M1: Magnetic Resonance Hardware

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and

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Electricity and Magnetism

Electromagnetism

Magnetic Resonance

MAXWELLS EQUATIONS

GAUSS' LAW

(i) ELECTROSTATICS

$\oint_S \underline{\mathbf{D}} \bullet d\underline{\mathbf{S}} = Q = \int_v \rho \, d\mathrm{v}$ or $\nabla \bullet \underline{\mathbf{D}} = \rho$

$\underline{\mathbf{D}}$ = Electric Displacement [Cm^{-2}]

(i) MAGNETOSTATICS

$\oint_S \underline{\mathbf{B}} \bullet d\underline{\mathbf{S}} = 0$ or $\nabla \bullet \underline{\mathbf{B}} = 0$

$\underline{\mathbf{B}}$ = Magnetic Flux Density [Tesla]

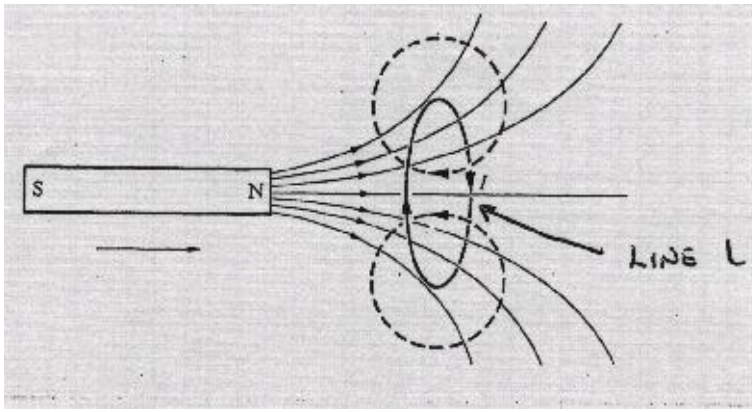
AMPERES CIRCUITAL LAW

$\oint_l \underline{\mathbf{H}} \bullet \mathrm{d}\underline{\mathbf{l}} = I = \int_S \underline{\mathbf{J}} \bullet \mathrm{d}\underline{\mathbf{S}}$ or $\nabla \times \underline{\mathbf{H}} = \underline{\mathbf{J}}$

$\underline{\mathbf{H}}$ = Magnetic Field [Am^{-1}]
$\underline{\mathbf{J}}$ = Current density [Am^{-2}]

FARADAY LAW OF ELECTROMAGNETIC INDUCTION

Oestred showed that an electrical current produces a magnetic field (1820). 1831 $\Rightarrow$ FARADAY found that a current was induced in a circuit when a magnetic field that links the circuit changes.



The EMF induced in a circuit (given by line l) is $\varepsilon_V = -\dfrac{\partial \Phi}{\partial t}$ (minus sign comes from Lenz's Law).

$$\Phi = \int_S \underline{\mathbf{B}} \bullet d\underline{\mathbf{S}}$$ (Any surface whose boundary is the line l)

$\Phi =$ MAGNETIC FLUX linked by the circuit [Tesla m^2 or Weber, Wb]

The induced EMF ε_V is equal the line integral of the induced $\underline{\mathbf{E}}$ [Vm^{-1}] electric field around the coil.

$$\oint_l \underline{\mathbf{E}} \bullet \mathrm{d}\underline{\mathbf{l}} = -\frac{\partial \Phi}{\partial t} = -\frac{\partial}{\partial t} \int_S \underline{\mathbf{B}} \bullet d\underline{\mathbf{S}}$$

Using Stokes Theorem $\oint_l \underline{\mathbf{E}} \bullet \mathrm{d}\underline{\mathbf{l}} = \int_S \nabla \times \underline{\mathbf{E}} \bullet \mathrm{d}\underline{\mathbf{S}}$

$$\int_S \nabla \times \underline{\mathbf{E}} \bullet \mathrm{d}\underline{\mathbf{S}} = -\int_S \frac{\partial \underline{\mathbf{B}}}{\partial t} \bullet d\underline{\mathbf{S}}$$

$$\therefore \ \nabla \times \underline{\mathbf{E}} = -\frac{\partial \underline{\mathbf{B}}}{\partial t}$$

CONSTITUTIVE RELATIONS

Ohms Law $V = IR$ $R = \frac{\rho_R l}{A}$ $\rho_R =$ Resistivity [Ω m]

$\sigma_C = \frac{1}{\rho_R}$ $\sigma_C =$ Conductivity [Ω^{-1} m^{-1}]

$I = \frac{V}{R} = V \frac{A}{\rho_R l} = \frac{V}{l}\sigma_C A$, re-arrange and we get $\frac{I}{A} = J = \sigma_C E$

Or in vector form (Homogeneous, isotropic media) $\underline{\mathbf{J}} = \sigma_C \underline{\mathbf{E}}$

So we now have:

$\underline{\boldsymbol{D}} = \varepsilon_0 \underline{\boldsymbol{E}} + \underline{\boldsymbol{P}}$ $\underline{\boldsymbol{D}} = \varepsilon_0 \varepsilon_r \underline{\boldsymbol{E}}$

$\underline{\mathbf{B}} = \mu_0 (\underline{\mathbf{H}} + \underline{\mathbf{M}})$ $\underline{\mathbf{B}} = \mu_0 \mu_r \underline{\mathbf{H}}$

$\underline{\mathbf{J}} = \sigma_C \underline{\mathbf{E}}$

BOUNDARY CONDITIONS IN MAGNETISM

We will consider boundaries between linear, isotropic and homogeneous media.

☺ THE TANGENTIAL COMPONENT OF **H** IS CONTINUOUS ACROSS A BOUNDARY PROVIDED THAT THERE IS NO SURFACE CURRENT ON THE BOUNDARY.

☺ THE NORMAL COMPONENT OF **B** IS CONTINUOUS ACROSS A BOUNDARY.

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ANY SOLUTION TO AN MAGNETOSTATICS PROBLEM MUST SATISFY THE BOUNDARY CONDITIONS.
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BOUNDARY CONDITIONS IN ELECTROSTATICS

☺ THE NORMAL COMPONENT OF ***D*** IS CONTINUOUS ACROSS A BOUNDARY PROVIDED THAT NO FREE CHARGE IS PRESENT ON THE BOUNDARY.

☺ THE TANGENTIAL COMPONENT OF ***E*** IS CONTINUOUS ACROSS A BOUNDARY.

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ANY SOLUTION TO AN ELECTROSTATICS PROBLEM MUST SATISFY THE BOUNDARY CONDITIONS.
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POWER DISSIPATION AND JOULE HEATING

Power is dissipated in the resistance *R* causing "Joule Heating".

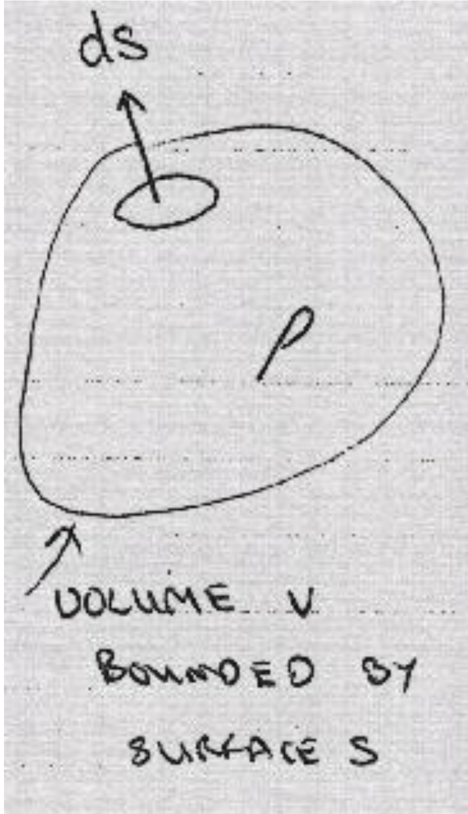
$$W = IV = \frac{V^2}{R} = I^2 R$$

$$W = J^2 A^2 \frac{l}{\sigma_C A} = \frac{\sigma_C E J}{\sigma_C} Al = EJ \times [Volume]$$

$$W = \int_v \underline{\mathbf{J}} \bullet \underline{\mathbf{E}} \, \mathrm{dv}$$

[Now works if $\underline{\mathbf{E}}$ and $\underline{\mathbf{J}}$ in different directions and/or vary with position]

THE EQUATION OF CONTINUITY



Imagine a volume of space v that at a given time contains a total charge Q, where

$$Q = \int_{\mathrm{v}} \rho \, \mathrm{dv}$$

If charge can flow out (or into) the volume then there is a current.

$$I = -\frac{\partial Q}{\partial t} = -\int_{\mathrm{v}} \frac{\partial \rho}{\partial t} \mathrm{dv} \qquad \text{but} \qquad I = \int_{\mathrm{S}} \underline{\mathbf{J}} \bullet \mathrm{d}\underline{\mathbf{S}}$$

[Think about the sign; charge decreasing implies current flowing out of surface and note the surface is closed]

Gauss' Theorem states $\int_{\mathrm{v}} \nabla \bullet \underline{\mathbf{J}} \, \mathrm{dv} = \oint_S \underline{\mathbf{J}} \bullet \mathrm{d}\underline{\mathbf{S}}$

So that $\int_{\mathrm{v}} \nabla \bullet \underline{\mathbf{J}} \, \mathrm{dv} = -\int_{\mathrm{v}} \frac{\partial \rho}{\partial t} \mathrm{dv}$ or $\nabla \bullet \underline{\mathbf{J}} = -\frac{\partial \rho}{\partial t}$

DISPLACEMENT CURRENT

In magnetostatics we found that $\oint_l \underline{\mathbf{H}} \bullet \mathrm{d}\underline{\mathbf{l}} = I$ and hence $\nabla \times \underline{\mathbf{H}} = \underline{\mathbf{J}}$

But $\nabla \bullet \nabla \times \underline{\mathbf{H}} = 0$ always (!) and $\nabla \bullet \underline{\mathbf{J}} \neq 0$ always!

