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Avalanching systems under intermediate driving rate

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Abstract
The paradigm of self-organized criticality (SOC) has found application in understanding scaling and bursty transport in driven, dissipative plasmas. SOC is, however, a limiting process that occurs as the ratio of driving rate to dissipation rate is taken to zero. We consider the more realistic scenario of finite driving rate. Similarity analysis reveals that there is a control parameter R_A which is analogous to the Reynolds number R_E of turbulence in that it relates to the number of excited degrees of freedom, that is, the range of spatio-temporal scales over which one finds scaling behaviour. However for avalanching systems the number of excited degrees of freedom is maximal at the zero driving rate, SOC limit, in the opposite sense to fluid turbulence. Practically, at finite R_E or R_A one observes scaling over a finite range which for turbulence, increases with R_E and for SOC, decreases with increasing R_A, suggesting an observable trend to distinguish them. We use the BTW sandpile model to explore this idea and find that whilst avalanche distributions can, depending on the details of the driving, reflect this behaviour, power spectra do not and thus are not clear discriminators of an SOC state.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The paradigm of self-organized criticality (SOC) [1] has found wide application to understanding 'bursty', scale free transport and energy release in confined, driven and dissipating plasmas (for a recent review, see e.g. [2]). In the astrophysical context models have been developed, and tested against data, for the dynamic magnetosphere ([3–6], see also the review [7]); for the dynamics of the solar corona (e.g. [8–11]); and for accretion discs (e.g. [12–14]). For magnetically confined laboratory plasmas there have been extensive efforts to construct relevant avalanche models and to establish signatures characteristic of SOC dynamics in experiments [15–30]. Many of the latter are based on power spectra of signals that capture fluctuations in either the internal state, or the flux out, of the confined plasma.

These plasma systems fall into a wider class that can be characterized as being driven, dissipative, out-of-equilibrium and having a conservation law or laws (see e.g. [31]). They have many degrees of freedom (d.o.f.), or excited modes, and long range correlations leading to scaling or multiscaling. This wider class includes both fully developed turbulence [32] and SOC [33]; indeed, since the original suggestion of Bak *et al* [1] that SOC '... could be considered a toy model of generalized turbulence' there has been continuing debate on the possible relationship between them ([34–36], see also [37] and references therein). The statistical signatures of turbulence, and systems in SOC, have similarities (e.g. [38, 39]) and it has been suggested that SOC signatures may emerge from MHD or reduced MHD models (e.g. in the solar corona context [40, 41]). In particular, it has recently been argued in the context of astrophysical plasmas that SOC and turbulence are aspects of a single underlying physical process ([42, 43] and references therein). However, the extent to which SOC uniquely captures the observed dynamics in astrophysical [44, 45], or magnetically confined laboratory [46, 47], plasmas, or is indeed consistent with it [48, 49] has been brought into question.

A central idea in physics is that the behaviour of extended (many d.o.f.) systems may be characterized by a few measurable macroscopic control parameters. In fluid turbulence, the Reynolds number R_E expresses the ratio of driving to dissipation, parametrizing the transition from laminar to turbulent flow. Control parameters such as the Reynolds number can be obtained from dimensional analysis [32, 50], without reference to the detailed dynamics. From this perspective the level of complexity is simply characterized by the number N of excited, coupled d.o.f. (or energy carrying modes) in the system. The transition from laminar to turbulent flow then corresponds to an (explosive) increase in N. The nature of this transition, the value of the R_E at which it occurs and the rate at which N grows with R_E all depend on specific system phenomenology. Dimensional arguments, along with the assumptions of steady state and energy conservation, are, however, sufficient to give the result that for Kolmogorov turbulence, N always grows with R_E [32, 51].

