SHORT QUASI-PERIODIC MHD WAVES IN CORONAL STRUCTURES

V. M. NAKARIAKOV*, D. J. PASCOE and T. D. ARBER
Physics Department, University of Warwick, Coventry, CV4 7AL, UK
*(*Author for correspondence: E-mail: v.nakariakov@warwick.ac.uk)*

(Received 29 July 2005; Accepted in final form 2 November 2005)

Abstract. The possibility of remote diagnostics of coronal structures with impulsively-generated short-period fast magnetoacoustic wave trains is demonstrated. An initially broad-band, aperiodic fast magnetoacoustic perturbation guided by a 1D plasma inhomogeneity develops into a quasi-periodic wave train with a well-pronounced frequency and amplitude modulation. The quasi-periodicity results from the geometrical dispersion of the modes, determined by the transverse profile of the loop, and hence contains information about the profile. Wavelet images of the wave train demonstrate that their typical spectral signature is of a "crazy tadpole" shape: a narrow spectrum tail precedes a broad-band head. The instantaneous period of the oscillations in the wave train decreases gradually with time, with a mean value of several seconds for typical coronal values. The period and the spectral amplitude evolution are determined by the steepness of the transverse density profile and the density contrast ratio in the loop, which offers a tool for estimation of the sub-resolution structuring of the corona.

Keywords: coronal seismology, coronal waves, MHD, solar corona

1. Introduction

Direct observational evidence of the wave and oscillatory activity of the solar corona is abundant in all observational bands and includes recent discoveries of propagating compressible waves in polar plumes and near loop footpoints, flare-generated global kink oscillations of loops, and longitudinal standing oscillations within loops, which are confidently interpreted in terms of magnetohydrodynamic (MHD) waves (see Aschwanden, 2004; Nakariakov and Verwichte, 2005 for recent comprehensive reviews). This observational breakthrough gave rise to the rapid development of a new method for the remote diagnostics of the solar coronal plasma, MHD coronal seismology, allowing for estimation of the absolute value of the magnetic field in coronal loops, Alfvén speeds, transport coefficients and other important coronal parameters. The above mentioned examples were found in the EUV band with spaceborne observational tools, where long cadence times restrict the shortest detectable periods to a few minutes at least. Those periods are comparable with the Alfvén and acoustic transit time along a coronal loop, but, however, are much longer than the transit time across the loop. The necessity to detect oscillations and waves with shorter periods, comparable with, at least, the transverse transit time is vital for the implementation of MHD coronal seismology to probing the sub-resolution structure of coronal active regions.

Space Science Reviews (2005) 121: 115–125
DOI: 10.1007/s11214-006-4718-8

Assuming that the typical transverse scale of coronal structuring is about a few megametres (which is likely to be just an upper limit) and taking the Alfvén speed of about a few Mm s^{-1}, one obtains that the resolution required should be shorter than a few seconds. Such short cadence time can be achieved, in particular, with ground-based observational facilities in the radio and visible light bands. Ground-based observations in the radio band have long revealed the presence of quasi-periodic pulsations in the radio emission produced in solar flares (see, e.g. Aschwanden, 1987). The typical periods detected in coronal radio pulsations are in the range from a few tenths of a second to a few minutes. In the visible light, the prolonged search for coronal oscillations in solar eclipse data has recently resulted in the discovery of rapidly propagating quasi-periodic wave trains (Williams *et al.*, 2001, 2002; Katsiyannis *et al.*, 2003).

The period of observed oscillations provides us with useful information about parameters of the plasma structure which supports the oscillation. To extract this information, we have to identify the physical mechanism responsible for the generation of the periodicity. There are several physical mechanisms which can be responsible for observed periodicities. Firstly, it is a geometrical resonance which determines frequencies of standing waves. This mechanism is believed to be associated with longer periodicities connected with global standing modes with periods about the longitudinal Alfvén and acoustic travel times (Aschwanden, 2004; Nakariakov and Verwichte, 2005). Secondly, the periodicity can be generated by some nonlinear processes, e.g. by certain dynamical regimes of magnetic reconnection (e.g., Kliem *et al.*, 2000) which are not well understood yet. Thirdly, the periodicity can be brought with the waves from other layers of the Sun, e.g. from the photosphere (De Pontieu *et al.*, 2005), so may be of non-coronal origin. Fourthly, quasi-periodic wave trains can be formed by dispersive evolution of initially broad-band signals. In a dispersive medium, impulsively generated waves evolve into a quasi-periodic wave train with a pronounced period and amplitude modulation. In the coronal context, it was pointed out by Roberts *et al.* (1983, 1984) that the periodicity of fast magnetoacoustic modes in coronal loops is not necessarily connected either with the wave source or with some resonances, but can also be created by the dispersive evolution of linear signals. The idea of the effect is that different spectral components propagate at different group speeds because of dispersion. In this paper we consider this mechanism in detail and discuss its possible implementation in the remote diagnostics of solar coronal structures.