$\nabla \bullet \underline{\mathbf{J}} = 0$ only when $\dfrac{\partial \rho}{\partial t} = 0$ i.e. STATICS

RESOLUTION OF THE PROBLEM

$$\nabla \bullet \underline{\mathbf{D}} = \rho \qquad \Rightarrow \qquad \nabla \bullet \frac{\partial \underline{\mathbf{D}}}{\partial t} = \frac{\partial \rho}{\partial t}$$

As $\nabla \bullet \underline{\mathbf{J}} = -\dfrac{\partial \rho}{\partial t} \qquad \Rightarrow \qquad \nabla \bullet \underline{\mathbf{J}} = -\nabla \bullet \dfrac{\partial \underline{\mathbf{D}}}{\partial t}$ or $\nabla \bullet \left(\underline{\mathbf{J}} + \dfrac{\partial \underline{\mathbf{D}}}{\partial t} \right) = 0$

Now we can see how we may amend Amperes Law $\nabla \times \underline{\mathbf{H}} = \underline{\mathbf{J}} + \dfrac{\partial \underline{\mathbf{D}}}{\partial t}$

$I = \int_S \underline{\mathbf{J}} \bullet \mathrm{d}\underline{\mathbf{S}}$ Conduction Current

$I = \int_S \dfrac{\partial \underline{\mathbf{D}}}{\partial t} \bullet \mathrm{d}\underline{\mathbf{S}}$ Displacement Current (not a real current)

AMPERE-MAXWELL LAW IN A DIELECTRIC WITH A FINTE CONDUCTIVITY

$$\nabla \times \underline{\mathbf{H}} = \underline{\mathbf{J}} + \frac{\partial \underline{\mathbf{D}}}{\partial t} \qquad \underline{\mathbf{J}} = \sigma_C \underline{\mathbf{E}} \qquad \underline{\mathbf{D}} = \varepsilon_0 \underline{\mathbf{E}} + \underline{\mathbf{P}}$$

$$\nabla \times \underline{\mathbf{H}} = \sigma_C \underline{\mathbf{E}} + \frac{\partial \underline{\mathbf{P}}}{\partial t} + \varepsilon_0 \frac{\partial \underline{\mathbf{E}}}{\partial t}$$

Conduction current
(Motion of free charges
through the medium)

Not related to a motion of
any sort of charge

Motion of the bound polarisation charges
in the vicinity of its nucleus.

In fact we have found that for time varying fields in vacuum ($\sigma_C = 0$, $\underline{\mathbf{P}} = 0$)

$$\nabla \times \underline{\mathbf{H}} = \varepsilon_0 \frac{\partial \underline{\mathbf{E}}}{\partial t}$$

We see a fundamental difference between dynamic and static electrical and magnetic fields.

STATICS:

$\underline{\underline{\mathbf{E}}}$ and $\underline{\underline{\mathbf{H}}}$ are completely independent of each other.

DYNAMICS (examples in vacuum):

When $\frac{\partial \underline{\underline{\mathbf{E}}}}{\partial t}$ is finite must also have a $\underline{\underline{\mathbf{H}}}$ field where $\nabla \times \underline{\underline{\mathbf{E}}} = -\mu_0 \frac{\partial \underline{\underline{\mathbf{H}}}}{\partial t}$

or when $\frac{\partial \underline{\underline{\mathbf{H}}}}{\partial t}$ is finite must also have a $\underline{\underline{\mathbf{E}}}$ field where $\nabla \times \underline{\underline{\mathbf{H}}} = \varepsilon_0 \frac{\partial \underline{\underline{\mathbf{E}}}}{\partial t}$

In dynamics $\underline{\underline{\mathbf{E}}}$ and $\underline{\underline{\mathbf{H}}}$ are coupled (cannot have one without the other).

MAXWELLS EQUATIONS

Gauss' Law in electricity and magnetism

$$\nabla \bullet \underline{\mathbf{D}} = \rho \qquad [M1]$$

$$\nabla \bullet \underline{\mathbf{B}} = 0 \qquad [M2]$$

Ampere-Maxwell Law

$$\nabla \times \underline{\mathbf{H}} = \underline{\mathbf{J}} + \frac{\partial \underline{\mathbf{D}}}{\partial t} \qquad [M3]$$

Faraday Law

$$\nabla \times \underline{\mathbf{E}} = -\frac{\partial \underline{\mathbf{B}}}{\partial t} \qquad [M4]$$

LINEAR AND ISOTROPIC MEDIA

$\underline{\mathbf{D}} = \varepsilon_0 \varepsilon_r \underline{\mathbf{E}}$ $\quad \underline{\mathbf{B}} = \mu_0 \mu_r \underline{\mathbf{H}}$ $\quad \underline{\mathbf{J}} = \sigma_C \underline{\mathbf{E}}$

$$\nabla \bullet \varepsilon_r \underline{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \bullet \mu_r \underline{\mathbf{H}} = 0$$

$$\nabla \times \underline{\mathbf{H}} = \sigma_C \underline{\mathbf{E}} + \varepsilon_0 \varepsilon_r \frac{\partial \underline{\mathbf{E}}}{\partial t}$$

$$\nabla \times \underline{\mathbf{E}} = -\mu_0 \mu_r \frac{\partial \underline{\mathbf{H}}}{\partial t}$$

LINEAR, ISOTROPIC AND HOMOGENEOUS MEDIA

ε_r and μ_r independent of position

$$\nabla \bullet \underline{\mathbf{E}} = \frac{\rho}{\varepsilon_r \varepsilon_0}$$

$$\nabla \bullet \underline{\mathbf{H}} = 0$$

$$\nabla \times \underline{\mathbf{H}} = \sigma_C \underline{\mathbf{E}} + \varepsilon_0 \varepsilon_r \frac{\partial \underline{\mathbf{E}}}{\partial t}$$

$$\nabla \times \underline{\mathbf{E}} = -\mu_0 \mu_r \frac{\partial \underline{\mathbf{H}}}{\partial t}$$

GENERAL WAVE EQUATION

Consider a medium in which $\rho = 0$, and that is LINEAR, ISOTROPIC and HOMOGENEOUS (ε_r and μ_r constants, independent of position)

$$\underline{\mathbf{D}} = \varepsilon_0 \varepsilon_r \underline{\mathbf{E}} \qquad \underline{\mathbf{B}} = \mu_0 \mu_r \underline{\mathbf{H}} \qquad \underline{\mathbf{J}} = \sigma_C \underline{\mathbf{E}}$$

$$\nabla \bullet \underline{\mathbf{E}} = 0 \qquad \nabla \bullet \underline{\mathbf{H}} = 0$$

$$\nabla \times \underline{\mathbf{H}} = \sigma_C \underline{\mathbf{E}} + \varepsilon_0 \varepsilon_r \frac{\partial \underline{\mathbf{E}}}{\partial t} \qquad \nabla \times \underline{\mathbf{E}} = -\mu_0 \mu_r \frac{\partial \underline{\mathbf{H}}}{\partial t}$$

Starting with $\nabla \times \underline{\mathbf{H}} = \sigma_C \underline{\mathbf{E}} + \varepsilon_0 \varepsilon_r \frac{\partial \underline{\mathbf{E}}}{\partial t}$

Take the curl of both sides

$$\nabla \times \nabla \times \underline{\mathbf{H}} = \sigma_C \nabla \times \underline{\mathbf{E}} + \varepsilon_0 \varepsilon_r \frac{\partial (\nabla \times \underline{\mathbf{E}})}{\partial t}$$

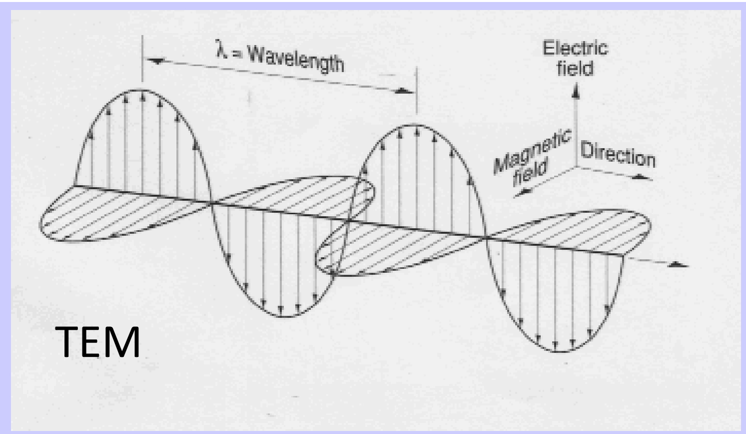
Using $\nabla \times \nabla \times \underline{\mathbf{F}} = \nabla \nabla \bullet \underline{\mathbf{F}} - \nabla^2 \underline{\mathbf{F}}$ and $\nabla \times \underline{\mathbf{E}} = -\mu_0 \mu_r \frac{\partial \underline{\mathbf{H}}}{\partial t}$

$$\nabla \nabla \bullet \underline{\mathbf{H}} - \nabla^2 \underline{\mathbf{H}} = -\sigma_C \mu_0 \mu_r \frac{\partial \underline{\mathbf{H}}}{\partial t} - \varepsilon_0 \varepsilon_r \mu_0 \mu_r \frac{\partial^2 \underline{\mathbf{H}}}{\partial t^2}$$

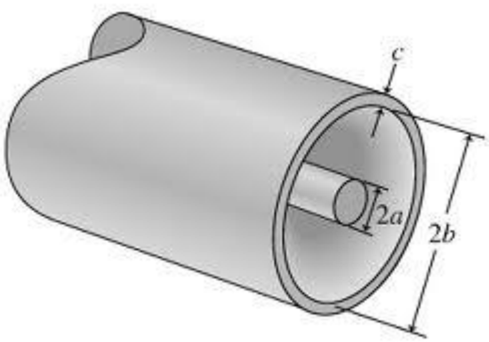
Since $\nabla \bullet \underline{\mathbf{H}} = 0$

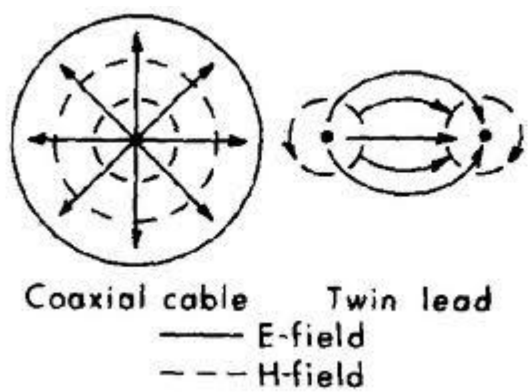
$$\nabla^2 \underline{\mathbf{H}} = \sigma_C \mu_0 \mu_r \frac{\partial \underline{\mathbf{H}}}{\partial t} + \varepsilon_0 \varepsilon_r \mu_0 \mu_r \frac{\partial^2 \underline{\mathbf{H}}}{\partial t^2}$$