It was originally argued ([1, 52], see also [53]) that avalanching systems self-organized to the SOC state without the tuning of a control parameter. Subsequent analysis has established a consensus [31, 54–56] that some tuning exists, in the sense that SOC is a limiting behaviour in the driving rate h and the dissipation rate ϵ, the 'slowly driven interaction dominated thresholded' (SDIDT) [33] limit. These findings are highly suggestive that h/ϵ plays the role of a control parameter. By arguing that distinct realizable avalanche sizes play the role of excited d.o.f. or energy carrying modes, we have recently [57] identified h/ϵ as a macroscopic control parameter R_A for SOC, by formal dimensional analysis. The rate at which N varies with R_A is dependent on the detailed phenomenology; however, similarity arguments, along with the assumptions of steady state and energy conservation, are sufficient to quite generally determine that N always decreases as R_A increases. The SOC state is then characterized by maximal N, that is, avalanches occurring on all length scales supported by the system. This SDIDT limit is reached by taking R_A to zero. Increasing the driving can then take the system away from the SOC state, towards order (smaller N) with few excited d.o.f. A key result of [57] is then that the SDIDT $R_A \to 0$ limit is in the opposite sense to fluid turbulence which maximizes N at $R_E \to \infty$. This suggests a possible means to distinguish observationally between turbulence and SOC—the spatio-temporal range over which scaling is observed expands with increasing R_E for turbulence, and contracts with increasing R_A for SOC. In a sufficiently large bandwidth system then, SOC phenomenology can quite generally persist under the more realistic conditions of finite driving rate. This has been seen in specific avalanche models [58–60]. Not all aspects of the dynamics will remain universal under intermediate driving rate, when we are no longer in the strict SDIDT limit. Scaling, taken as a signature of SOC can, as we shall see, also be a signature of the driving itself. We will explore here which observables are the most appropriate to distinguish 'finite driving rate SOC'.

2. Control parameters from the Π-theorem

Formal dimensional analysis, Buckingham's Π-theorem ([61], see also [50]), provides a general framework for obtaining control parameters. The essential idea is that the system's behaviour is captured by a general function F which only depends on the *relevant* variables $Q_{1\ldots V}$ that describe the system. Since F is dimensionless it must be a function of the possible dimensionless groupings, the $\Pi_{1\ldots M}(Q_{1\ldots V})$, which can be formed from the $Q_{1\ldots V}$. The (unknown) function $F(\Pi_1, \Pi_2 \ldots \Pi_M)$ is universal, describing all systems that depend on the $Q_{1\ldots V}$ through the $\Pi_{1\ldots M}(Q_{1\ldots V})$ and the relationships between them. If one then has additional information about the system, such as a conservation property, the $\Pi_{1\ldots M}(Q_{1\ldots V})$ can be related to each other to make F explicit. If the V macroscopic variables are expressed in W physical dimensions (i.e. mass, length, time) then there are $M = V - W$ dimensionless groupings. For a wide class of systems including turbulence and SOC, we have that $V = 4$; there are always four relevant macroscopic variables to consider [57].

This framework leads directly to the Reynolds number R_E as a function of N for Kolmogorov (K41) turbulence (see e.g. [57] for details, for a detailed discussion of the universal scaling properties of K41 turbulence and their origin in the Navier–Stokes equations see e.g. [32]). For K41 the four relevant macroscopic variables are the driving length scale L_0 and the dissipation length scale η (both dimension $[L]$), the bulk (driving) flow speed U (dimension $[L][T]^{-1}$) and the viscosity ν (dimension $[L]^2[T]^{-1}$). These form two dimensionless groups:

$$\Pi_1 = \frac{UL_0}{\nu} = R_E, \qquad \Pi_2 = \frac{L_0}{\eta} = f(N), \tag{1}$$

where Π_1 is just the Reynolds number R_E of the flow, and the ratio of length scales Π_2 is related to the number of d.o.f. N that can be excited. We relate R_E to $f(N)$ by relating Π_1 to Π_2. The procedure is to assume an on average steady state and conservation so that the rate of energy injection is balanced by the energy dissipation rate on the ensemble average. We insist on scaling so that $N \sim (L_0/\eta)^\alpha$. This leads to (e.g. [57])

$$R_E \sim \left(\frac{L_0}{\eta}\right)^\beta \sim N^{\beta/\alpha}. \tag{2}$$

The value of the exponents α and β depends on the detailed phenomenology of the turbulent flow. Importantly, the only property of turbulence with which we are concerned here is that both $\beta > 0$ and $\alpha > 0$ so that $\beta/\alpha > 0$. This identifies the Reynolds number as the control parameter for a process (turbulence) which simply excites more active modes or d.o.f. as we increase R_E.