2. Dispersive Evolution of Fast Modes

In a uniform medium, ideal MHD waves are dispersionless. However, structuring of the plasma brings a characteristic spatial scale and appearance of guided magnetoacoustic modes. Fast magnetoacoustic modes with wave lengths comparable with the characteristic scale of structuring are highly dispersive. Typical dispersive

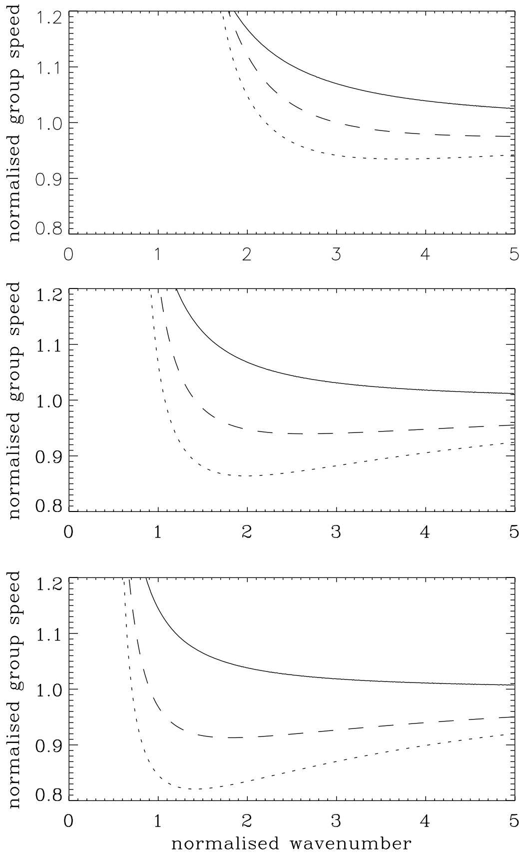

Figure 1. The dependence of group speeds of fast sausage magnetoacoustic modes upon the parallel wave number in a magnetic slab. The speeds are measured in the Alfvén speed at the axis of the slab and the wave number is normalised to the half width of the slab. The upper panel corresponds to the density contrast $\rho_0/\rho_\infty = 2$, the middle to 5 and the lower to 10. The solid lines correspond to the profiles with the steepness $p = 1$ (*symmetric Epstein profile*), the dashed lines to $p = 2$, and the dotted lines to $p \to \infty$ (*step-function profile*).

curves of fast sausage modes of a magnetic slab, which is a standard model for study of the interaction of MHD waves with a plasma inhomogeneity, are shown in Figure 1. For the wave lengths comparable with the slab width (the normalised wave number is about unity), the group speed depends upon the wave number very sharply.

Studying the dispersive evolution of propagating sausage modes of a straight magnetic cylinder with a step-function transverse profile, Roberts *et al.* (1984) qualitatively predicted that the development results in a characteristic quasi-periodic wave train with three distinct phases. Such an evolution scenario is determined by the presence of a minimum in the group speed dependence upon the wave number.

An estimation of the characteristic period, provided in Roberts *et al.* (1984), is

$$P_{\text{prop}} \approx \frac{2\pi w}{j_0 C_{A0}} \sqrt{1 - \frac{\rho_e}{\rho_0}}, \tag{1}$$

where w is the cylinder half-width, $j_0 \approx 2.40$ is the first zero of the Bessel function $J_0(x)$, C_{A0} is the Alfvén speed in the cylinder, and ρ_e and ρ_0 are the plasma densities outside and inside the cylinder, respectively. In loops with the large contrast ratio $\rho_e \ll \rho_0$, Equation (1) reduces to $P_{\text{prop}} \approx 2.6w/C_{A0}$. The typical periods are close to the transverse Alfvén transit time. We would like to point out that expression (1) provides us only with the estimation of the range of the periods, as the intrinsic feature of the discussed effect is the variation of the period with time. The *initial* stage of the fast sausage pulse evolution in a straight magnetic slab was numerically modelled by Murawski and Roberts (1993), Murawski *et al.* (1998), and was found to be consistent with the analytical prediction. Wave trains with signatures qualitatively similar to the theoretically predicted were found in 303 and 343 MHz coronal data recorded by the digital Icarus spectrometer (Roberts *et al.*, 1984).

