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Time signatures of impulsively generated coronal fast wave trains

V. M. Nakariakov,[1★] T. D. Arber,[1] C. E. Ault,[1] A. C. Katsiyannis,[2] D. R. Williams[3] and F. P. Keenan[2]

[1]*Physics Department, University of Warwick, Coventry CV4 7AL*
[2]*Department of Pure and Applied Physics, Queen's University Belfast, Belfast BT7 1NN*
[3]*Mullard Space Science Laboratory, University College London, Holmbury St Mary, Dorking, Surrey RH5 6NT*

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ABSTRACT

Impulsively generated short-period fast magneto-acoustic wave trains, guided by solar and stellar coronal loops, are numerically modelled. In the developed stage of the evolution, the wave trains have a characteristic quasi-periodic signature. The quasi-periodicity results from the geometrical dispersion of the guided fast modes, determined by the transverse profile of the loop. A typical feature of the signature is a tadpole wavelet spectrum: a narrow-spectrum tail precedes a broad-band head. The instantaneous period of the oscillations in the wave train decreases gradually with time. The period and the spectral amplitude evolution are shown to be determined by the steepness of the transverse density profile and the density contrast ratio in the loop. The propagating wave trains recently discovered with the Solar Eclipse Coronal Imaging System (SECIS) instrument are noted to have similar wavelet spectral features, which strengthens the interpretation of SECIS results as guided fast wave trains.

Key words: MHD – Sun: corona – Sun: oscillations – stars: coronae – stars: flare.

1 INTRODUCTION

Periodic perturbations of the solar corona have been observed for a few decades (e.g. Aschwanden et al. 1999; Nakariakov 2003, for reviews). Recent progress in the observational detection and study of coronal oscillations and waves provides us with the basis for a new method for the determination of coronal physical parameters – magnetohydrodynamic (MHD) coronal seismology (Nakariakov et al. 1999; Nakariakov & Ofman 2001).

The kink (Aschwanden et al. 1999; Nakariakov et al. 1999) and longitudinal (Wang et al. 2002; Ofman & Wang 2002) oscillations of coronal loops are observed to be standing modes of the structures, with the wavelength determined by the length of the loop. Consequently, the periods of these oscillations are given by the ratio of wavelength to phase speed of the waves, i.e. the kink speed in the case of the kink oscillations and the tube speed in the case of the longitudinal oscillations. In particular, the analysis of kink oscillations of coronal loops gives an estimation of the magnetic field in loops (Nakariakov & Ofman 2001).

Another intensively studied coronal phenomenon, propagating slow magneto-acoustic waves (e.g. DeForest & Gurman 1998; Berghmans & Clette 1999; De Moortel et al. 2002, and references therein), also demonstrates clear quasi-periodicity, which has not yet been explained. It is believed (e.g. Ofman, Nakariakov & DeForest 1999; Nakariakov et al. 2000) that the periodicity of these waves is prescribed by the source.

It has been theoretically predicted (Roberts, Edwin & Benz 1983, 1984) that propagating disturbances associated with another MHD wave mode, the fast magneto-acoustic wave, can be guided by solar coronal structures and that these waves can form quasi-periodic wave trains. It was pointed out by Edwin & Roberts (1983) and Roberts et al. (1983, 1984) that periodicity of fast magneto-acoustic modes in coronal loops is not necessarily connected with the wave source, but can be created by the dispersive evolution of an impulsively generated signal. Roberts et al. (1984) analytically predicted that the development of the propagating sausage pulse forms a characteristic quasi-periodic wave train with three distinct phases. That analysis was restricted to the case of the slab with sharp boundaries and to sausage modes only. The initial stage of the pulse evolution has been numerically modelled by Murawski & Roberts (1993, 1994) and Murawski, Aschwanden & Smith (1998), and was consistent with the analytical prediction. However, the developed stage of the pulse evolution, when the pulse develops into the theoretically predicted quasi-periodic wave train, has not previously been modelled. Also, effects of the smooth Alfvén speed profile and density contrast on the wave train signature have not yet been studied.