We now consider a generic avalanche model in a system of size L_0, in Euclidean dimension D where the height of sand is specified on a grid, with nodes at spacing δl. Sand is added to individual nodes, that is, on length scale δl at an average time rate h per node. On intermediate length scales $\delta l < l < L_0$, sand is conservatively transported via avalanches. Sand is then lost to the system (dissipated) at a time rate ϵ over the system size L_0 (see also [62–64]). In our discussion here we follow [52] and assume that the transport timescale is fast, so that avalanches occur instantaneously and do not overlap. There must be some detail of the internal evolution of the pile that maximizes the number of length scales l on which avalanches can occur. For avalanche models this is the property that transport can only occur locally if some local critical gradient is exceeded; as a consequence the pile evolves through many metastable states. If these length scales represent excited d.o.f. then the number N of d.o.f. available will be bounded by L_0 and δl.

The four relevant variables for the avalanching system are then the system size L_0 and grid size δl (both dimension $[L]$), and the average driving rate per node h and the system averaged dissipation or loss rate ϵ (both dimension $[S][T]^{-1}$, where $[S]$ are the dimensions of the sand). These give two dimensionless groups:

$$\Pi_1 = \frac{h}{\epsilon} = R_A, \qquad \Pi_2 = \frac{L_0}{\delta l} = f(N). \tag{3}$$

We next insist on scaling: $N \sim (L_0/\delta l)^\alpha$ with $D \geqslant \alpha \geqslant 0$ for $D > 1$ (the value of α depends on the details and may be fractional). Conservation on the average implies that $h(L_0/\delta l)^D \sim \epsilon$ so that

$$R_A = \frac{h}{\epsilon} \sim \left(\frac{\delta l}{L_0}\right)^D \sim N^{-D/\alpha} \tag{4}$$

and importantly $-D/\alpha < 0$. We then have that the number of excited d.o.f. *decreases* as we increase the control parameter $R_A = h/\epsilon$. Thus we recover the SDIDT limit for SOC, namely $R_A \to 0$, but now explicitly identify this limit with maximizing the number of excited d.o.f.

Both R_E and R_A increase with driving of the system, but in the case of turbulence and SOC the system's response is quite different. As we increase the driving in K41 turbulence, the smallest length scale η can decrease (via Navier–Stokes) to provide the necessary dissipation to maintain a steady state, and since we have assumed scaling the system simply excites more modes or d.o.f. On the other hand, in the avalanche models both the smallest and largest length scales are fixed; increasing the driving will ultimately introduce sand at a rate that exceeds the rate at which sand can be transported by the smallest avalanches, as we discuss next.

3. Signatures of SOC-like behaviour under intermediate driving

For avalanching to be the dominant mode of transport of sand, there are conditions on the microscopic details of the system. Specifically, there must be a separation of timescales such that the relaxation time for the avalanches must be short compared with the time taken for the driving to accumulate sufficient sand locally to trigger an avalanche. Avalanches are triggered when a critical value for the local gradient is exceeded. The critical gradient can be a random variable but provided it has a defined average value g, we have that on average, we would need to add $g\delta l$ sand to a single cell of an initially flat pile to trigger redistribution of sand. The number of time steps that this would take to occur would on average be $(g\delta l)/(h\delta t)$ where δl is the cell size and δt is the time step. This gives the condition for avalanching to dominate transport on all length scales in the grid $[\delta l, L_0]$, so that avalanches only occur after many grains of sand have been added to any given cell in the pile (the strict SDIDT [54, 55] limit), to be $h\delta t \ll g\delta l$. There will be a regime of intermediate driving where we are far from SDIDT but where the sandpile is large enough to on average accommodate the avalanches (see also [57, 59]):

$$1 < \frac{h\delta t}{g\delta l} \ll \left(\frac{L_0}{\delta l}\right)^D. \tag{5}$$