The transverse profile of the Alfvén speed in the slab, directly connected in a low-β coronal plasma with the density profile, affects the fast wave dispersion and, consequently, the wave train signature. This effect was studied by Nakariakov and Roberts (1995) by taking the density profile as the generalised symmetric Epstein function

$$\rho_0 = \rho_{\max} \operatorname{sech}^2\left[\left(\frac{x}{w}\right)^p\right] + \rho_\infty, \tag{2}$$

where $\rho_{\max}$, ρ_∞, w are constant and the density gradient is in the x-direction, which is perpendicular to the direction of the straight, uniform magnetic field. Here, the parameter $\rho_{\max}$ is the density enhancement at the centre of the inhomogeneity, ρ_∞ is the density at $x = \infty$ and w is a parameter governing the inhomogeneity width. The power index p determines the steepness of the profile. The cases when the power index p equals either unity or infinity correspond to the symmetric Epstein profile or to the step function profile, respectively. In the zero plasma-β case, both profiles give known analytical solutions in the eigenvalue problem describing guided fast modes. Figure 1 demonstrates the dependence of the fast sausage wave dispersion upon the steepness and the depth of the transverse profile. It was established that the group speed has a minimum for all profiles with the power index greater than unity, which, in other words, are steeper than the symmetric Epstein profile. Thus, the properties of the profile affect the dispersion shape, and can be estimated by the analysis of wave trains formed by the dispersion.

Numerical simulations of the *developed* stage of the dispersive evolution of a fast wave train, propagating along the magnetic field in a smooth straight slab of a low-β plasma (Nakariakov *et al.*, 2004) confirmed the qualitative prediction of Roberts *et al.* (1983) that development of an impulsively generated pulse leads to the formation of a quasi-periodic wave train with the mean wavelength comparable with the slab width. In agreement with the analytical theory, wave trains have pronounced period and amplitude modulation. In this situation, a convenient analytical tool is a wavelet transform technique. In particular, it was found (Nakariakov *et al.*, 2004) that the dispersive evolution of fast wave trains leads to the appearance of characteristic "tadpole" wavelet signatures (or, rather a "crazy tadpole" as it comes tail-first). Similar wavelet signatures were found for the propagating wave trains detected with SECIS (Katsiyannis *et al.*, 2003; Nakariakov *et al.*, 2004). At a certain distance from the excitation location, the arrival time of the beginning of the wave train is determined by the travel time at the external Alfvén speed. For the density profiles with the power indices $p \leq 1$, the end of the wave train passes the observation point at the time determined by the travel time at the internal Alfvén speed, while for steeper profiles the highest amplitudes are detected after that time. Thus, the numerical modelling of (Nakariakov *et al.*, 2004) demonstrated that the wave trains indeed were affected by the steepness and the contrast ratio of the density profile across the structure.

Another parameter which can be responsible for the wave train signature is the structure of the initial perturbation, in particular its localisation in space or initial spatial spectrum. In Figure 2 we present results of numerical modelling of fast sausage wave trains formed at a certain distance from the location of an impulsive energy deposition in a plasma slab. The calculations were performed with LARE2D code (Arber *et al.*, 2001). The guiding plasma slab is formed by a symmetric one-dimensional plasma density inhomogeneity in the x-direction across a straight magnetic field in the z-direction. Since we are considering sausage modes, which are not strongly dependent on the choice of slab or cylindrical geometry, we may ignore the third, y-direction. The density profile is described by the symmetric Epstein function (given by Equation (2) with $p = 1$), with the density enhancement corresponding to the Alfvén speed minimum at the axis of the slab. The ratio of the density at the profile top to the background density was taken to be 5. The initial perturbation is of the Gaussian shape,