The dispersion relation for the fast magneto-acoustic modes of a coronal loop modelled by a straight plasma cylinder, surrounded by a plasma with a magnetic field, was derived and discussed in Edwin & Roberts (1983) and Roberts et al. (1983, 1984). In particular, it was found that the modes are highly dispersive if their wavelengths

★E-mail: valery@astro.warwick.ac.uk

are about the radius of the cylinder. The analysis of the group speeds of these modes shows that the long-wavelength spectral components propagate faster than the medium- and short-wavelength ones. This suggests that an impulsively generated fast wave train has a characteristic wave signature with three distinct phases: the periodic phase, the quasi-periodic phase and the decay phase (Roberts et al. 1983; Roberts et al. 1984). Nakariakov & Roberts (1995) showed theoretically that the time signature of a fast wave train contains information about the transverse Alfvén speed profile in the loop. In particular, it was shown that, in the zero-β plasma case, the decay phase of the wave train can be absent if the density profile is smooth enough.

Recently, Williams et al. (2001, 2002) and Katsiyannis et al. (2003) reported the observational discovery of rapidly propagating compressible waves in coronal loops with the Solar Eclipse Coronal Imaging System (SECIS) instrument. As the observed speed was estimated at approximately 2100 km s^{-1}, these waves were interpreted as fast magneto-acoustic modes. The waves were observed to have a quasi-periodic pattern with a mean period of about 6 s. In Williams et al. (2001, 2002), Katsiyannis et al. (2003) and Cooper et al. (2003b), interpretation of these observations in terms of fast wave trains was suggested. To strengthen or reject this interpretation, it is necessary to compare observationally determined parameters of the wave trains with theoretical modelling of time signatures of fast magneto-acoustic wave trains.

The aim of this paper is to study, for the first time, the developed stage of the fast wave evolution, simulating the formation of the quasi-periodic wave train predicted by Roberts et al. (1983, 1984) and Nakariakov & Roberts (1995) for density profiles of different steepness. Also, applying the wavelet technique, we shall demonstrate that the theory is relevant to interpretation of the compressive wave trains observed by SECIS (Katsiyannis et al. 2003) and show that observations, together with the theory, provide us with a potential tool for the determination of the internal structure of coronal loops.

2 DISPERSIVE EVOLUTION OF FAST WAVE TRAINS

The formation of a quasi-periodic wave train is governed by wave dispersion. More precisely, the shape of the wave train is determined by the dependence of the wave group speed upon the wavenumber. These dependences, determined from dispersion relations for sausage modes of a magnetic slab with the step-function density profile (e.g. Roberts et al. 1983) and the symmetric Epstein profile (see Cooper et al. 2003 for details), are shown in Figs 1 and 2. In the long-wavelength part of the spectrum, the waves experience cut-offs. The cut-off wavenumbers of sausage modes of a magnetic slab are

$$k_c = \frac{1}{w}\sqrt{\frac{C_{A0}^2}{C_{Ae}^2 - C_{A0}^2}}\begin{cases}\pi/2 & \text{(step function)}\\ \sqrt{2} & \text{(Epstein function)},\end{cases} \qquad (1)$$

where w is the semiwidth of the density profile, and C_{A0} and C_{Ae} are the Alfvén speeds inside and outside the slab, respectively. [However, the analytical solution for the symmetric Epstein profile could be found in the case of a cold (zero-β) plasma only.] Near the cut-off, the group speed is about C_{Ae} and with increasing wavenumber, the group speed sharply decreases to C_{A0}. The significant difference between the step-function profile case and the smooth Epstein profile case is the behaviour of the group speed near C_{A0}. For all

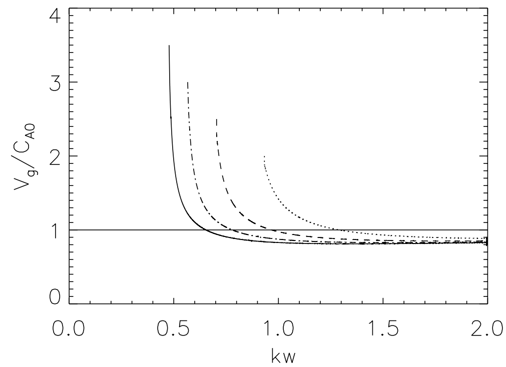

Figure 1. Group speeds of sausage modes in a magnetic slab with step-function density profile. The solid line corresponds to the Alfvén speed ratio of 4, the dash-dotted curve 3, the dashed curve 2.5 and the dotted curve 2. The thin horizontal line shows the Alfvén speed inside the slab.