For a given physical realization of the sandpile, that is, fixed box size L_0 and grid size δl, successively increasing $h\delta t$ above $g\delta l$ then successively increases the sizes of the smallest avalanches. If the driving is at a localized region then, for all except the smallest avalanches, the avalanching process will be on length scales that are well separated from that of the driving. If the avalanching process is self-similar, we would then anticipate that intermediate

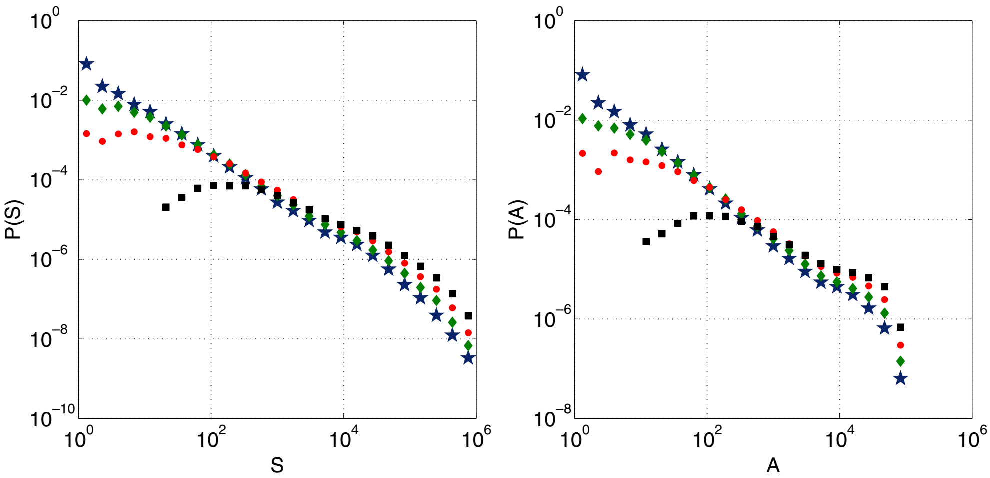

Figure 1. Probability density functions of avalanche size S and area A for four runs of an open boundary 2D BTW [1, 52] $L_0/\delta l = 400$ and $h\delta t = 1$ (blue star), 4 (green diamond), 8 (red circle) and 16 (black square) with no randomness on the driving rate h.

sized avalanches, that is, on length scales between that of the driving and of the system, will still show power law statistics sharing approximately the same exponent as at the SDIDT limit.

We now explore this in more detail with simulations of the BTW [52] sandpile in 2D, where the driving is spatially restricted to the centre of the pile. In all cases shown, the sandpile is of size $L_0 = 400\delta l$, is centrally driven, all boundaries are open and the critical gradient (threshold for avalanching) is $g = 4\delta l^{-1}$ (so that the addition of four grains to a flat pile would just trigger redistribution). The sandpile is initialized in a random state where each cell is assigned a uniformly distributed random height in the range [1, 4]. Statistics are shown for the time taken for of order 10^4 avalanches to occur, starting from a time after which the total sand in the sandpile has reached an on average steady value. We will compare simulations in which the driving rate (average number of grains added per time step) is sucessively increased. We first plot in figure 1 normalized distributions of the number of topplings per avalanche S (a measure of avalanche size [52]) and the total area A of sites perturbed by each avalanche. In these runs the driving rate $h\delta t^{-1}$ is [1, 4, 8, 16]. We can see from the figure that both these avalanche distributions clearly show that as we increase the driving, the sizes of the smallest avalanches become successively larger. As the largest realizable avalanches are constrained by the system size, the dynamic range of avalanche sizes is thus successively reduced. Avalanche distributions can then provide a reasonable diagnostic of an SOC system under intermediate driving rate.

