Compressibility in Solar Wind Plasma Turbulence

Bogdan Hnat,* Sandra C. Chapman, and George Rowlands
Physics Department, University of Warwick, Coventry, CV4 7AL, United Kingdom
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Incompressible magnetohydrodynamics is often assumed to describe solar wind turbulence. We use extended self-similarity to reveal scaling in the structure functions of density fluctuations in the solar wind. The obtained scaling is then compared with that found in the inertial range of quantities identified as passive scalars in other turbulent systems. We find that these are not coincident. This implies that either solar wind turbulence is compressible or that straightforward comparison of structure functions does not adequately capture its inertial range properties.

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The supersonic and super-Alfvénic flow of the solar wind offers a unique opportunity to investigate the properties of the magnetized and turbulent plasma. The transition from a laminar to turbulent flow requires a large Reynolds number $\mathrm{Re} = LU/\nu$, and its magnetic counterpart $R_m = LU/\eta$, where L is the energy injection scale length, U is the velocity difference on a scale L, ν is the viscosity, and η is the magnetic diffusivity. Estimates of Re and R_m in the solar wind exceed 10^8 [1,2] compared with 10^4 achieved in direct numerical simulations (DNS) [3] and just a few hundreds in some magnetized liquid laboratory experiments [4]. The presence of turbulence in the solar wind is strongly suggested by numerous observations. These include power law power spectra with $-5/3$ Kolmogorov-like slopes in the kinetic and magnetic energy densities (e.g., [5–7]) and non-Gaussian probability density functions (PDFs) (e.g., [8–11]) found for fluctuations in the velocity and the magnetic field.

These observations imply that solar wind turbulence shares many statistical properties with incompressible isotropic hydrodynamic turbulence [12–14]. As a result, the turbulent dynamics of the solar wind is often modeled assuming incompressibility. This assumption is particularly convenient in analytical and numerical studies of magnetohydrodynamic (MHD) turbulence and also appears to be in good agreement with the results of compressive MHD simulations where a generation of compressive modes from Alfvénic turbulence was found to be suppressed [2]. In the context of the solar wind, incompressibility has been suggested to be a reasonable approximation for plasma in fast wind streams [10,15,16]. Considerable progress has been made by treating the solar wind as dominated by Alfvénic fluctuations (e.g., [17]) and with nearly incompressible magnetohydrodynamic theory [18].

However, statistical features associated with turbulence have also been identified in the density fluctuations derived from solar wind observations. A $k^{-5/3}$ scaling in the omnidirectional wave number spectrum of electron and proton density has been reported as early as the 1970s (see [19] and references therein). Models of the solar wind can offer possible mechanisms for generating strong density fluctuations close to the heliospheric current sheets [20]. Recent studies suggest that, at least in the slow solar wind, the coupling of Alfvénic and magnetosonic MHD modes do not fully describe the nature of these fluctuations. For example, the ratio $\delta\rho/\langle\rho\rangle$ was found to be nearly constant and independent of the amplitude of the magnetic field fluctuations $\delta B/\langle B\rangle$ in the slow solar wind [21]. More recently the density fluctuations were also found to exhibit nontrivial, approximately self-similar scaling [11].

In this Letter we examine the scaling properties of the proton density ρ in fast and in slow solar wind. We focus on a comparison of the scaling properties of fluctuations in density with that of various passive scalars identified in experiments and DNS of fluid turbulence. Previously, these passive scalars have been shown to exhibit scaling that corresponds closely to that of the magnitude of the magnetic field B in the solar wind, although the data interval was not ordered by solar wind speed [22]. By assuming incompressibility it can be argued [22] that B should act as a passive scalar. We will for completeness also compare the scaling properties of magnetic field magnitude in fast and slow solar wind.