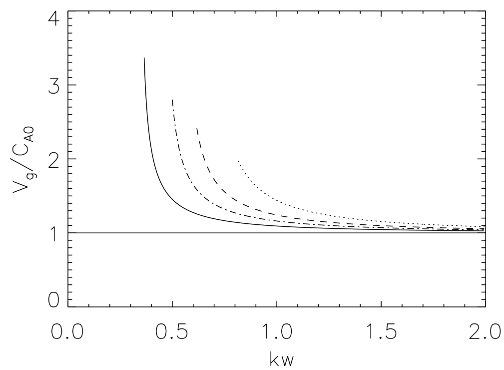

Figure 2. The same as in Fig. 1, but for the symmetric Epstein profile.

profiles steeper than the Epstein profile [that can be approximated by the parabolic $1 - (x/w)^2$-function near the centre of the slab], the group speed spectrum has a minimum (Nakariakov & Roberts 1995). For smoother profiles, including the Epstein profile, this minimum does not exist, cf. Figs 1 and 2. The variation of the group speed with wavenumber, thus, depends critically on the steepness of the Alfvén speed across the loop and the ratio of the Alfvén speed inside and outside the loop. These two limiting cases discussed above provide us with the reference basis for the consideration of the general case. In the next section it is shown how these variations manifest themselves in the signatures of fast wave trains.

3 WAVELET SIGNATURES OF FAST WAVE TRAINS

We study characteristic signatures of impulsively generated fast magneto-acoustic wave trains propagating along coronal loops by modelling them with the 2D full MHD LARE2D numerical code (Arber et al. 2001). The loop is modelled as a 1D enhancement of the plasma density ρ_0 across the field B_0; B_0 is parallel to the z-axis. The density profile is

$$\rho(x) = \rho_\infty + (\rho_{max} - \rho_\infty)\mathrm{sech}^2\left[\left(\frac{|x|}{w}\right)^p\right], \qquad (2)$$

where ρ_{max}, ρ_∞, p and w are constant. Here, the parameter ρ_{max} is the density at the centre of the inhomogeneity, ρ_∞ is the density at $x = \infty$ and w is a parameter governing the inhomogeneity width. The power index p controls the steepness of the density profile.

When the power p tends to infinity, the profile becomes a step function, and when $p = 1$, the profile is the symmetric Epstein profile (Nakariakov & Roberts 1995). These two limiting cases are the only cases with known analytical solutions. In the numerical experiments, plasma β was kept small, but finite, so the magnetic field did not change much across the profile and the Alfvén speed had a minimum at the axis of the profile. Such a structure is a fast magneto-acoustic refractive waveguide.

The equilibrium is perturbed by a pulse situated at the centre of the numerical domain,

$$V_x = A_0 x \exp\left(-\frac{x^2}{\lambda_x^2}\right) \exp\left(-\frac{z^2}{\lambda_z^2}\right), \quad (3)$$

where A_0, λ_x and λ_z are the pulse amplitude and widths in the transverse and longitudinal directions, respectively. In the numerical runs described, we restrict ourself to the case of linear amplitudes, $A_0 = 0.01$. The pulse width is comparable to the loop width with $\lambda_x = 1.5w$ and $\lambda_z = 3w$. The transverse structure of the pulse, given by equation (3), is odd and generates sausage mode perturbations.

