \geq n \). This represents a reasonable and easily accessible measure of the volume filling factor of the regions satisfying the corresponding condition. These diagnostics explicitly illustrate the nonuniform and patchy character of the work done since the normalized conditional averages of \( D_p \) strongly increase with smaller volume fraction. This results in a mean \( D_p \) for the threshold \( J \geq 2.4J_\mathrm{ms} \) that is about 3–6 times the global average, despite such high local current density regions occupying less than 2% of the data. Hence, regions of higher electric current density are increasingly rare, but make disproportionately large contributions to the total work done.

4. DISCUSSION

We have used Cluster multispacecraft data to examine the structure within a previously identified magnetic reconnection jet in the terrestrial magnetotail. In particular, the intermittent nature of the magnetic field component increments \( \delta B_x, \delta B_y, \delta B_z \), and the work done, \( J \cdot E \), on particles by the electromagnetic fields, is characterized. The significance of this result is seen by recalling the structure of the KRSH for hydrodynamic turbulence: \( e_\nu \sim \epsilon_1^{1/3} \nu^{1/3} \). It is postulated that the longitudinal velocity increments \( \epsilon_1 \) on scale \( \ell \) are related statistically to the dissipation rate \( e_\nu \) averaged over a volume of size \( \ell^3 \). For a low-density plasma such as the magnetospheric plasma sheet, a formal statement of refined similarity has not been elucidated. However, our analysis is tantamount to a statistical examination of both principle elements of a putative analogous relation for plasmas (e.g., Merrifield et al. 2005; Chandran et al. 2015), assuming that the time increments employed here are comparable to spatial increments, which is reasonable for a form of random sweeping.

The magnetic field fluctuations (increments) within a reconnection jet exhibit a multifractal non-self-similar scaling of higher-order moments in the inertial range, which transitions to a self-similar, but still non-Gaussian, monofractal scaling in the kinetic range. In addition, we find that \( J \cdot E \) within the same jet is highly non-Gaussian, with heavy tails in the probability distribution. Regions of strong \( J \cdot E \) are non-space-filling, with indications that the large transfer of random energy to particles is almost certainly highly concentrated in small volumes that contain atypically large electric current density. Thus, even if the magnetotail plasma is not ohmic in nature, its dissipation is statistically associated with regions of high current density. This finding is consistent with results from the explicit examination of electromagnetic work in two- and three-dimensional collisionless plasma simulations (Wan et al. 2012, 2015; Karimabadi et al. 2013), and from less direct inference in observations of the solar wind (Osman et al. 2014), magnetopause (Mozer et al. 2002), and magnetosheath (Sunkvist et al. 2007; Chasapis et al. 2015). The conclusion seems increasingly certain that intermittent dissipation is as typical in large low-density plasma systems as it is in high-Reynolds number hydrodynamic turbulence.

It should be noted that the flow speed ranges from 204 to 1360 km s\(^{-1}\) within the earthward magnetic reconnection jet. Hence, this variability will reduce the accuracy of estimates of \( D_p \) and the proton gyroradius. However, we adjust for this variability in flow speed by normalizing \( D_p \) by its standard deviation or expressing it as scaled with another representative parameter. While this is not ideal, (Wan et al. 2012) showed that distributions of \( J \cdot E \) in different fluid frames are all similar but the tails become less heavy as the electron frame is approached. Thus, an error in frame transformation will likely result in a larger value of \( J \cdot E \), but with a similar distribution. Hence, variability in flow speed is unlikely to have a significant impact on the validity of our conclusions. This could be related to our result that kinetic turbulence is monofractal, and thus the PDFs are related by a scale transformation.

While our analysis focuses on turbulence within a magnetic reconnection outflow jet, the reconnection process itself occurs within a small diffusion region. The NASA Magnetospheric Multi-Scale mission will make high-resolution plasma and magnetic field measurements, and thus should allow the statistical nature of \( J \cdot E \) to be probed within the proton and electron diffusion regions. This will improve our understanding of magnetic reconnection and turbulent dissipation. Indeed, as diagnostics for examining intermittent dissipation become more well understood, we may anticipate that a more refined expectation may be emerging regarding the heating and intermittency that will likely be observed by the upcoming ESA Solar Orbiter, NASA Solar Probe Plus, and proposed ESA THOR missions as they seek to understand how the solar corona is heated and the solar wind is accelerated, leading to the emerging structure of the entire plasma heliosphere.

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