For a physical system where typically a time series of some variable is available, an observable that requires less processing is the Fourier spectrum of some characteristic time series. Indeed, SOC was originally invoked to explain $1/f$ noise in naturally observed power spectral densities, although in practice, special conditions are required to see $1/f$ in the BTW sandpile [33]. One observable is the flux over the edge of the pile, and in figure 2 we show the Power Spectral Density (PSD) for the four runs shown in figure 1. We can see that there is a small region at lower frequencies which is roughly a power law, with slope having an exponent close to -2; a line of this slope is drawn on the figure for comparison. At high frequencies the spectrum is flat, indicating white noise. The clearest indicator of increasing driving rate is that the crossover from $\sim 1/f^2$ to white noise shifts to sucessively higher frequencies, the range does not vary strongly. Thus without some *a priori* knowledge of the system the PSD

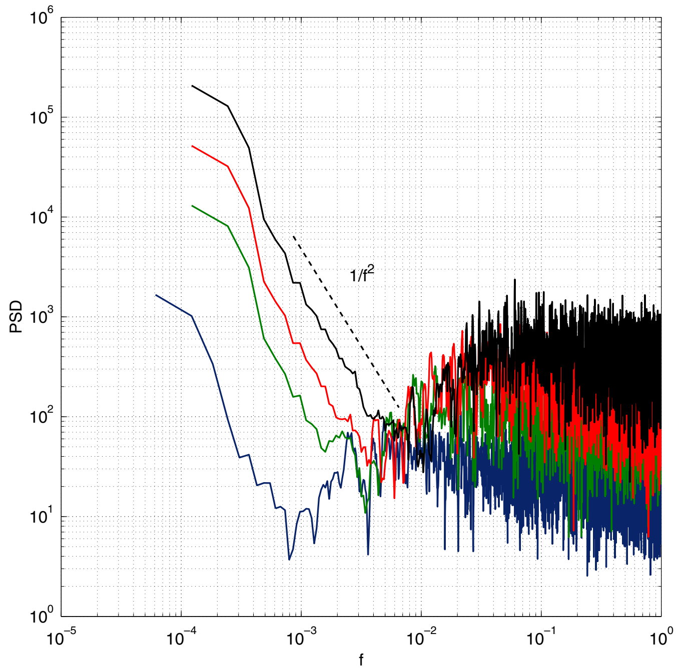

Figure 2. PSD for the total flux over the edge for the four runs shown in figure 1 ($\delta t = 1$). The PSD was computed using the Matlab periodogram routine with a rectangular window in the time domain and a five point moving average taken in spectral space.

would, under experimental conditions, be a weak discriminator of SOC. This $\sim 1/f^2$ is most straightforwardly explained as arising from a time series of uncorrelated random pulses.

Finally, we explore the effect of randomness on the driving rate h. We show in figure 3 the avalanche PDFs for runs with the same system size as before. Now the the driving rate is an i.i.d. number in the range $[0, \bar{h}\delta t]$ with $\bar{h} = 1, 4, 8, 16$. Although these plots still do show a trend with increasing driving rate, the clear separation of scales between small (suppressed) and large (unsuppressed) avalanches is much harder to discern. Indeed, one could easily interpret these plots as showing a set of approximate power law slopes with an exponent that varies with the driving rate. These power law slopes are not indicative of the system's proximity to an SOC state however; rather they are in some sense the result of integrating the white noise driving signal (cf models for interface growth [65]).

4. Conclusions

A subset of high dimensional, driven, dissipating, out-of-equilibrium systems including both turbulence and SOC have a single control parameter which expresses the ratio of the driving to the dissipation and which can be related to the number of excited d.o.f. N, which corresponds to the dynamic range over which these systems show scaling. For avalanche models that can exhibit SOC, the control parameter is $R_A = h/\epsilon$. The limit $R_A \to 0$ is just the well-known SDIDT limit of SOC. Specific avalanching systems will have different values of the scaling exponent for the avalanche distribution but will all share the essential property that N is maximal under the limit of vanishing driving. This is in the opposite sense to Kolmogorov