Part of the initial pulse energy is guided along the axis of the structure in the z-direction. This guided fast magneto-acoustic pulse evolves according to the dispersion relation illustrated by Figs 1 and 2. Different spectral components propagate at different group speeds. Characteristic time signatures of the developed fast magneto-acoustic wave trains guided by a coronal loop are presented in Figs 3, 4 and 5. Different figures correspond to different Alfvén speed profiles and density contrasts. The upper panels show time signatures of the signals. As the signals clearly demonstrate frequency modulation, a natural tool for their analysis is the wavelet transform. The lower panels show the wavelet spectra of the signals. The wavelet power transform of these data also allows direct comparison with observational data, as the wavelet transform is intensively used in observational data processing.

According to the lower panels of Figs 3, 4 and 5, wavelet transforms of fast wave trains guided by loops have a common feature: in all cases the spectra have a tadpole shape. The wavelet tadpole consists of a tail of gradually decreasing oscillation period, and a pronounced thick head. The relatively thin tail is formed by the

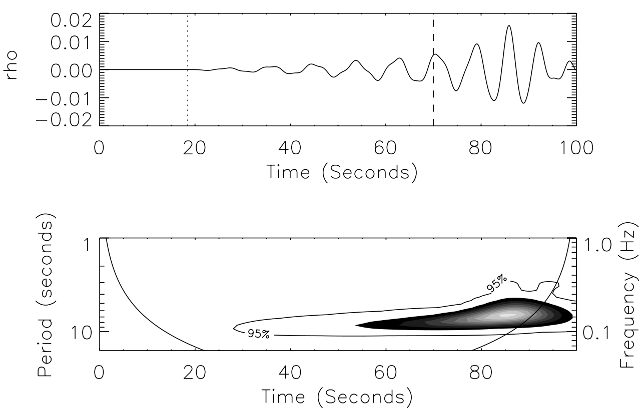

Figure 3. Numerical simulation of an impulsively generated fast magneto-acoustic wave train propagating along a coronal loop with a density contrast ratio of 14.3 and profile steepness power index equal to 8. Upper panel: the characteristic time signature of the wave train at $z = 70w$, where w is the loop semiwidth, from the source point. The vertical lines show the pulse arrival time if the density was uniform; the dotted line using the external density; and the dashed line the density at the centre of the structure. Lower panel: wavelet transform analysis of the signal, demonstrating the characteristic tadpole wavelet signature.

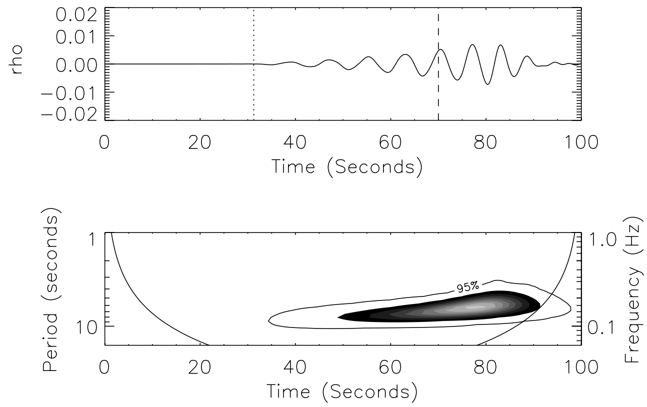

Figure 4. The same as in Fig. 3, but for the density contrast ratio equal to 5.

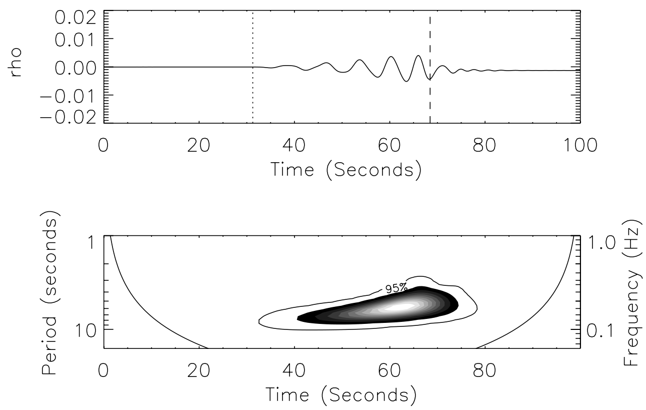

Figure 5. The same as in Fig. 3, but for the density profile power index equal to 0.7.

sharply decreasing spectral dependence of the group speed, while the head is formed by the almost dispersionless part of the group speed spectrum near the value C_{A0}. The spectrum is broad there, as the spectral components in that band propagate at almost the same group speed.

