

# Parallel, High Frequency Waves in Cold Magnetized Plasmas

We restrict our attention to the case of waves that propagate *parallel* to the magnetic field.

The dispersion relation for the waves parallel to the magnetic field describes three wave modes:

- $\varepsilon_3 = 0$ , or

$$1 - \frac{\omega_{pe}^2}{\omega^2} = 0,$$

the electrostatic wave.

- $(\varepsilon_1 - N_z^2)^2 - \varepsilon_2^2 = 0$ , which has two roots:

$$k = \frac{\omega}{c} \left( 1 - \frac{\omega_{pe}^2}{\omega(\omega_{ce} + \omega)} \right)^{1/2}, \quad (1)$$

which describes left hand circularly polarized EM waves and

$$k = \frac{\omega}{c} \left( 1 + \frac{\omega_{pe}^2}{\omega(\omega_{ce} - \omega)} \right)^{1/2}, \quad (2)$$

which describes right hand circularly polarized EM waves.

Applying the condition  $k = 0$ , we can find the cut-off frequencies as the solutions of Eqs. (1) and (2). The cut-off frequency of left hand circularly polarized EM waves is

$$\omega_1 = \frac{\omega_{ce}}{2} \left( \sqrt{1 + \frac{4\omega_{pe}^2}{\omega_{ce}^2}} - 1 \right),$$

and the cut-off frequency of right hand circularly polarized EM waves is

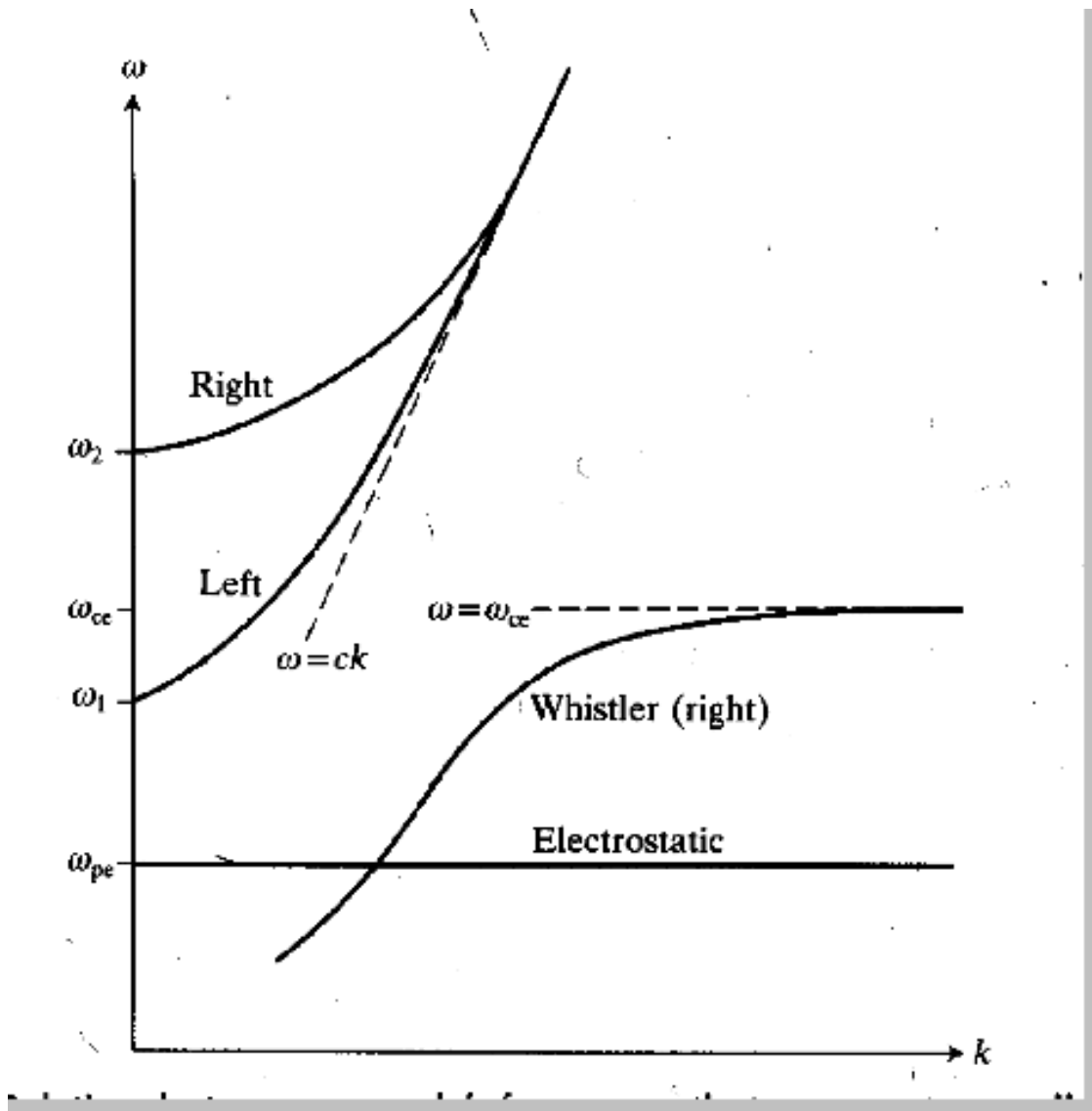
$$\omega_2 = \frac{\omega_{ce}}{2} \left( \sqrt{1 + \frac{4\omega_{pe}^2}{\omega_{ce}^2}} + 1 \right).$$

A wave mode can propagate in the plasma with given  $\omega_{ce}$  and  $\omega_{pe}^2$  if the wave frequency is higher than the cut-off frequency for this wave mode.

Also, there is another right hand circularly polarized mode, with the frequency  $\omega < \omega_{ce}$ . This wave mode is called a whistler mode.

When the frequency  $\omega$  of the right hand circularly polarized mode approaches  $\omega_{ce}$  from below, the wave number  $k$  tends to infinity. This phenomenon is called resonance. In this case, the resonance occurs because the frequency of the right hand circularly polarized waves approaches the Larmor frequency of electron gyration, which is also a right-handed motion. The phenomenon is called electron cyclotron resonance.

On the  $\omega k$ -plane, the dispersion curves are (in the case  $\omega_{pe}/\omega_{ce} < \sqrt{2}$ ):



Note, that this description is incomplete, because there are other waves modes well below the electron plasma frequency  $\omega_{pe}$  and the electron Larmor frequency  $\omega_{ce}$ . To describe these low frequency wave modes, we should take into account the motion of ions.