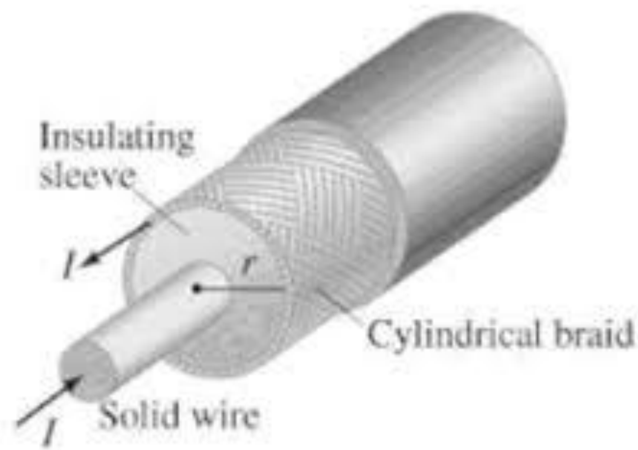
Guided EM Waves



$$\nabla^2 \underline{\mathbf{E}} = \mu\sigma_C \frac{\partial \underline{\mathbf{E}}}{\partial t} + \mu\varepsilon \frac{\partial^2 \underline{\mathbf{E}}}{\partial t^2}$$







Hollow Waveguides Cannot Support TEM modes

Rectangular hollow waveguide with field patterns for a propagating TE_{10} mode. Solid lines indicate electric field and broken lines magnetic field; ×'s indicate energy E pointing down; solid circles indicate E pointing up. (After MIT Radar School Staff, Principles of Radar, 1952)

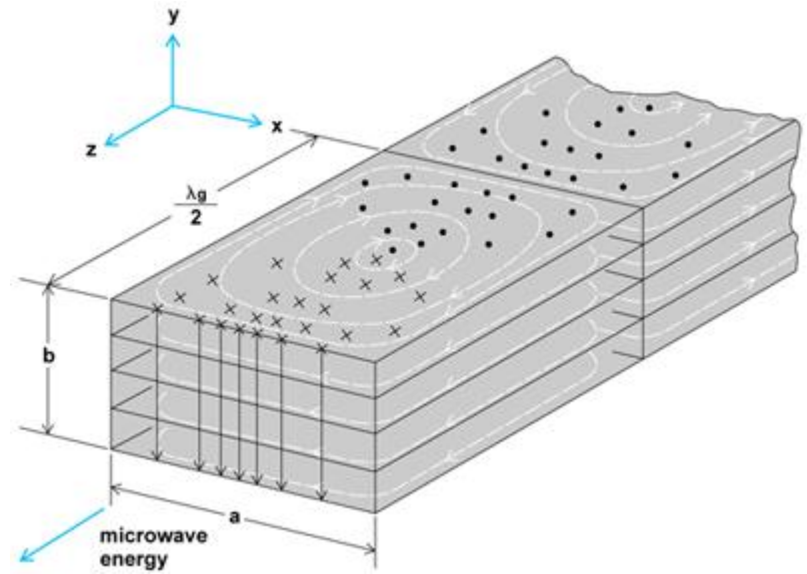
$$H_x = \left(\frac{\gamma k_x}{k_c^2}\right) H_0 \sin(k_x x)\cos(k_y y)$$

$$H_y = \left(\frac{\gamma k_y}{k_c^2}\right) H_0 \cos(k_x x)\sin(k_y y)$$

$$H_z = H_0 \cos(k_x x)\cos(k_y y)$$

$$E_x = Z_{TE} H_y$$

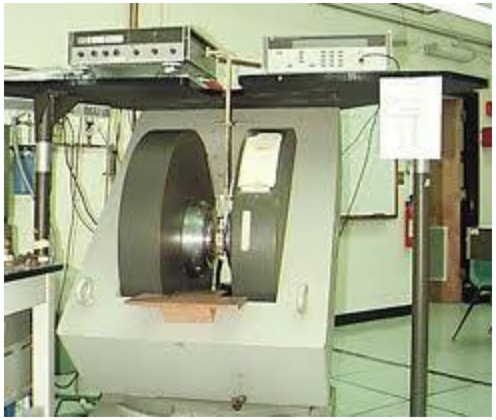
$$E_y = -Z_{TE} H_x$$



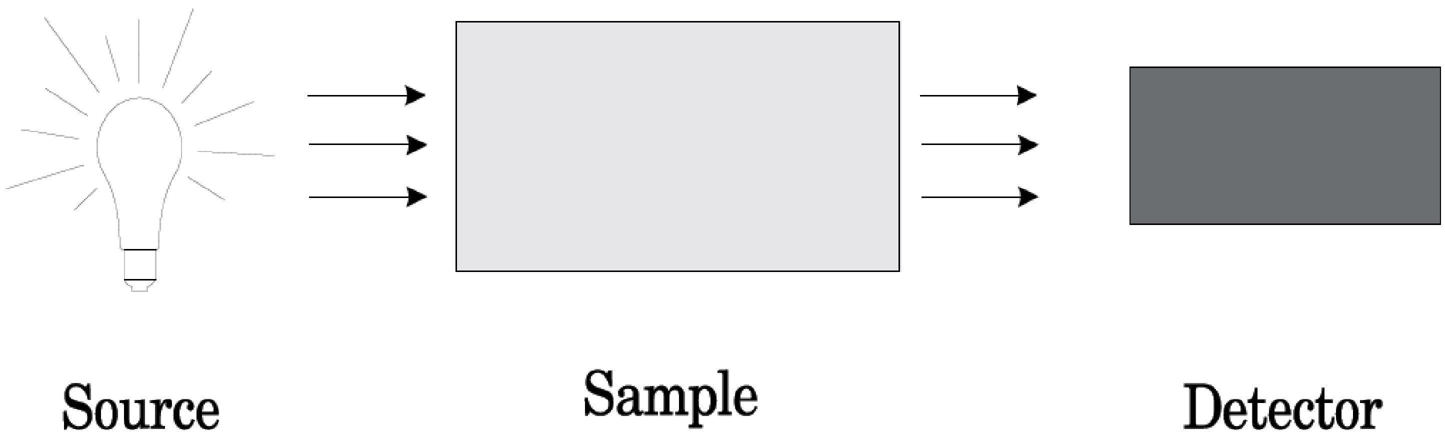
$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} \qquad \gamma = \sqrt{\left(k_c^2 - k^2\right)} \qquad k = \frac{2\pi}{\lambda_{\text{Free Space}}} \qquad Z_{TE} = \left(\frac{\mu}{\varepsilon}\right)\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$k_x = \frac{m\pi}{a}; \quad k_y = \frac{n\pi}{b}; \quad k_c = \frac{2\pi}{\lambda_c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad f_c = c\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}$$

Magnetic Resonance Spectrometers



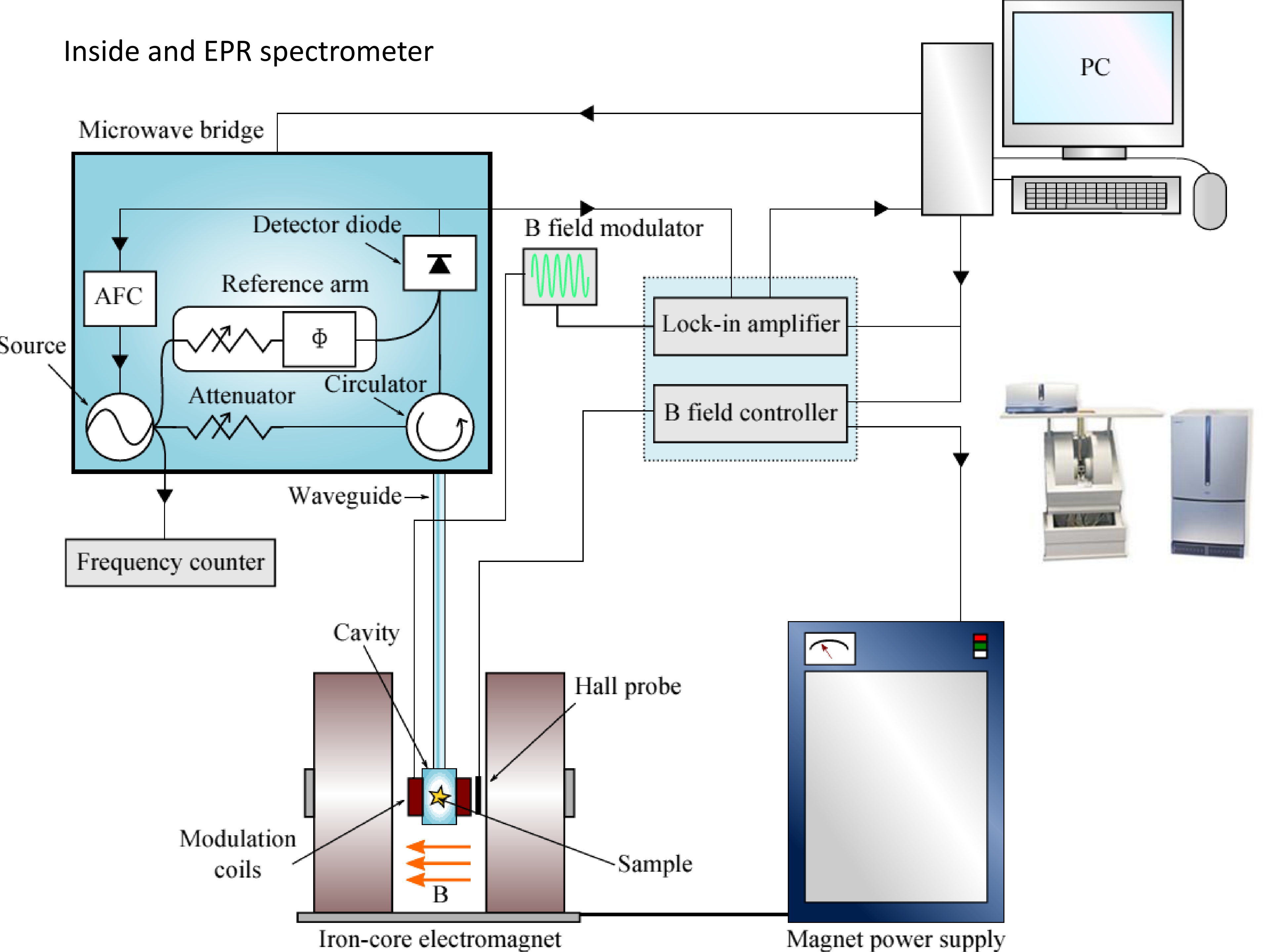
Why do we use resonators?