$$V_x = \frac{A_0}{\lambda_z} x \exp\left(-\frac{x^2}{4}\right) \exp\left(-\frac{z^2}{\lambda_z^2}\right) \tag{3}$$

where x and z are the transverse and longitudinal axis, respectively, the amplitude $A_0 = 0.1$, and λ_z is the longitudinal width of the pulse; all spatial variables are measured in the units of the slab half-width w. Because of the symmetry of the initial perturbation, only sausage modes were excited. The perturbation develops into a quasi-periodic fast wave train, recorded as a density perturbation at a distance of approximately 15 slab widths from the initial position along the slab. Different panels of the figure demonstrate the wave trains developed from the initial Gaussian perturbations of different longitudinal widths, and their wavelet spectra. Apparently,

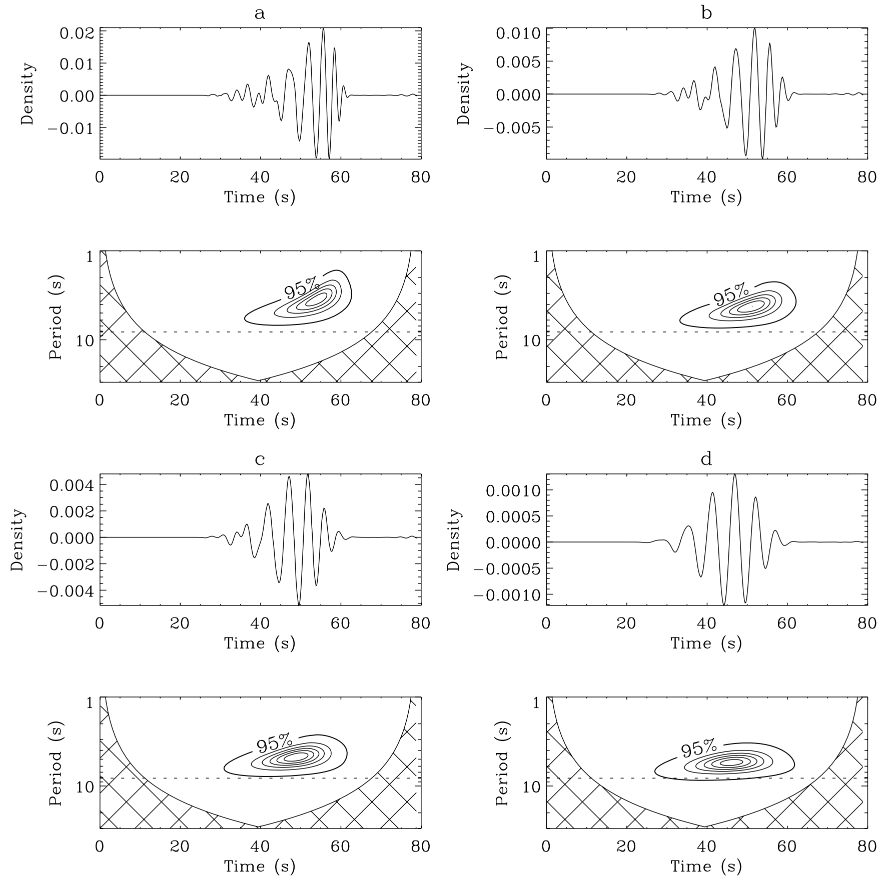

Figure 2. Density perturbations in impulsively generated fast wave trains and their wavelet transforms, for different initial longitudinal wave lengths: (a) $\lambda_z = w/2$ (b) $\lambda_z = w$ (c) $\lambda_z = 3w/2$ (d) $\lambda_z = 5w/2$, where w is the slab half-width. The horizontal dotted line shows the value of the longest possible period of a guided sausage mode, which in the plasma profile considered is 8.2 s.

in all cases, the energy of the wave trains has periods *shorter* than the cut-off period of the guided sausage mode,

$$P_c = \frac{\sqrt{2}\pi w \sqrt{C_{A\infty}^2 - C_{A0}^2}}{C_{A\infty} C_{A0}}, \tag{4}$$

where $C_{A\infty}$ is the Alfvén speed far from the slab where the density is ρ_∞. Consequently, the spectral components with the periods longer than the cut-off value leak out the slab very efficiently and are not present in the spectrum of the guided wave trains at all. The effect of the leakage of fast waves from coronal loops has been discussed in several papers, see, e.g. (Cally, 1986).