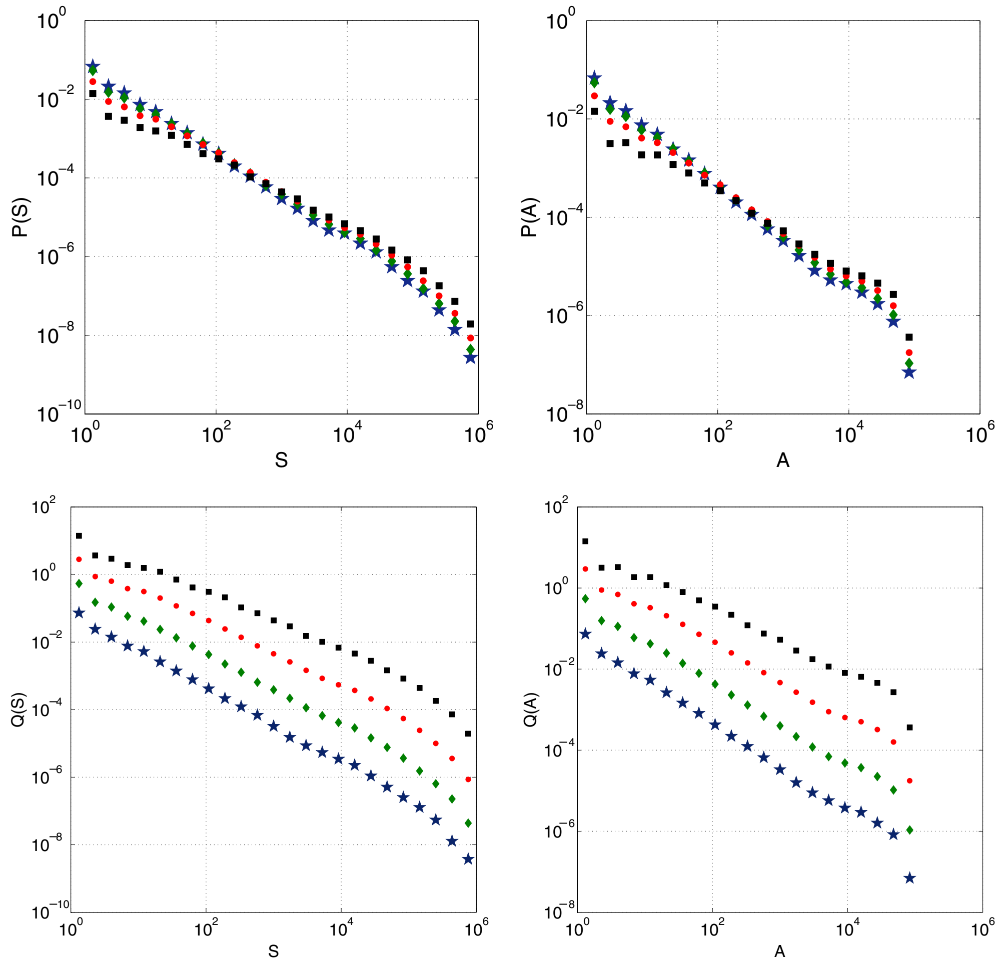

Figure 3. Avalanche size S and area A normalized distributions for four runs of an open boundary 2D BTW [1, 52] $L_0/\delta l = 400$ and $h\delta t = 1$ (blue star), 4 (green diamond), 8 (red circle) and 16 (black square), with randomness on the driving rate h with the same amplitude. Upper panels: probability densities; lower panels: the same curves shifted up progressively by a factor of ten for clarity.

turbulence which maximizes the number of excited d.o.f. N under maximal (infinite) driving. This establishes an essential distinction between turbulence and SOC. Practically speaking, it can for example arise because if we fix the outer, driving scale in Kolmogorov turbulence, the dissipation scale can simply adjust as we increase the driving. Since the system shows scaling, this acts to increase the available d.o.f. Avalanching, on the other hand, is realized in a finite sized domain and driven on a fixed, smallest scale, so increasing the driving beyond a certain point simply swamps the smallest spatial scales, thus reducing the available d.o.f. Since the number of excited d.o.f. corresponds to the bandwidth over which the system shows scaling, in principle one could distinguish SOC from turbulence observationally by testing how this varies with the driving rate. We have demonstrated this here with the BTW sandpile and find that avalanche distributions rather than power spectra are sensitive to the reduction in the number of d.o.f. for an SOC system as we increase the driving rate. However, this signature can be masked if the driving is also broadband and scale free (as in a white or coloured noise); in this case the system acts as a scale free integrator and shows SOC-like power law avalanche distributions even when the system is far from the SOC limit.

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