It is instructive to recall that the dynamics of a passive scalar $T \equiv T(\mathbf{x}, t)$ in a velocity field $\mathbf{v}(\mathbf{x}, \mathbf{t})$ is given by the advection equation

$$\partial_t T = -(\mathbf{v}\cdot\nabla)T + \kappa\nabla^2 T, \tag{1}$$

where κ is the diffusivity. Dynamical and statistical properties of a passive scalar are more tractable analytically, as compared to active fields like velocity, since the Eq. (1) is linear in T. In the case of hydrodynamics, theoretical predictions have also been verified, to some extent, experimentally using tracer particles that do not disturb the flow [23]. Let us consider the compressible MHD equations for the evolution of the magnetic field and the density:

$$\partial_t \mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \eta\nabla^2\mathbf{B} \quad \text{and} \quad \partial_t\rho + \nabla\cdot(\mathbf{v}\rho) = 0. \tag{2}$$

Given the assumption of incompressibility ($\nabla\cdot\mathbf{v} = 0$), it immediately follows that ρ should behave as a passive scalar in the turbulent solar wind flow:

$$\partial_t\rho = -(\mathbf{v}\cdot\nabla)\rho. \tag{3}$$

To investigate the scaling properties of ρ and B we use 64 s averaged data from the Advanced Composition Explorer (ACE) spacecraft [24] spanning from 01/01/1998 to 12/31/2001. This interval includes dates previously considered in Ref. [22]. The slow and fast solar wind density are known to exhibit distinct scaling (e.g., [6]). We thus split the data into slow and fast solar wind sets using 450 km/s wind speed as a separation criteria. The resulting data sets consist of $\sim 1 \times 10^6$ samples for the slow wind and $\sim 0.6 \times 10^6$ for the fast wind. We apply structure function [25] analysis to these data sets. Generalized structure functions S_m of fluctuations in, say, the density $\rho(t)$, on time scale τ are defined as moments m through $S_m(\tau) = \langle |\rho(t+\tau) - \rho(t)|^m \rangle$ where the ensemble $\langle \ldots \rangle$ is taken in the time domain [26]. If scaling is present in the time series we expect these to show a power law dependence on the temporal scale τ, i.e., $S_m \propto \tau^{\zeta(m)}$.

Finite, experimental data sets include a small number of extreme events (outliers) that, due to poor statistics, may obscure the correct scaling of the high order moments. Here, we will exclude these events by the use of *conditioning* [14]. This approach puts a limit on the range of fluctuations used in computing structure functions. This limit is varied with the temporal scale τ to account for the growth of range with temporal scale in the signal. In our case we defined this threshold as $15\sigma(\tau)$, where $\sigma(\tau)$ is a standard deviation of fluctuations on temporal scale τ. We stress that conditioning improves the scaling where it already exists but does not enforce it on the investigated data if the applied threshold is sufficiently large. In practice, for the limit chosen here, we eliminate less then 1% of the data points. These typically correspond to coherent structures in the solar wind [27].

Figure 1 shows the structure functions S_m plotted versus τ on logarithmic axes for orders $1 \leq m \leq 4$ for fluctuations in density in the slow solar wind. The plot shows a scaling region extending from $\tau \sim 10$ min to $\tau \sim 3$ h ($\sim$ 1.5 decades on the logarithmic scale). The quality of the scaling deteriorates when fast wind streams are considered. The scaling regions can be still identified but they extend only <1 decade from ~ 10 min to 1 h.

It has been empirically shown that extended self-similarity (ESS) can considerably extend the region of scaling in structure functions [28]. The method seeks a scaling $S_m(\tau) \propto S_p^{\eta(m)}(\tau)$ which should emerge on a plot of S_m versus S_p. We plot S_m versus S_3 on logarithmic axes for fluctuations in ρ, in the slow and fast wind, respectively, in Figs. 2 and 3. These figures demonstrate that ESS extends scaling in the density in both cases to over two decades. We use this extended scaling range to obtain a revised estimate of the exponents $\tilde{\zeta}(m) = \zeta(p)\eta(m)$. Plots of S_3 versus τ for ρ give $\zeta(3) = [1.16 \pm 0.08, 1.01 \pm 0.07]$ in the slow and fast solar wind, respectively. Figures 2 and 3 then give the $\eta(m)$ and the resulting values of $\tilde{\zeta}(m)$ which differ from the $\zeta(m)$ by $\sim 3\%$.