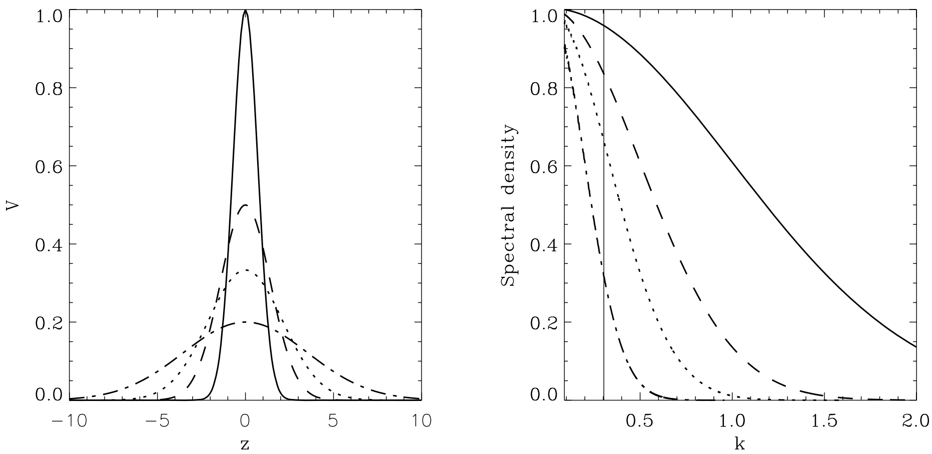

Figure 3. The left panel: the longitudinal profile of the initial pulse; the solid line corresponds to the longitudinal width $\lambda_z = w/2$, the dashed line to $\lambda_z = w$, the dotted line to $\lambda_z = 3w/2$ and dash-dotted line to $\lambda_z = 5w/2$. *The right panel* shows spatial spectra of the initial pulses. The spectral densities are normalised to the peak value of the $\lambda_z = w/2$ case. The thin solid vertical line shows the normalised cut-off wave number of the guided sausage mode, which is equal to 0.3 in the slab considered.

From Figure 2 it is also clear that the wave trains generated by longer initial pulses have less pronounced "crazy tadpole" spectra and are more monochromatic. The explanation of this effect is connected with the initial spectra of the pulses, shown in Figure 3. Longer initial pulses have narrower spatial spectra with the main part of the energy localised at the wave numbers smaller than the cut-off wave number. Consequently, for longer pulses, the guided part of the initial spectral energy is localised just above the cut-off wave number, giving the narrower time spectrum, which is consistent with the numerical results shown in Figure 2.

Thus, the signature of the frequency modulation is directly connected with the dependence of the group speed of the mode upon the frequency, upon the along-the-loop distance from the energy deposition location to the detection point, and upon the properties of the initial source. The observed modulation is also connected with the initial spectrum of the source.

3. Perpendicularly Propagating Fast Wave Trains

Another possible manifestation of the dispersive generation of quasi-periodic wave train can occur for fast magnetoacoustic pulses propagating in the direction perpendicular to a bundle of magnetic loops or threads. Random structuring of the Alfvén speed in the transverse direction, formed by the variation of loop radii, densities, temperatures and the magnetic field strength, causes dispersion of fast waves. The role of the characteristic spatial scale in this case is played by the correlation length of the structuring, which is comparable with the mean minor radius of coronal loops or a filling factor.

A simple 1D model describing the interaction of a perpendicular fast magnetoacoustic pulse with a randomly structured bundle of coronal loops filled by a zero-β plasma was developed by Murawski *et al.* (2001). In this model, the equilibrium plasma density is taken as $\rho_0 = \rho_{00} + \rho_r(x)$, where ρ_r is a random function of the transverse coordinate x, with the mean value equal to zero. The magnetic field is taken to be straight and constant everywhere. The analytically derived dispersion relation for fast waves in this model is

$$\omega^2 - C_A^2 k^2 = \omega^4 \int_{-\infty}^{+\infty} \frac{E(k - \bar{k})}{\omega^2 - C_A^2 \bar{k}^2} \, \mathrm{d}\bar{k}, \tag{5}$$