Figs 3 and 4 show that increasing the density contrast, for fixed profile shape, increases the amplitude of the wave train. This result is obvious as the excitation of the guided wave train is more efficient in the case of a higher density contrast. More specifically, a part of the energy of the initial pulse remains inside the loop in the form of the guided fast wave trains, while the remaining energy leaks out the loop in the form of fast magneto-acoustic waves. The partition of the energy between the leaky and guided modes depends upon the steepness of the profile and the density contrast. In the limiting case when the external and internal densities are equal to each other, there are no guided modes.

The time signatures of the wave trains are consistent with Fig. 1. The wave train arrival times show that it propagates at the group speed between C_{A0} and C_{Ae}, or below. The fact that, for quite a broad range of spectral components, the group speed can be below the minimal Alfvén speed coincides with the analytical curves shown in Fig. 1. A wave train guided by a loop with a smoother profile is shown in Fig. 5. The profile power index is smaller than 1, and all spectral components propagate at the group speeds between C_{A0} and C_{Ae}. There is no maximum in the group speed, which is consistent with Fig. 2. Thus, the wave train time signature contains information about the steepness of the transverse density profile in the loop.

In all cases the tadpole structure is evident in the wavelet spectra. Thus, recognition of such a signature in observational data would allow us to identify not only the waves as fast magneto-acoustic waves, but also that these waves are probably trapped in a waveguide and were formed from an impulsive, or other broad-band, source. Furthermore, the analysis of the observed wave train may give us information on the structure of the waveguide.

4 AN OBSERVATIONAL ILLUSTRATION AND DISCUSSION

Williams et al. (2001, 2002) and Katsiyannis et al. (2003) have detected a number of tadpole-shaped wave trains in coronal loops in active region NOAA AR 8651 during the 1999 August total solar eclipse using SECIS. They used CCD imaging to obtain observations of the Fe XIV 5303 Å line (formed at $\sim 2.0 \times 10^6$ K) at a rate of 44 frames per second and applied the same wavelet analysis to detect oscillations in coronal loops. Fig. 6 is an example of their detections illustrating the similarity between the theoretically predicted shape and the observational detections. Indeed, both the quasi-harmonic tail and the broad-band head are clearly seen in the wavelet. This supports the interpretation of the propagating disturbances observed by SECIS as fast magneto-acoustic wave trains, suggested by Williams et al. (2001, 2002), Cooper, Nakariakov & Tsiklauri (2003a) and Cooper, Nakariakov & Williams (2003b). The observational confirmation of the theoretical predictions provides us with a potential tool for the observational determination of the loop cross-section structure and density filling factor, which are not open to direct observations. However, efficient application of this method requires that additional features are taken into account, e.g. the line-of-sight effects discussed by Cooper et al. (2003a,b).

