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Guiding Centre Drift Due to Non-Magnetic Forces

Suppose there is a non-magnetic force $\mathbf{F}$ acting in the plane perpendicular to $\mathbf{B}$.

Dynamics of a charged particle is described by **the equation of motion**,

$$\dot{\mathbf{V}} = \frac{q}{m}\mathbf{V}\times\mathbf{B} + \frac{\mathbf{F}}{m} \tag{1}$$

The particle velocity $\mathbf{V}$ can be considered as a sum of two components,

$$\mathbf{V} = \mathbf{V}_\perp + \mathbf{V}_{||}, \tag{2}$$

perpendicular and parallel to the magnetic field, respectively.

The parallel component of the equation of motion:

$$\dot{\mathbf{V}}_{||} = 0, \tag{3}$$

and, consequently, $\mathbf{V}_{||}$ is constant.

The perpendicular component of the equation of motion:

$$\dot{\mathbf{V}_\perp} = \frac{q}{m}\mathbf{V}_\perp\times\mathbf{B} + \frac{\mathbf{F}}{m}. \tag{4}$$

Let us define a new variable,

$$\mathbf{u} = \mathbf{V}_\perp - \frac{\mathbf{F}\times\mathbf{B}}{q|B|^2}, \tag{5}$$

or

$$\mathbf{u} = \mathbf{V}_\perp - \mathbf{V}_D, \tag{6}$$

where

$$\mathbf{V}_D = \frac{\mathbf{F}\times\mathbf{B}}{q|B|^2}. \tag{7}$$

Using $\mathbf{u}$, we can re-write Eq. (4) as

$$\dot{\mathbf{u}} = \frac{q}{m}\mathbf{u}\times\mathbf{B} \tag{8}$$

(To prove it, let us substitute Eq. (6) into Eq. (8):

$$\dot{\mathbf{V}}_\perp - \dot{\mathbf{V}}_D = \frac{q}{m}\left(\mathbf{V}_\perp\times\mathbf{B} - \frac{(\mathbf{F}\times\mathbf{B})\times\mathbf{B}}{q|B|^2}\right). \tag{9}$$

Using that $\dot{\mathbf{V}}_D = 0$ *(it is a constant!) and*

$$-(\mathbf{F}\times\mathbf{B})\times\mathbf{B} = \mathbf{B}\times(\mathbf{F}\times\mathbf{B}) = B^2\mathbf{F} - (\mathbf{F}.\mathbf{B})\mathbf{B}, \tag{10}$$

where the last term is zero (as $\mathbf{F} \perp \mathbf{B}$*), we obtain*

$$\dot{\mathbf{V}}_{\perp} = \frac{q}{m}\mathbf{V}_{\perp} \times \mathbf{B} + \frac{\mathbf{F}}{m}. \tag{11}$$

So, Eq. (8) is correct.)

Eq. (8) coinsides with the equation of motion for a particle in a constant magnetic field with no non-magnetic forces, discussed before. The solution of the equation is the gyration with the Larmor frequency.

Thus, the solution for $\mathbf{u}$ gyrates with the Larmor frequensy. Returning back to the laboratory frame of reference (with the use of Eq. (6)), we see that the particle experiences
• the gyration with the Larmor frequency and
• the drift with the drift velocity $\mathbf{V}_D$.

As $\mathbf{V}_D \propto \mathbf{B} \times \mathbf{F}$, the drift motion is perpendicular to both the magnetic field B and the non-magnetic force ($\perp$ B).

`Example 1`
`The non-magnetic force is caused by the static electric field` $\mathbf{E} \perp \mathbf{B}$.

`In this case` $\mathbf{F} = q\mathbf{E}$`, and the drift velocity is`

$$\mathbf{V}_D = \frac{\mathbf{E} \times \mathbf{B}}{|B|^2}, \tag{12}$$

`perpendicular to both` $\mathbf{B}$ `and` $\mathbf{E}$`. Note that the drift velocity is independent of the charge, so both the electrons and ions drift in the same direction.`

`Example 2`
`The non-magnetic force is gravitational,` $\mathbf{F} = m\mathbf{g}$.

`The drift velocity is`

$$\mathbf{V}_D = \frac{m}{q}\frac{\mathbf{g} \times \mathbf{B}}{|B|^2}, \tag{13}$$

`perpendicular to both` $\mathbf{B}$ `and` $\mathbf{g}$`. Note that the drift velocity depends upon the charge, so the electrons and ions drift in the opposite direction.`

Drift → Charge separation → Electric current