where C_A is the regular part of the Alfvén speed, and E is the Fourier transform of the correlation function of structuring (see Murawski *et al.*, 2001 for details), which actually prescribes the dispersion. Consequently, different spectral components of the fast wave propagate at different speeds and form a quasi-periodic wave train. It was numerically demonstrated that an initially aperiodic (Gaussian in the model) pulse evolves into a quasi-periodic wave train. The characteristic wave length in the train was about several correlation lengths of structuring. Assuming that a typical Alfvén speed in coronal active regions is about one Mm s^{-1}, and taking the correlation length of about the mean minor radius of EUV coronal loops (about 1 Mm), we conclude that the generated period is about a few seconds. This effect has not yet been confirmed observationally. However, if observed, it would allow us to estimate the correlation length of coronal structuring – the filling factor.

Here, we preliminary describe new results obtained by numerical modelling of the interaction of a fast pulse with a 1D randomly structured plasma, performed with LARE2D code. The random density structure is constructed by superposition of N harmonics of the computational domain

$$\rho_r(x) = \sum_{n=1}^{N} \frac{A_n}{k_0} \sin(k_n x + \phi_n) \exp(-k_n/k_0) \tag{6}$$

where A_n and ϕ_n are a randomly generated amplitude and phase respectively for the n-th harmonic, k_n is the wave number of the harmonic, and k_0 is a constant determining the density correlation length. In the numerical simulations described, the number of harmonics was $N = 4000$. The correlation length of this structure, l_x is defined as the FWHM of the density autocorrelation function.

The fast wave train was launched as a propagating Gaussian pulse with the amplitude 0.001 and the width equal to $2l_x$. The pulse gradually evolved into a quasi-periodic wave train, analysed in Figure 4. In the analysis, it was assumed that the density correlation length is of 1 Mm (a typical size of TRACE loops) and the Alfvén speed is 1 Mm s^{-1}. This gives characteristic periods of about several seconds, which is qualitatively consistent with the results of Murawski *et al.* (2001). Also, a

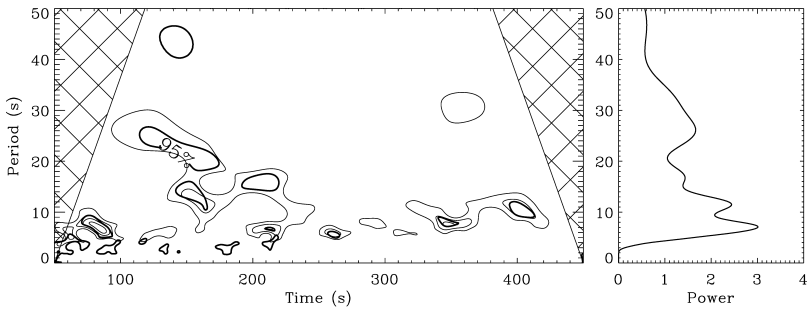

Figure 4. Spectral structure of quasi-periodic pulsations generated in a one-dimensionally randomly structured plasma after the passage of a perpendicular fast magnetoacoustic pulse. The *left* panel shows the wavelet spectrum of the velocity perturbations in the wave train. The *right* panel shows the global wavelet spectrum. The velocity was recorded at a distance of 40 correlation lengths from the initial pulse position.

pronounced modulation of the period is clearly seen in the wavelet panel. Further analysis of these results, in particular calculation of the modulation with the use of Equation (5), will be published elsewhere.

A similar estimation was obtained by Uralov (2003), who considered analytically the interaction of perpendicular fast magnetoacoustic pulses with a *regularly* periodically structured plasma.

We would like to point out that 2D effects, e.g. connected with structuring or the presence of boundaries in the direction of the magnetic field, can produce quasi-periodic wave trains too. In particular, impulsively excited fast waves, which propagate perpendicularly to the field along the axis of a coronal arcade and interact with a single loop, exhibit a quasi-period behaviour (Terradas *et al.*, 2005). However, in that study, the mean period of the oscillations is connected with the longitudinal length of the structure, i.e. with the resonant period of a standing mode.

4. Conclusion

This paper reviews recent findings on the dispersive mechanism for the generation of quasi-periodicity in the solar corona. Structuring of the coronal plasma in the Alfvén speed makes the corona a dispersive medium for propagating fast magnetoacoustic waves. Both waves guided along the magnetic field by regular coronal structures (e.g., loops) and freely propagating through the structured medium across the field, are subject to the dispersion. The dispersion is strongest at the wave lengths comparable with the characteristic scale of structuring – the loop radii, filling factors, correlation lengths, etc. One of the consequences of the dispersion is formation of quasi-periodic wave trains, with the periods about the transverse Alfvén transit times.