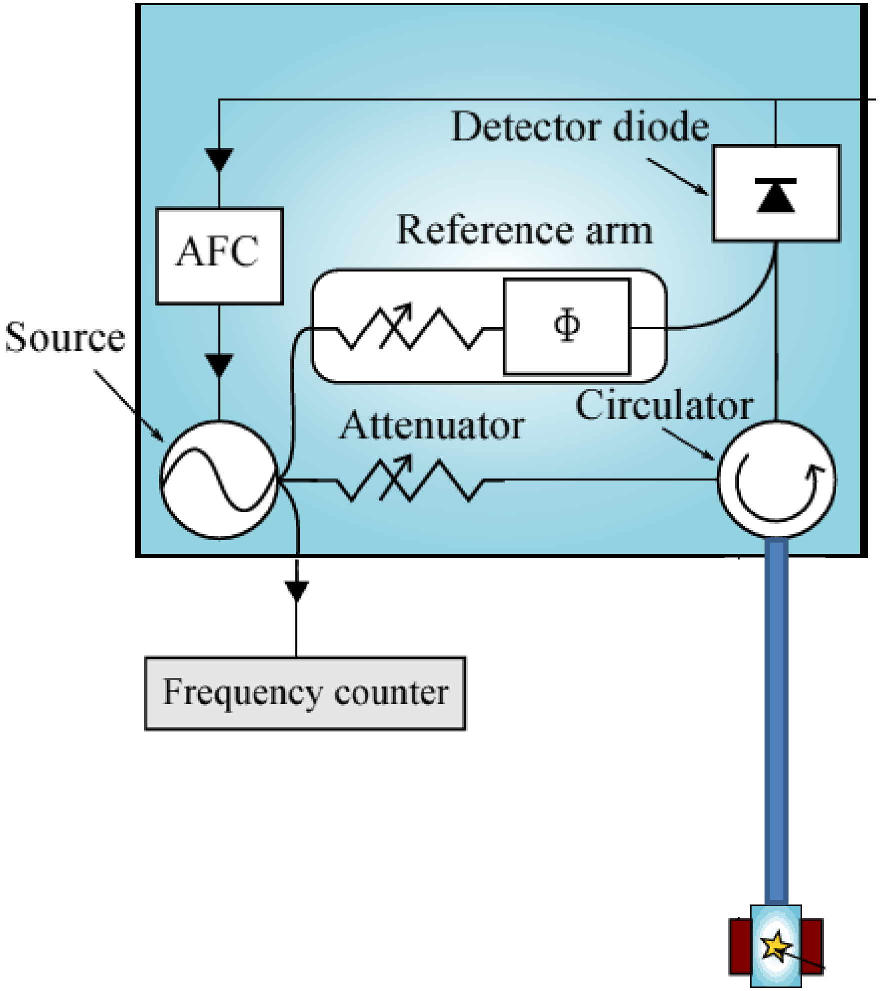
A simple Optical Absorption Spectrometer