ESS was previously applied to the magnitude of magnetic field in an undifferentiated interval of solar wind [22] and for completeness we give an analysis in slow and fast wind here. Figures 4 and 5 show S_m versus S_3 on logarithmic axes for fluctuations in B and we recover the extended range of scaling [22]. We obtain for B, $\zeta(3) = [0.85 \pm 0.06, 0.79 \pm 0.05]$ in the slow and fast wind, respectively, and use this, with the $\eta(m)$ from Figs. 4 and 5, to determine $\tilde{\zeta}(m)$ exponents for comparison with that of ρ. Here, the values of $\tilde{\zeta}(m)$ differ from $\zeta(m)$ by $\sim 4\%$.

We now directly compare the scaling found for the density ρ with that identified for quantities acting as passive scalars and that of the magnetic field magnitude B. The resulting functional form of the scaling exponents $\tilde{\zeta}(m)$ for ρ and B in slow and fast solar wind are shown in Figs. 6 and 7, respectively. For comparison, the scaling exponents obtained for passive scalars from the DNS [29] and the wind tunnel experiment [30] are also shown. We immediately see that whereas the exponents for magnetic field magnitude and the passive scalars fall close to each

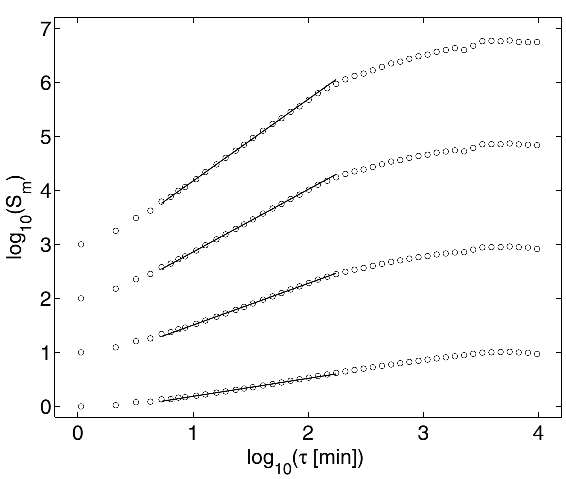

FIG. 1. Conditioned structure functions of fluctuations in the density ρ for slow solar wind (orders $m = 1 - 4$ are shown).

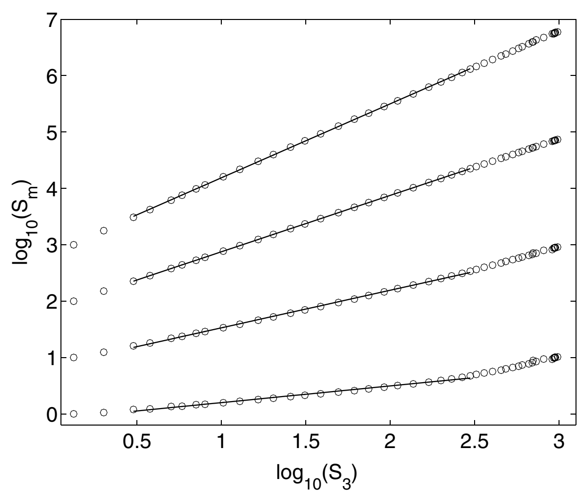

FIG. 2. ESS of conditioned structure functions of fluctuations in the density ρ for slow solar wind.

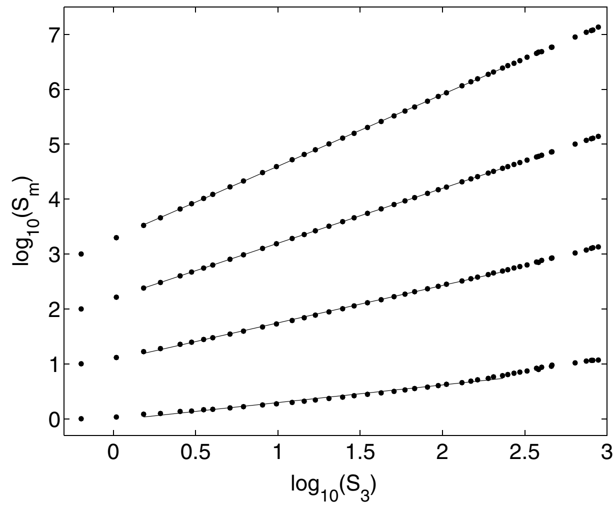

FIG. 3. ESS of conditioned structure functions of fluctuations in the density ρ for fast solar wind.

