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Dispersion Relations for High Frequency Electromagnetic Waves in a Cold Magnetized Plasma

Dispersion relations connect frequencies ω and wave vectors $\mathbf{k}$ of a wave (it is assumed that the wave is linear and plane, and

$$\propto \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t).$$

Also, we assume that the wave fields are of high frequency, so we can neglect motions of ions. The ambient magnetic field is directed along $\hat{z}$. It is convenient to introduce the vector

$$\mathbf{N} = \frac{c\mathbf{k}}{\omega}.$$

(The absolute value of $\mathbf{N}$ gives us the refractive index.)

Dispersion relations of high frequency electromagnetic waves in a cold magnetized plasma can be derived using the dielectric tensor for a cold magnetized plasma,

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix},$$

where

$$\varepsilon_1 = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2 - \omega^2}, \qquad \varepsilon_2 = \frac{\omega_{ce}}{\omega}\frac{\omega_{pe}^2}{\omega_{ce}^2 - \omega^2}, \qquad \varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega^2}.$$

This gives us the dispersion relations for high frequency electromagnetic waves in a cold magnetized plasma:

$$\mathrm{Det}(\hat{M}) = 0,$$

where

$$\hat{M} = \frac{\omega^2}{c^2}\begin{bmatrix} \varepsilon_1 - N_y^2 - N_z^2 & -i\varepsilon_2 + N_xN_y & N_xN_z \\ i\varepsilon_2 + N_xN_y & \varepsilon_1 - N_x^2 - N_z^2 & N_yN_z \\ N_xN_z & N_yN_z & \varepsilon_3 - N_x^2 - N_y^2 \end{bmatrix},$$

where $N_{x,y,z}$ are corresponding components of the vector $\mathbf{N}$.

The dispersion relations shows that propagation of electromagnetic waves in magnetized plasmas is *anisotropic* (depends upon the angle between the wave vector $\mathbf{k}$ and the ambient magnetic field $\mathbf{B}$).

For waves parallel to the field ($\mathbf{k} \,||\, \mathbf{B} \,||\, \hat{z}$):
$N_x = N_y = 0$, and the dispersion relation reduces to

$$\varepsilon_3[(\varepsilon_1 - N_z^2)^2 - \varepsilon_2^2] = 0.$$

For waves perpendicular to the field ($\mathbf{k} \perp \mathbf{B} \,||\, \hat{z}$):
Let $\mathbf{k} \,||\, \hat{x}$, then $N_y = N_z = 0$ and the dispersion relation is

$$(\varepsilon_3 - N_x^2)[\varepsilon_1(\varepsilon_1 - N_x^2) - \varepsilon_2^2] = 0.$$