Cavity, Loop Gap, Dielectric, Coil……

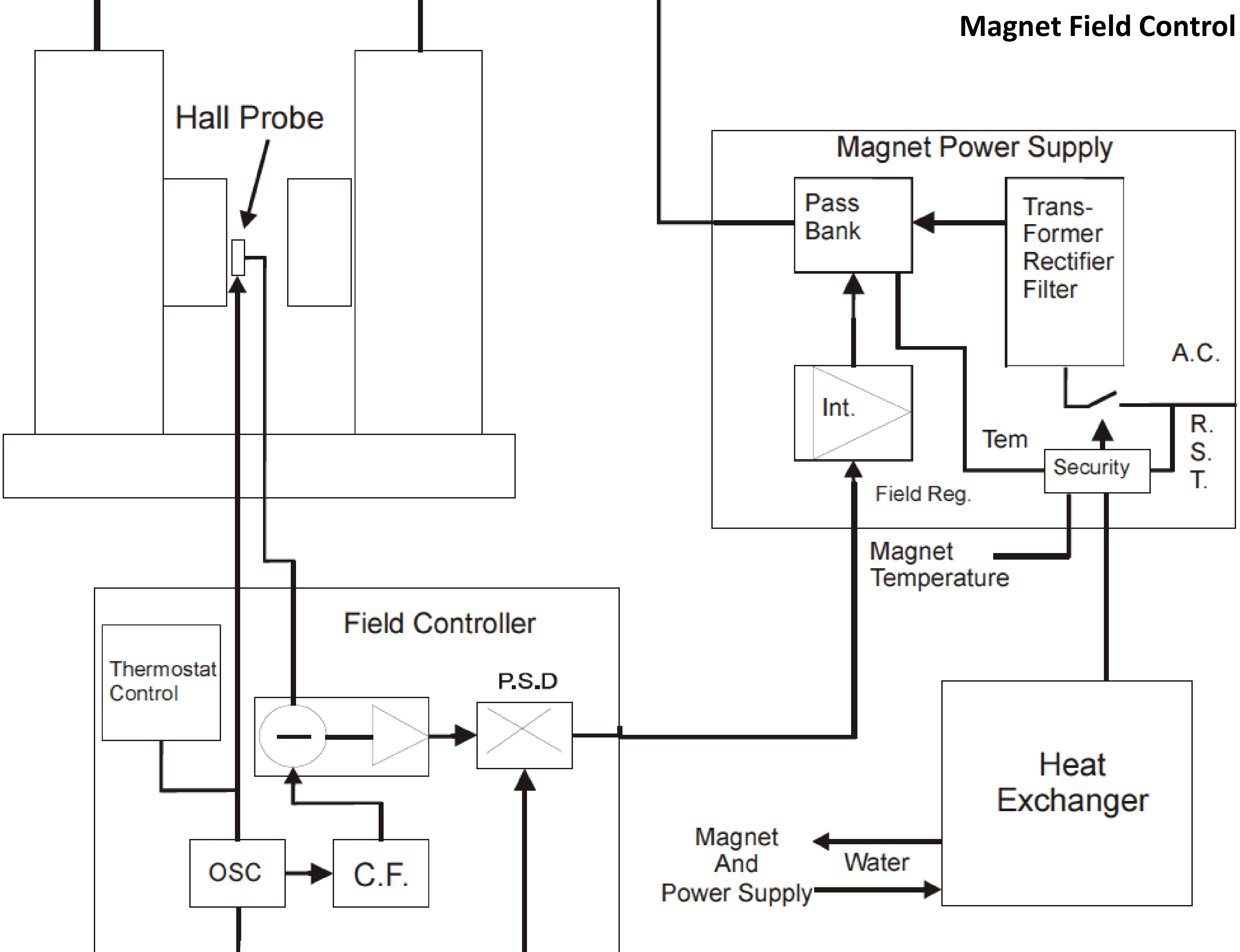
Inside and EPR spectrometer



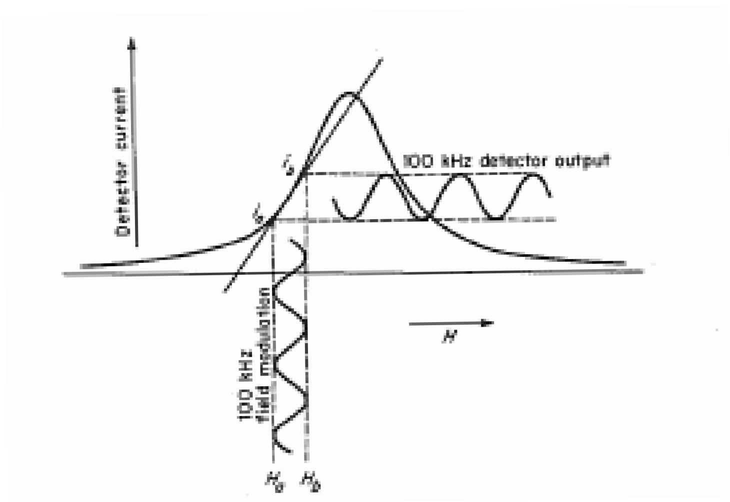
Automatic Frequency Control

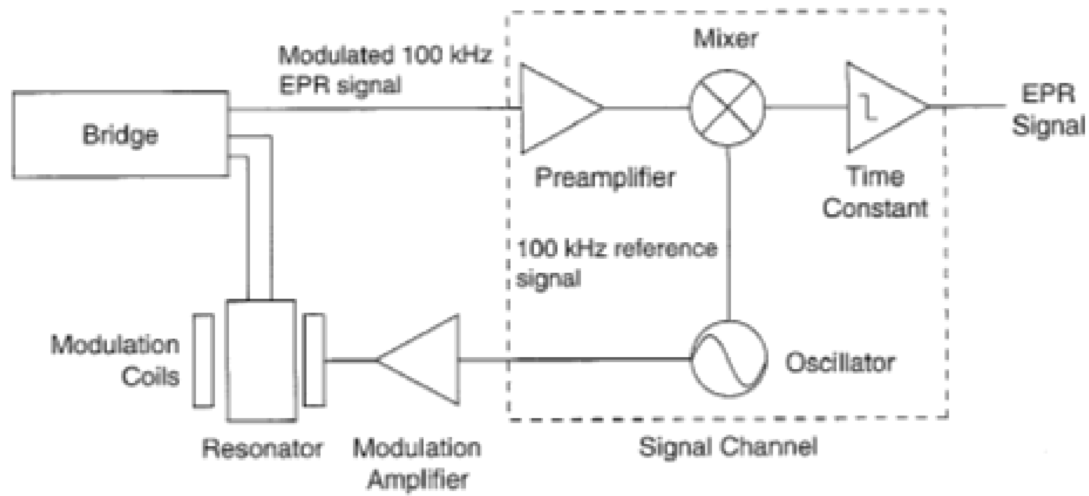


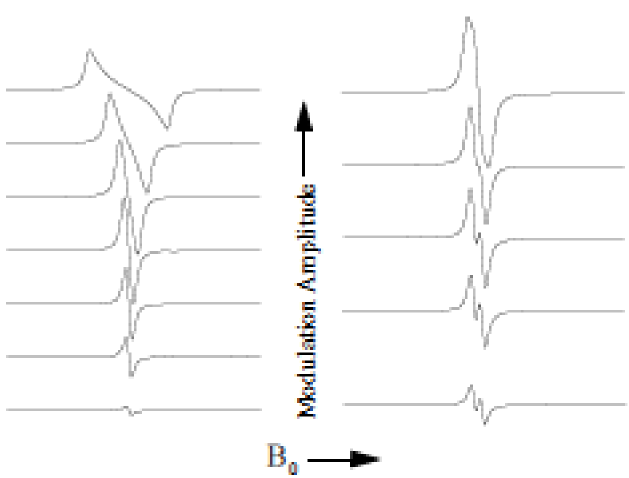
Magnet Field Control



Field Modulation







$$EPR_{Signal} = \chi'' \eta Q_L \sqrt{Z_0 P}$$

χ'' Imaginary part of paramagnetic susceptibility

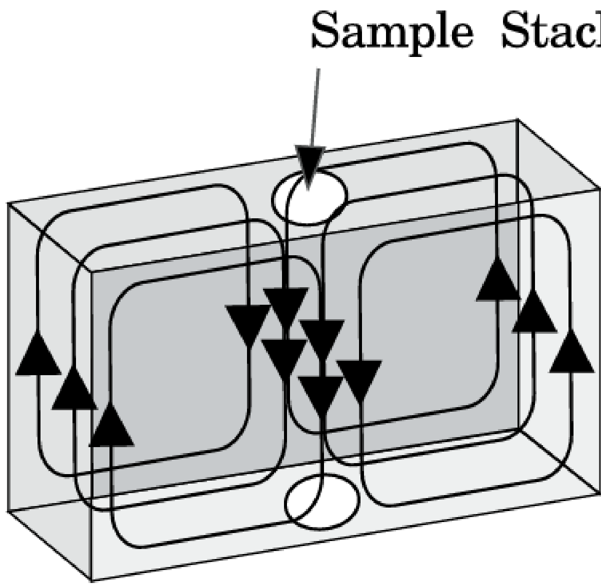
η Filling Factor

$$\eta = \frac{\oint_{\text{Sample Volume}} B_1^2 \sin^2\theta \, dV}{\oint_{\text{Resonator Volume}} B_1^2 \, dV}$$

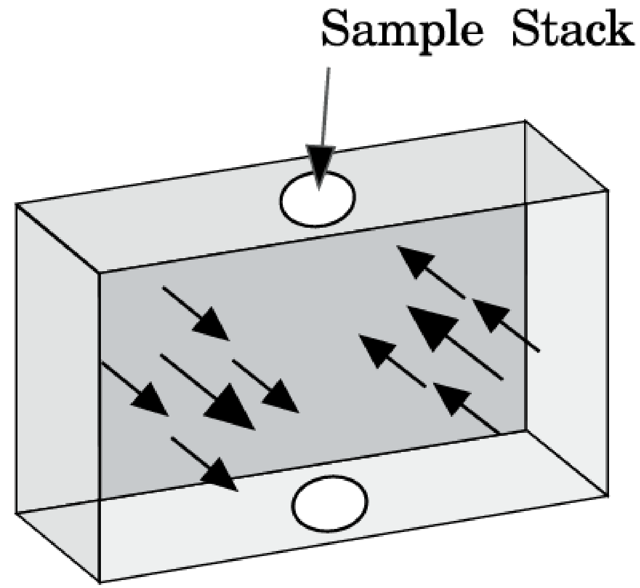
Q_L Loaded cavity Q

Z_0 Impedance of transmission line

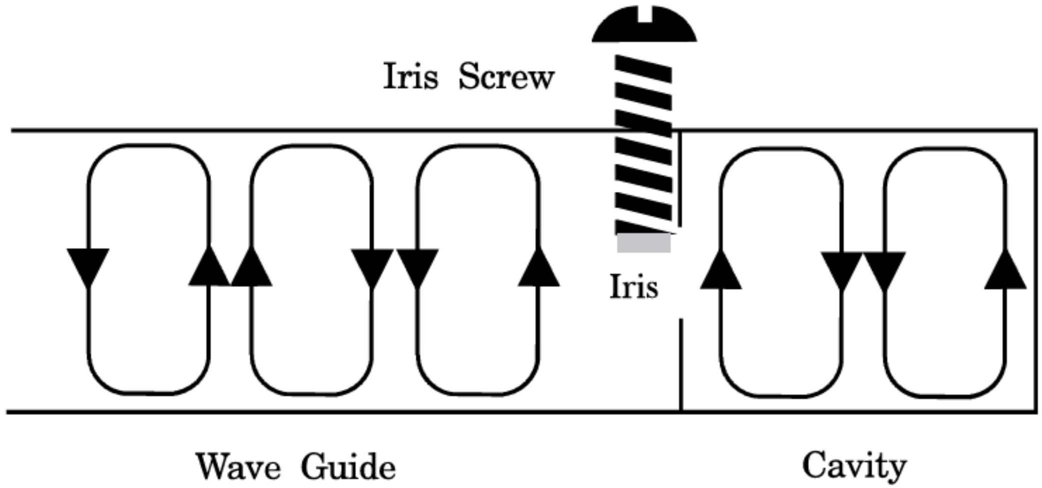
P Microwave power delivered to resonator



Microwave Magnetic Field



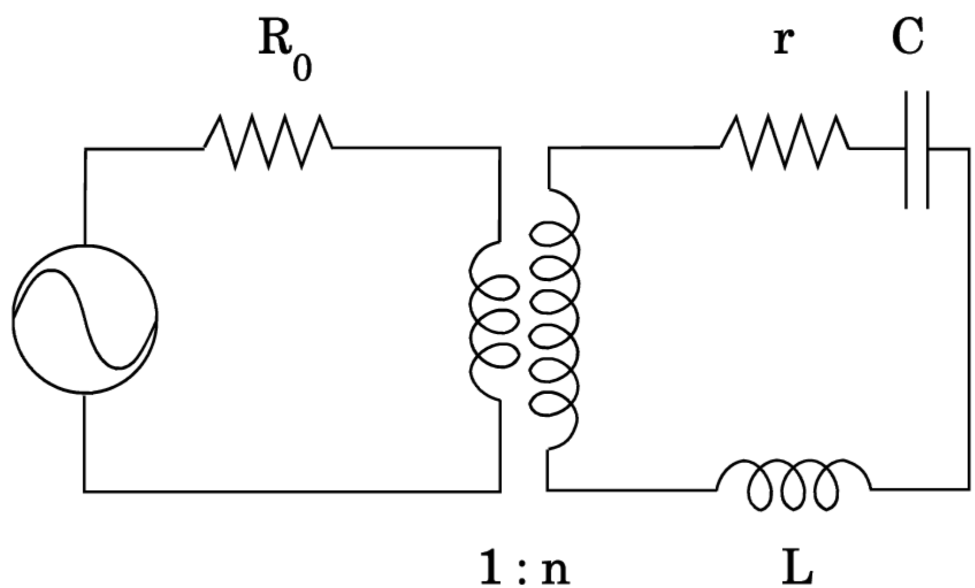
Microwave Electric Field



$$Q_u = \frac{\omega L}{r}$$

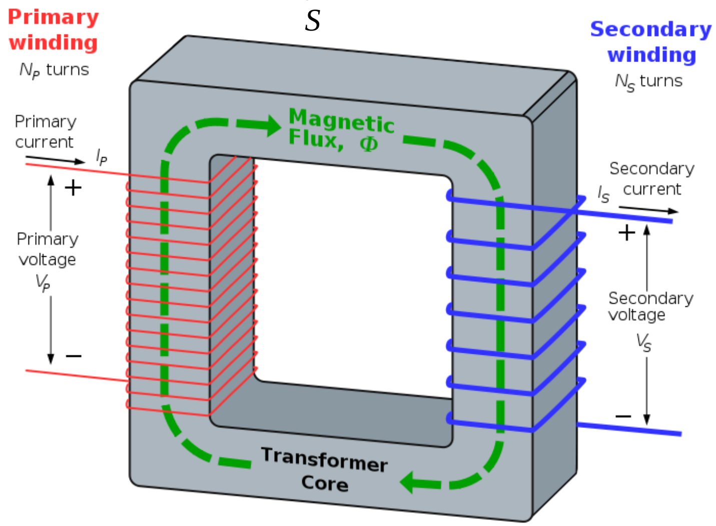
$$Q_L = \frac{\omega L}{R_0 n^2 + r}$$

$$\beta = \frac{R_0 n^2}{r}$$



The transformer is based on two principles: first, that an electric current can produce a magnetic field (electromagnetism), and, second that a changing magnetic field within a coil of wire induces a voltage across the ends of the coil (electromagnetic induction). Changing the current in the primary coil changes the magnetic flux that is developed. The changing magnetic flux induces a voltage in the secondary coil.

$$\Phi = \int_S \underline{B} \bullet d\,\underline{S}$$



Induction law

The voltage induced across the secondary coil may be calculated from Faraday's law of induction, which states that:

$$V_S = -N_S \frac{d\Phi}{dt}$$

Since the same magnetic flux passes through both the primary and secondary coils in an ideal transformer, the instantaneous voltage across the primary winding equals:

$$V_P = -N_P \frac{d\Phi}{dt}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