Our modelling of the evolution of a broad-band initially localised sausage fast magnetoacoustic pulses, propagating along the magnetic field in plasma inhomogeneities, demonstrates that, at the developed stage, the signal has a characteristic "crazy tadpole" wavelet signature. It is shown that the specific shape of the wavelet spectrum depends upon the initial spectrum. The spectrum is restricted by the cut-off period. The spectral components below the cut-off period are found to leak out the structure very efficiently.

Perpendicularly propagating fast pulses are shown to develop into quasi-periodic wave trains too, because of random structuring of the active region medium. The wave trains have a pronounced period modulation. The mean period is found to be connected with the structuring correlation length.

Fine structuring of coronal active regions and, in particular, the actual transverse size of loops and the transverse profiles of the density and the Alfvén speed, remain unrevealed. As the dispersive formation of fast wave trains is based upon those properties of loops, the fast wave trains provide us with a natural tool for their diagnostics.

Acknowledgements

V.M.N. acknowledges the support of a Royal Society Leverhulme Trust Senior Research Fellowship. D.J.P. was supported by a PPARC studentship. Wavelet software was provided by C. Torrence and G. Compo, and is available at URL: http://paos.colorado.edu/research/wavelets/.

References

Arber, T. D., Longbottom, A. W., Gerrard, C. L., and Milne, A. M.: 2001, *J. Comp. Phys.* **171**, 151–181.

Aschwanden, M. J.: 1987, *Solar Phys.* **111**, 113–136.

Aschwanden, M. J.: 2004, *Physics of the Solar Corona*, Springer Praxis Books, Berlin.

Cally, P. S.: 1986, *Solar Phys.* **103**, 277–298.

De Pontieu, B., Erdélyi, R., and De Moortel, I.: 2005, *Astrophys. J.* **624**, L61–L64

Katsiyannis, A. C., Williams, D. R., McAteer, R. T. J., Gallagher, P. T., Keenan, F. P., and Murtagh, F.: 2003, *Astron. Astrophys.* **406**, 709–714.

Kliem, B., Karlický, M., and Benz, A. O.: 2000, *Astron. Astrophys.* **360**, 715–728.

Murawski, K., Aschwanden, M. J., and Smith, J. M.: 1998, *Solar Phys.* **179**, 313–326.

Murawski, K., Nakariakov, V. M., and Pelinovsky, E. N.: 2001, *Astron. Astrophys.* **366**, 306–310.

Murawski, K. and Roberts, B.: 1993, *Solar Phys.* **144**, 101–112.

Nakariakov, V. M. and Verwichte, E.: 2005, *Living Rev. Solar Phys.* **2**, 3. URL: http://www.livingreviews.org/lrsp-2005-3.

Nakariakov V. M., Arber T. D., Ault, C. E., Katsiyannis, A. C., Williams, D. R., and Keenan, P.: 2004, *Mon. Not. R. Astron. Soc.* **349**, 705–709.

Nakariakov, V. M. and Roberts, B.: 1995, *Solar Phys.* **159**, 399–402.

Roberts, B., Edwin, P. M., and Benz, A. O.: 1983, *Nature* **305**, 688–690.
Roberts, B., Edwin, P. M., and Benz, A. O.: 1984, *Astrophys. J.* **279**, 857–865.
Terradas, J., Oliver, R., and Ballester, J. L.: 2005, *Astron. Astrophys.* **441**, 371–378.
Uralov, A. M.: 2003, *Solar Phys.* **218**, 17–27.
Williams, D. R., Phillips, K. J. H., Rudawy, P., Mathioudakis, M., Gallagher, P. T., O'Shea, E., Keenan, F. P., Read, P., and Rompolt, B.: 2001, *Mon. Not. R. Astron. Soc.* **326**, 428–436.
Williams, D. R., Mathioudakis, M., Gallagher, P. T., Phillips, K. J. H., McAteer, R. T. J., Keenan, F. P., Rudawy, P., and Katsiyannis, A. C.: 2002, *Mon. Not. R. Astron. Soc.* **336**, 747–752.