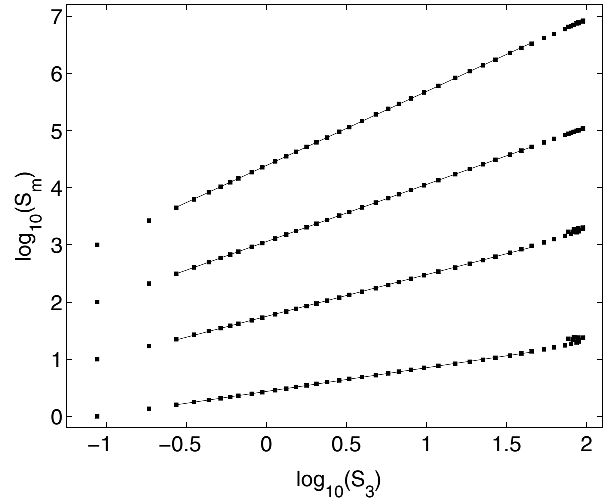

FIG. 5. ESS of conditioned structure functions of fluctuations in the magnetic field magnitude B for fast solar wind.

other on these plots (as also reported in [22]), they are distinct from those obtained for the solar wind density in both slow and fast solar wind. The fluctuations in B and that of the passive scalars exhibit multifractal scaling. Intriguingly, the density fluctuations are nearly self-similar [$\tilde{\zeta}(m) = \alpha m$] with exponents $\alpha^{\rho}_{\rm slow} = 0.39 \pm 0.03$ and $\alpha^{\rho}_{\rm fast} = 0.33 \pm 0.03$.

This suggests one of two possible conclusions. The first is that the turbulent solar wind is compressible and Eq. (3) does not hold, so that the density is an active quantity. This is rather surprising and calls into question the significant body of work on MHD turbulence in the solar wind that relies on the assumption of incompressibility (for example, [17]). This also invalidates the arguments in Ref. [22] required to cast the advection equation for B in the form of a passive scalar which requires incompressibility, yielding:

$$\partial_t B = -(\mathbf{v} \cdot \nabla) B + \eta \nabla^2 B + \lambda B. \tag{4}$$

The second possibility is that commonality of scaling (as measured by structure functions) does not imply shared phenomenology, and as a corollary, an absence of such a commonality does not imply distinct phenomenology. The fact that several quantities share the same structure functions is then coincidental rather than expressing some universality of fluid turbulence. This implies that the structure functions of a single quantity do not fully capture the phenomenology of a given turbulent system. This has implications for analysis of turbulence which has been achieved through comparison between (multifractal) models of turbulence and the data, via the structure functions (see, for example, [26]).

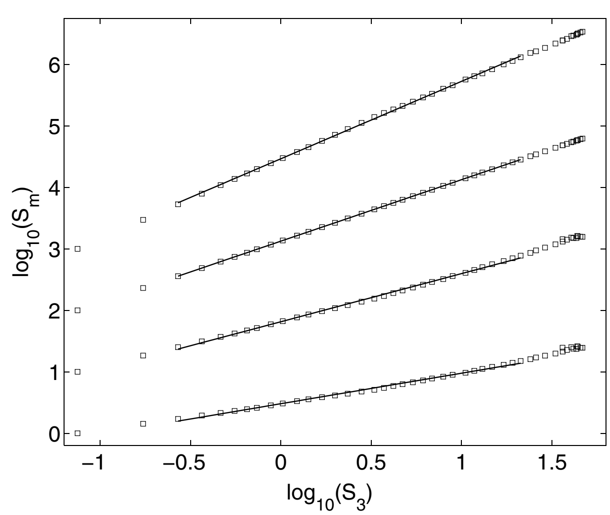

FIG. 4. ESS of conditioned structure functions of fluctuations in the magnetic field magnitude B for slow solar wind.