If the secondary coil is attached to a load that allows current to flow, electrical power is transmitted from the primary circuit to the secondary circuit. Ideally, the transformer is perfectly efficient. All the incoming energy is transformed from the primary circuit to the magnetic field and into the secondary circuit. If this condition is met, the input electric power must equal the output power:

$$P_{IN} = I_P V_P = P_{OUT} = I_S V_S$$

giving the ideal transformer equation

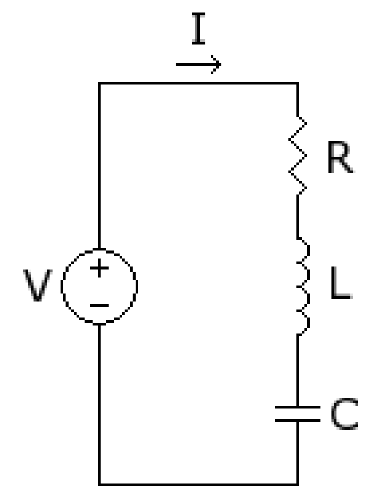
$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

$$v_R + v_L + v_C = V(t)$$

$$Ri + L\frac{di}{dt} + \frac{Q}{C} = V(t)$$

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = \frac{dV}{dt}$$

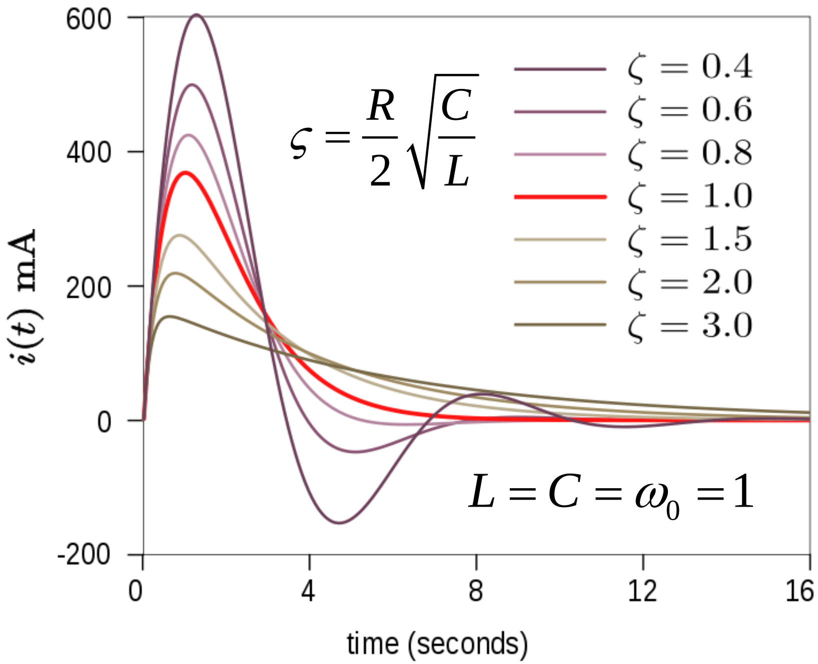
$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{1}{L}\frac{dV}{dt}$$



$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

$$m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$$

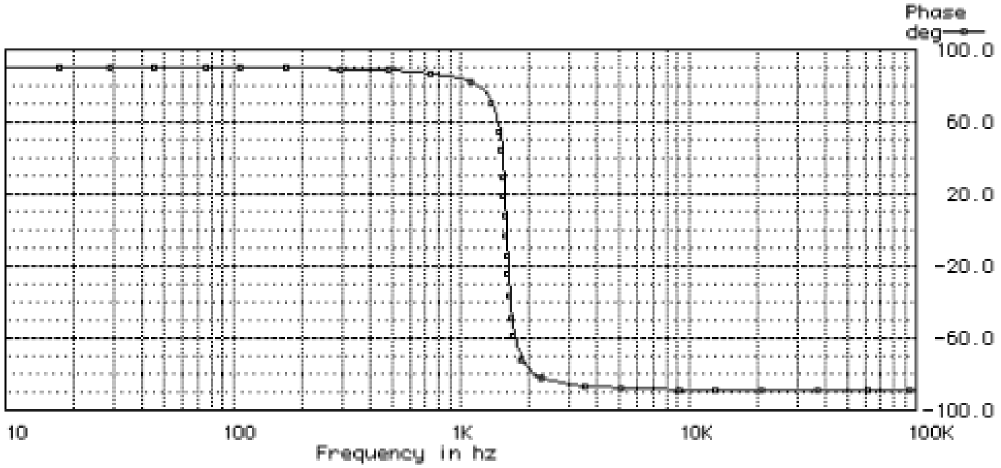
$$i(t) = A_1 \exp(m_1 t) + A_2 \exp(m_2 t)$$



$$Z = R + \frac{1}{j\omega C} + j\omega L$$

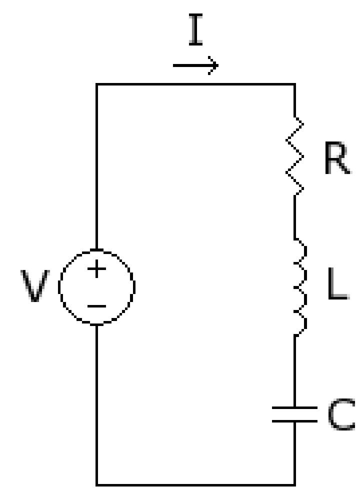
$$V_{IN} = V_0 \cos(\omega t)$$

$$V_R = \frac{V_{IN} R}{R + \frac{1}{j\omega C} + j\omega L}$$



$$V_R = \frac{V_0 R \cos(\omega t - \phi)}{|Z|}$$

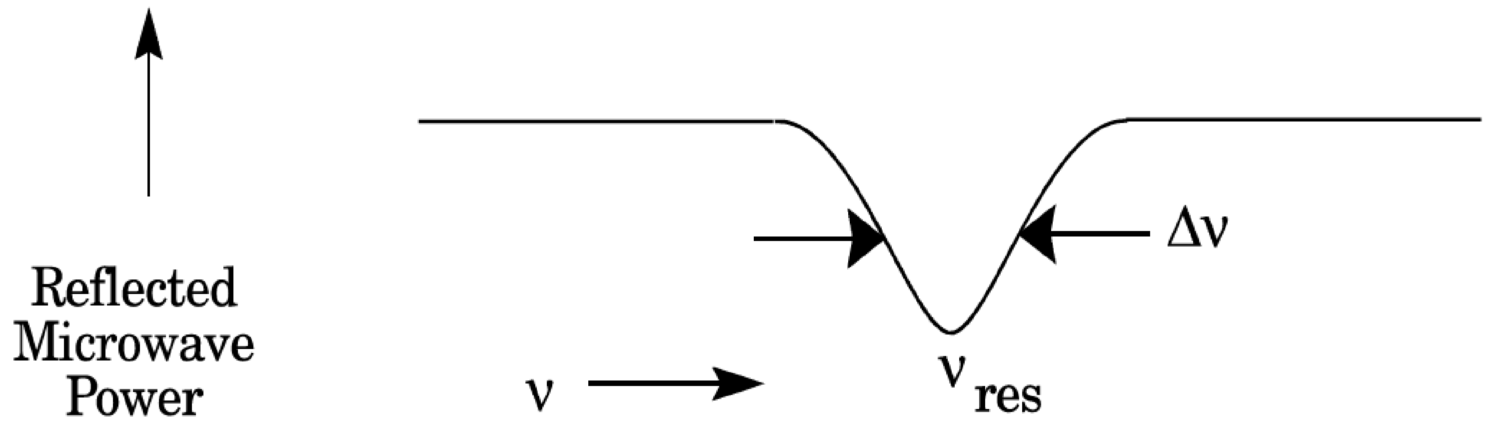
$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} \qquad |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



In general $V_C(t)$, $V_R(t)$, and $V_L(t)$ are all out of phase with the applied voltage.

I(t) and $V_R(t)$ are in phase in a series RLC circuit.

The amplitude of V_C, V_R, and V_L depend on ω.



$$Q = \frac{2\pi(\text{energy stored})}{\text{energy dissipated per cycle}}$$

$$Q = \frac{\nu_{res}}{\Delta\nu}$$

Q (quality factor) of a circuit: Determines how well the RLC circuit stores energy

Q = 2π (max energy stored)/(energy lost) per cycle

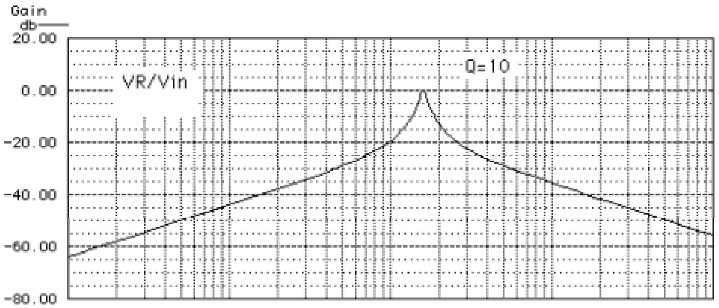
The maximum energy stored in the inductor is $\dfrac{LI_{Max}^2}{2}$

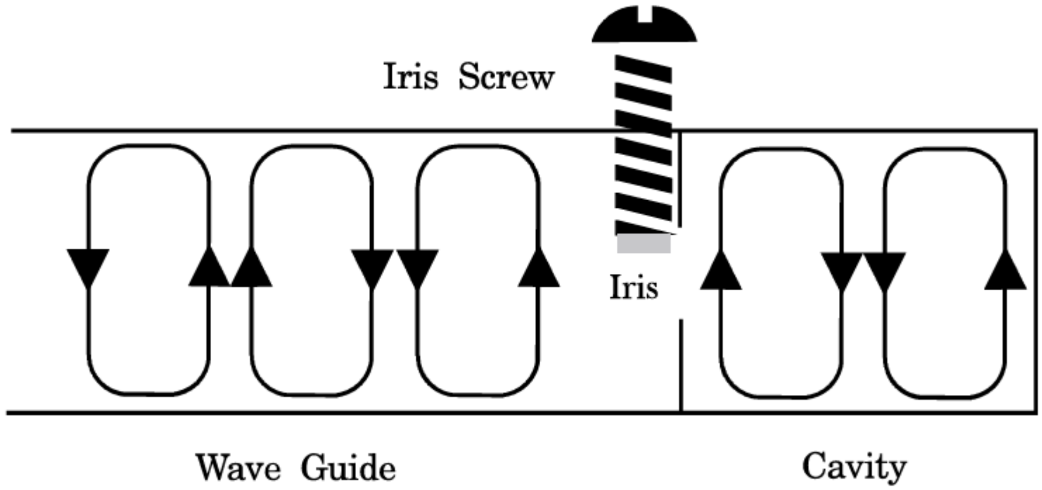
There is no energy stored in the capacitor at this instant because I and V_C are 90° out of phase.