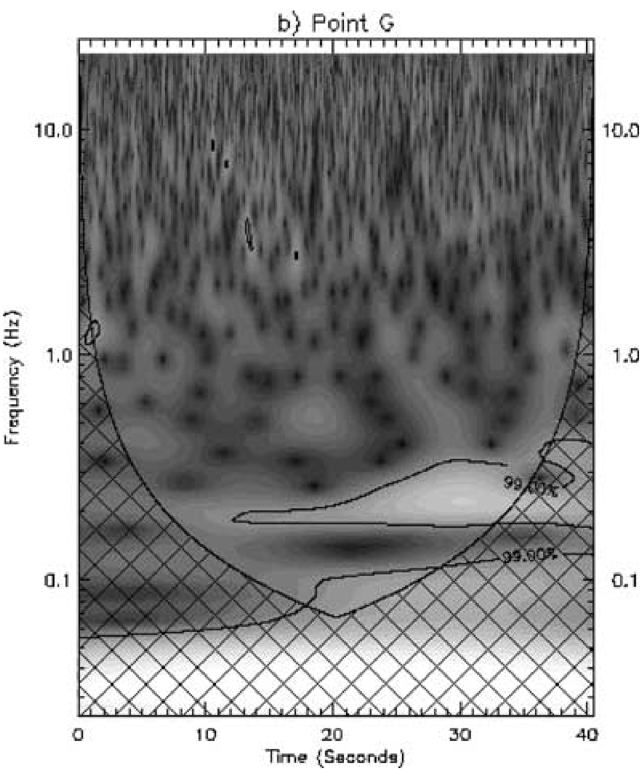

Figure 6. A typical signature and wavelet spectrum of a wave train observed with SECIS instrument (from Katsiyannis et al. 2003). The wavelet spectrum has the characteristic tadpole signature predicted by the theory (cf. e.g. Fig. 5).

The practical diagnostic application of this method requires the quantitative study of the tadpole wavelet signature of fast wave trains and will be the subject of future investigations. A question which naturally arises from the Williams et al. (2002) analysis is where the wave-driver impulse was initiated. It would certainly be of interest as to whether this wave might have been generated by a pulse situated at the loop footpoints [e.g. by an explosive event: see Marik & Erdélyi (2002) and Mendoza-Briceño et al. (2002)], or further up the loop by e.g. a reconnection event. Such work may also reveal more information about the frequency spectrum of the initial impulse which generated the fast magneto-acoustic wave train, and help us to speculate about its origin.

The same mechanism may be responsible for the formation of quasi-periodic pulsations with periods of about a second, observed in association with stellar flares [see e.g. Bastian (1990) for a review of radio band and multi-wavelength observations]. Obviously, the lack of spatial resolution makes observation of the fast wave trains difficult, as the emission from positive and negative semi-periods of the wave may cancel each other. However, observational conditions, such as the LOS-effect discussed by Cooper et al. (2003a,b) or stratification, may restrict the observable area to a small segment of the loop. Therefore, the effectively resolved region may be small enough to provide the detection of the wave trains passing through that segment of the loop.

5 CONCLUSIONS

We summarize our findings as follows.

(i) Dispersive evolution of impulsively generated fast magneto-acoustic waves guided by a solar coronal loop, filled by a finite-β plasma, was simulated for realistic loop lengths, up to 70 loop semi-widths. It was found that development of the initial pulse leads to formation of a quasi-periodic wave train. The mean wavelength of the simulated wave trains is comparable to the loop width. These distances from the wave source allowed us to observe the fully developed wave trains with the time signatures theoretically predicted by Roberts et al. (1983, 1984). For the first time, the characteristic quasi-periodic fast wave train time signature predicted by Roberts et al. (1983, 1984) is confirmed by full-MHD 2.5D numerical simulations.

(ii) The theoretical prediction of Nakariakov & Roberts (1995) that the fast wave train signature is affected by the steepness of the loop density profile is confirmed by direct numerical modelling. It is demonstrated that the wave train signature contains information about the steepness of the profile. This remains true in the realistic case of the finite-β plasma.

(iii) It is demonstrated that the wavelet transform provides us with an efficient tool for studying quasi-periodic fast wave trains formed by dispersion. In particular, it is found that the dispersive evolution of fast wave trains leads to the appearance of characteristic tadpole wavelet signatures.

(iv) It is demonstrated that the wavelet spectra of propagating disturbances of the coronal green line intensity, discovered with SECIS, have the tadpole features. The agreement of theory and observations strengthens the interpretation of SECIS waves as impulsively generated fast magneto-acoustic wave trains guided by coronal structures.

Further development of this study, both in theoretical modelling and in observational searches for fast wave trains in coronal loops, may provide the basis for determining the transverse structure of coronal loops.

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