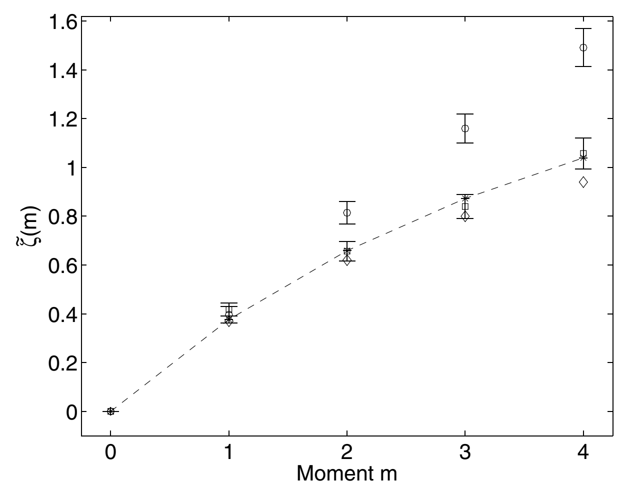

FIG. 6. Scaling exponents of the conditioned structure functions for δB (squares) and $\delta\rho$ (circles) in slow solar wind. Dashed line and diamonds correspond to DNS [29] and experimental [30] results, respectively.

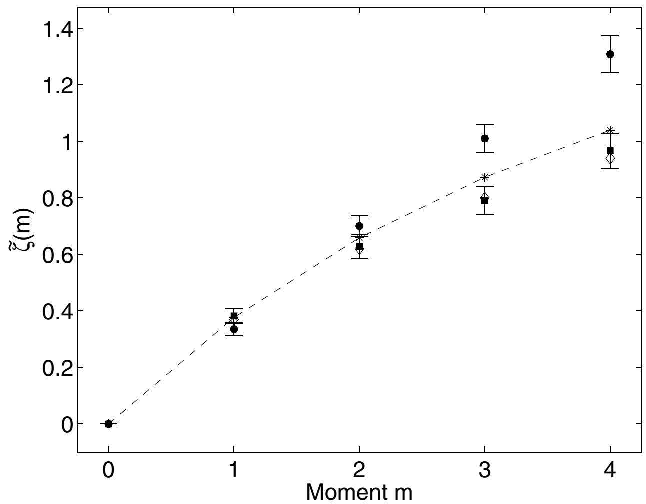

FIG. 7. Scaling exponents of the conditioned structure functions for δB (squares) and $\delta\rho$ (circles) in fast solar wind. Other symbols are the same as in Fig. 6.

A possible resolution may be found in the form of Eqs. (3) and (4). It is known that the molecular diffusivity can change the inertial range scaling properties of a passive scalar [31]. The observed differences could then be due to the presence of the diffusive term in the magnetic field Eq. (4) and its absence in the density Eq. (3). This may account for the different scaling found in these quantities.

Intriguingly, Eq. (3), when written for the moments of fluctuations in ρ, has no explicit dependence on the order of the moment [31]

$$[\partial_t + \mathbf{v}(\mathbf{x}, \mathbf{t}) \cdot \nabla + \mathbf{v}(\mathbf{x}', \mathbf{t}) \cdot \nabla'](\delta\rho)^n = 0, \qquad (5)$$

where $(\delta\rho)^n \equiv [\rho(\mathbf{x}, t) - \rho(\mathbf{x}', t)]^n$. In this case fluctuations in density should be simply those imposed on the initial condition $\delta\rho(\mathbf{x}, 0)$. These may be mediated via large scale coherent structures (shocks and coronal mass ejections). This may suggest a solar origin of these fluctuations in the density [32]. It may thus be informative to attempt to relate the scaling found in the density fluctuations in the solar wind with that of the solar corona. The power law scaling of x-ray flux from solar flares [33] is intriguing in this regard.

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*Electronic address: hnat@astro.warwick.ac.uk

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