The energy lost in one cycle is (Power)x(time for cycle)

$$= I_{RMS}^2 R \times \frac{2\pi}{\omega_0} = \left(\frac{RI_{Max}^2}{2}\right)\frac{2\pi}{\omega_0}$$

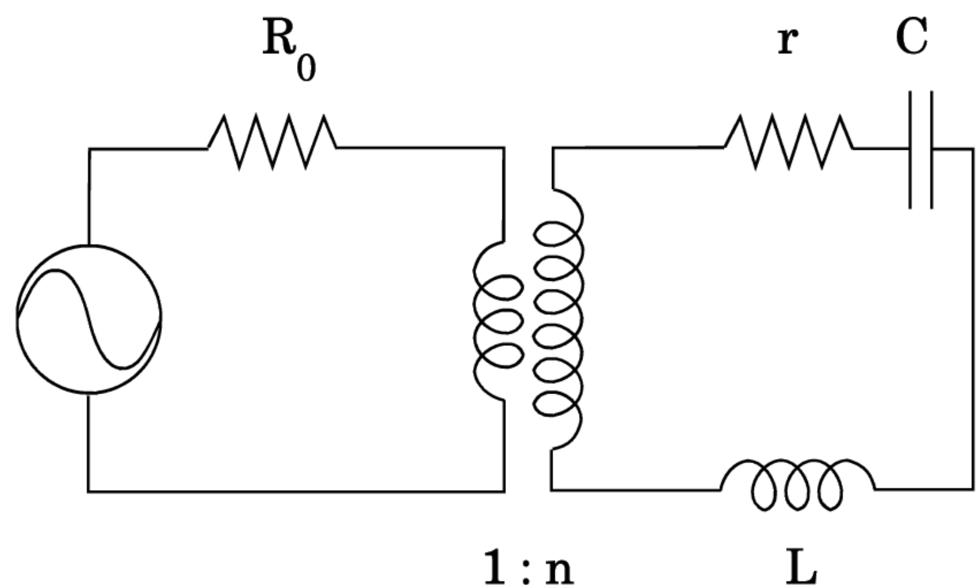
$$Q = \frac{2\pi \dfrac{LI_{Max}^2}{2}}{\dfrac{2\pi}{\omega_0}\left(\dfrac{RI_{Max}^2}{2}\right)} = \frac{\omega_0 L}{R}$$





$$Q_u = \frac{\omega L}{r}$$

$$Q_L = \frac{\omega L}{R_0 n^2 + r}$$



$$\beta = \frac{R_0 n^2}{r}$$

$$\Gamma = \frac{VSWR-1}{VSWR+1}$$

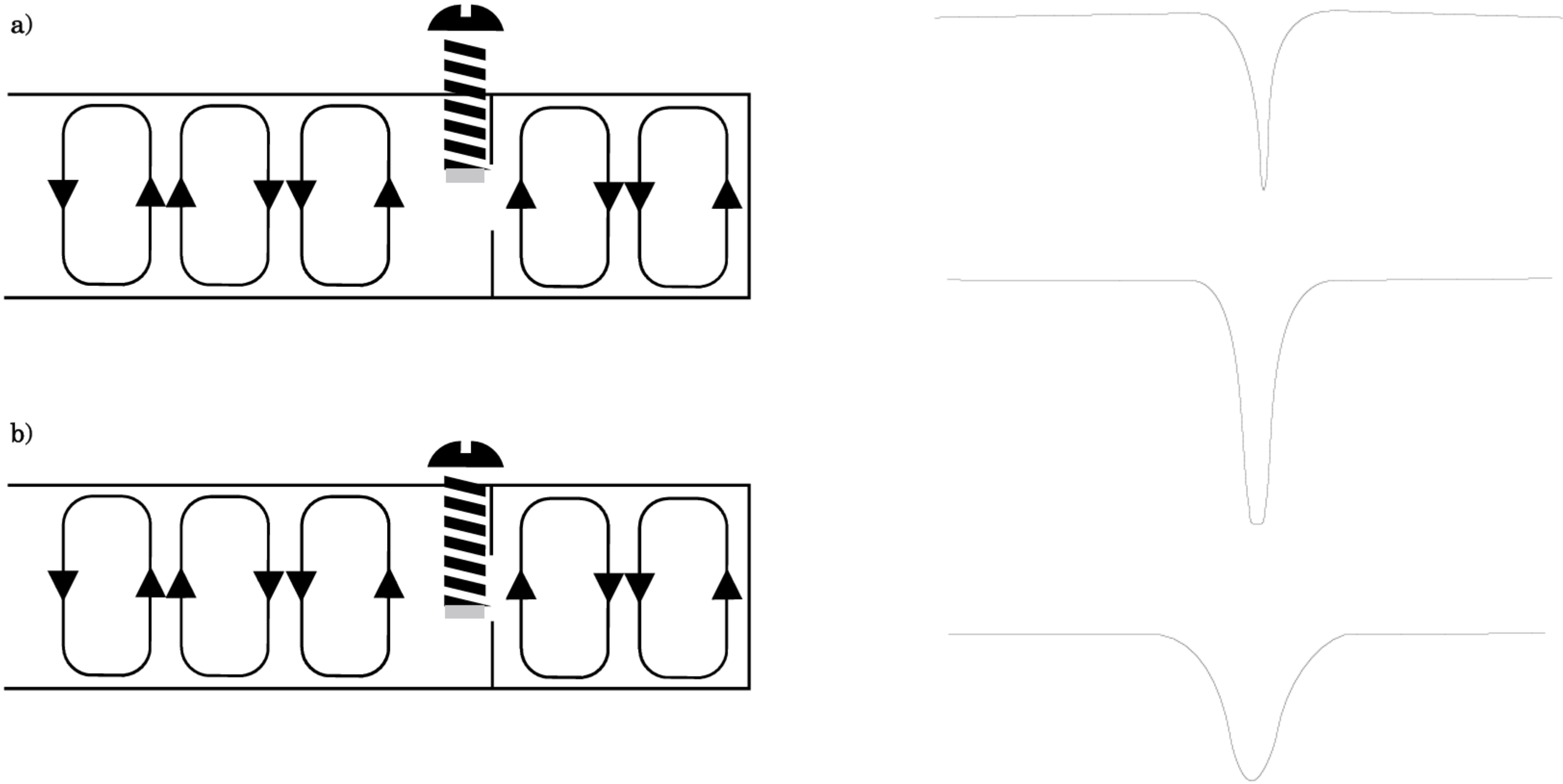
The coupling coefficient conveniently defines three conditions.

β= 1 corresponds to a critically coupled or "matched" resonator. This corresponds to maximum power transfer from the microwave source to the cavity.
Maximum EPR sensitivity is achieved in this condition.

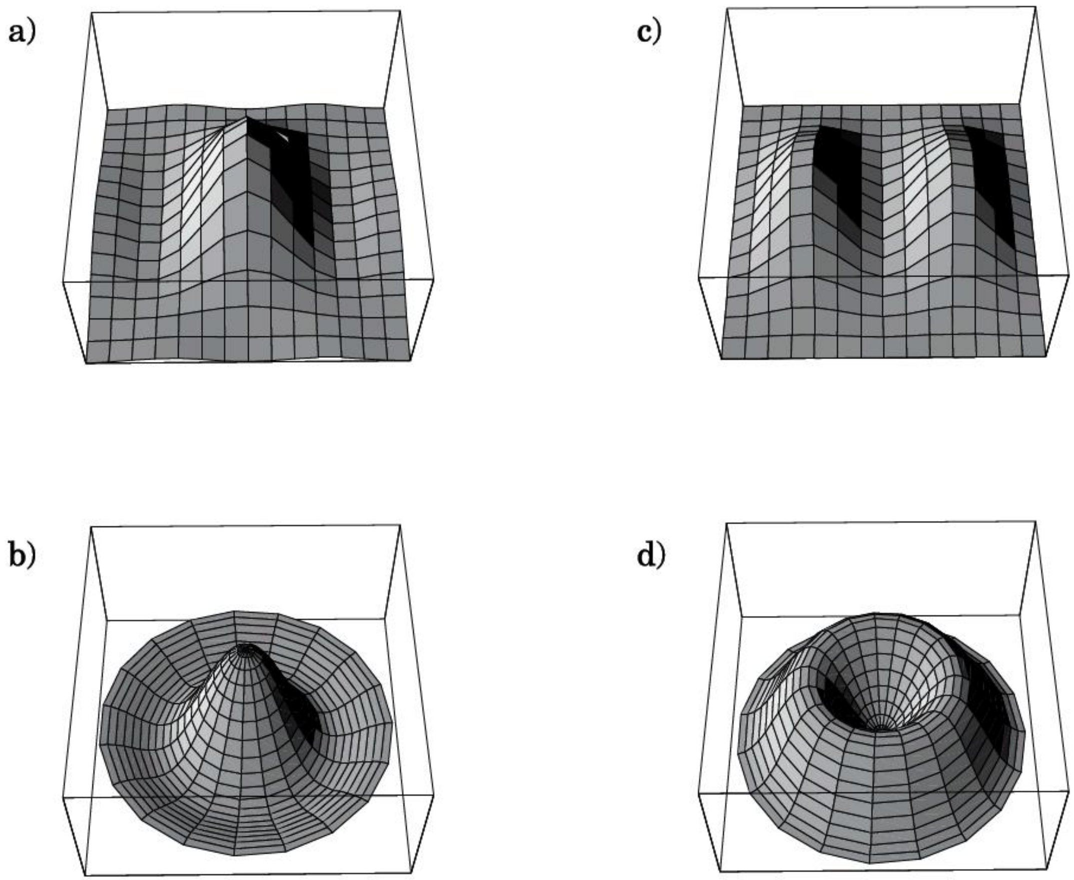
The VSWR (Voltage Standing Wave Ratio) is equal to 1, which corresponds to a reflection coefficient Γ, of zero. This means when we are matched, no microwaves are reflected from the cavity. It also means that the loaded Q is half the unloaded Q.

$\beta < 1$ corresponds to an undercoupled cavity with VSWR = 1/ β and $\Gamma > 0$. This means that microwaves are reflected from the cavity. The Q is somewhat higher than for a matched cavity.

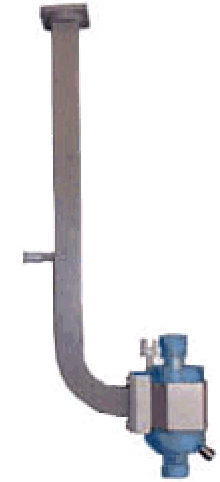
$\beta > 1$ corresponds to an overcoupled cavity with VSWR = β and $\Gamma < 0$. Microwaves are reflected from the cavity with a 180 degree phase shift. The Q is lower than for a matched cavity.

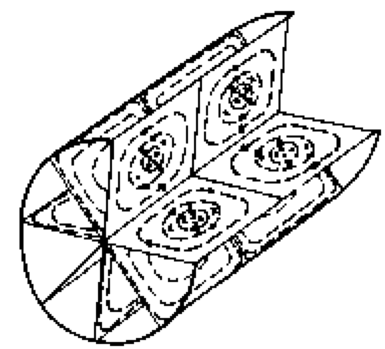

a)
b)

TE_{011} cylindrical cavity

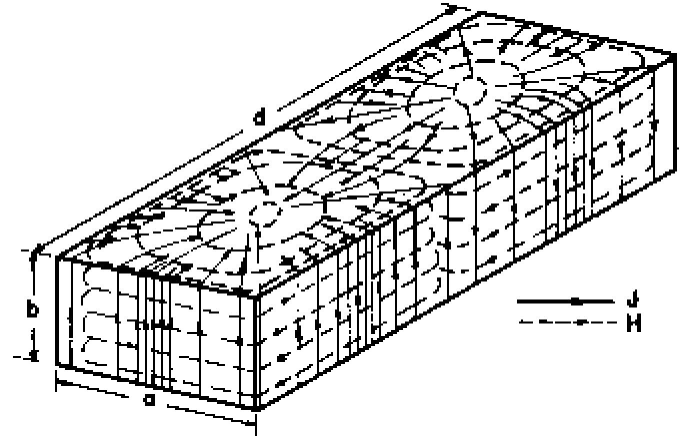


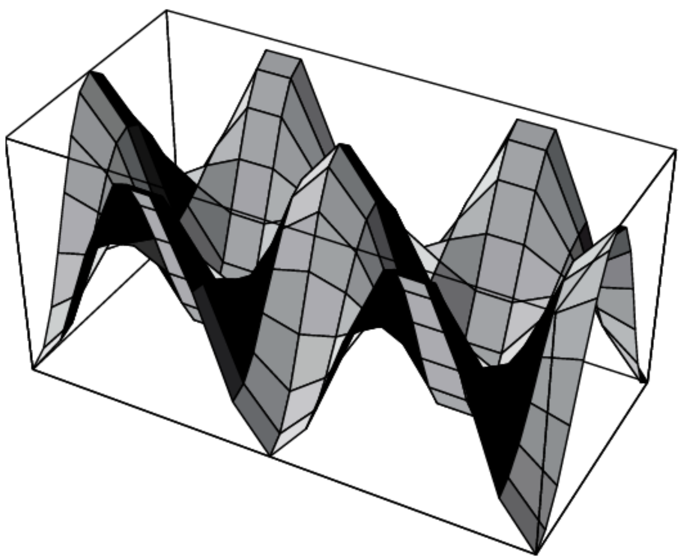
B_1^2 E_1^2



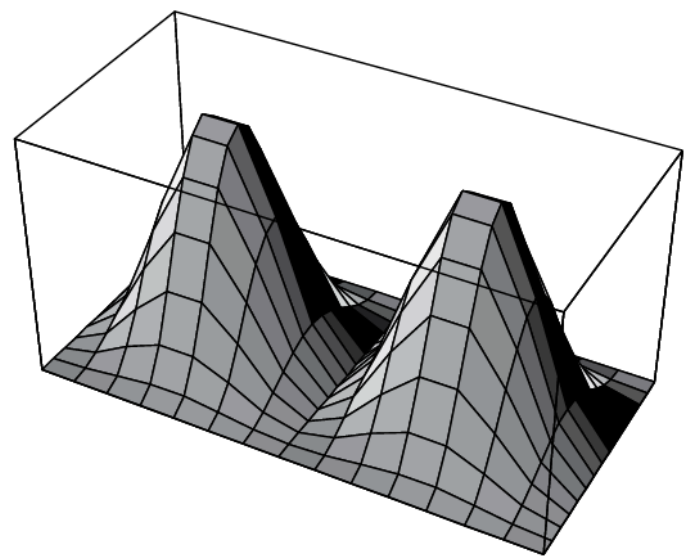


TE_{102} rectangular cavity



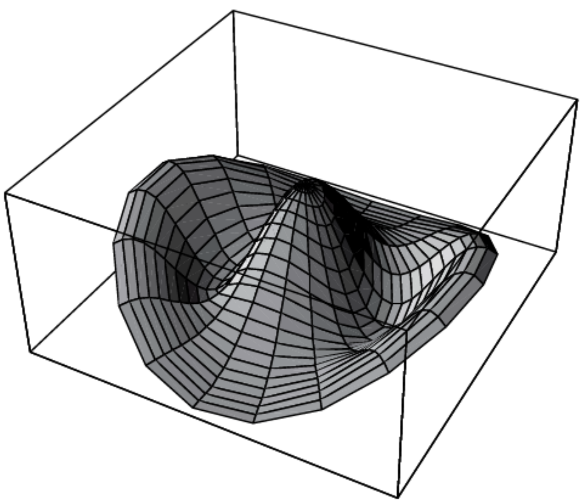


B_1^2

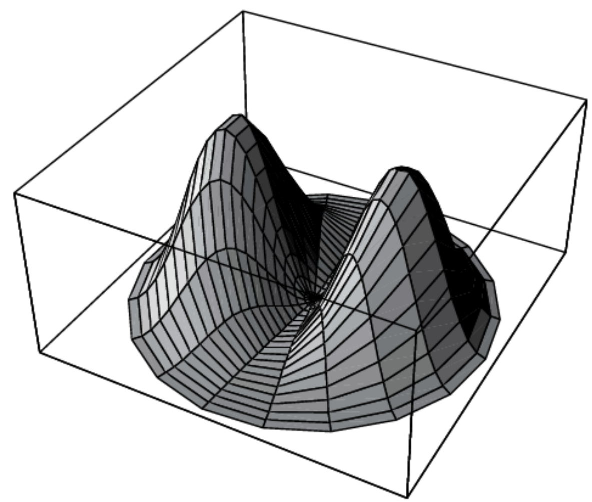


E_1^2

TM_{110} cylindrical cavity



B_1^2



E_1^2

Self-Inductance and the Solenoid

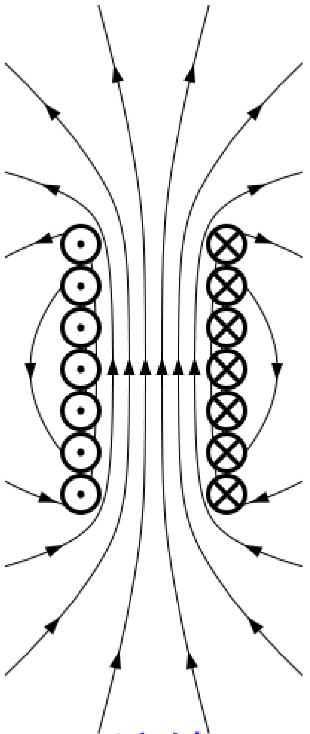
$$v = -\frac{d\Phi}{dt} = -L\frac{di}{dt}$$

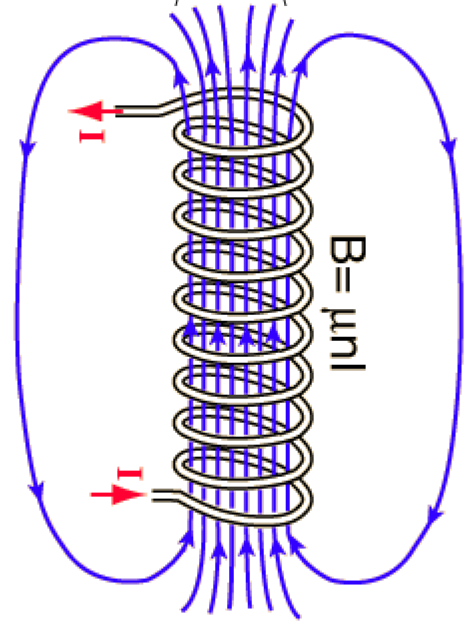
$$B = \frac{\mu_0 N i}{l}$$

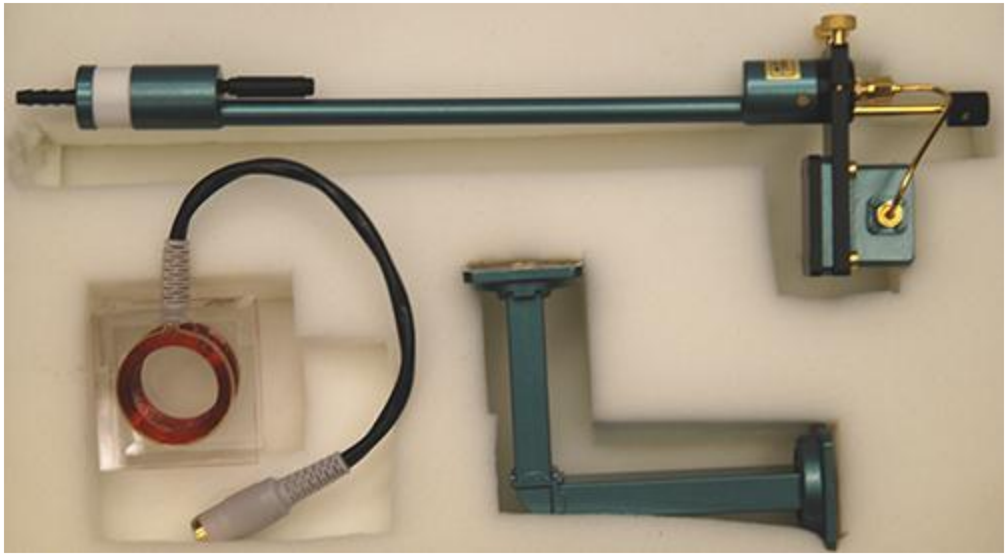
$$\frac{d\Phi}{dt} = \frac{\mu_0 N \pi r^2}{l}\frac{di}{dt}$$

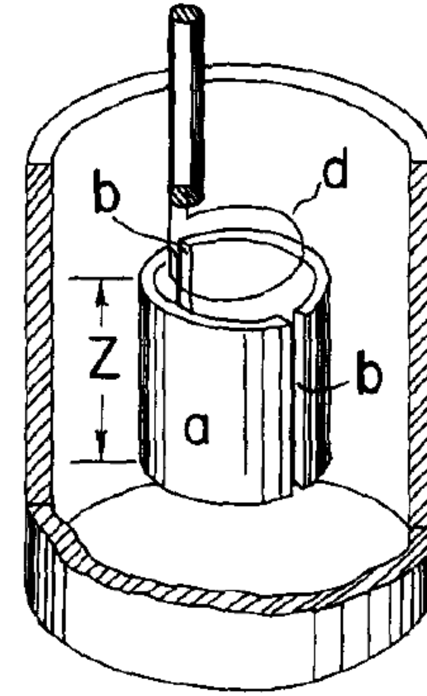
$$A = \pi r^2$$

$$L = \frac{\mu_0 N \pi r^2}{l}$$









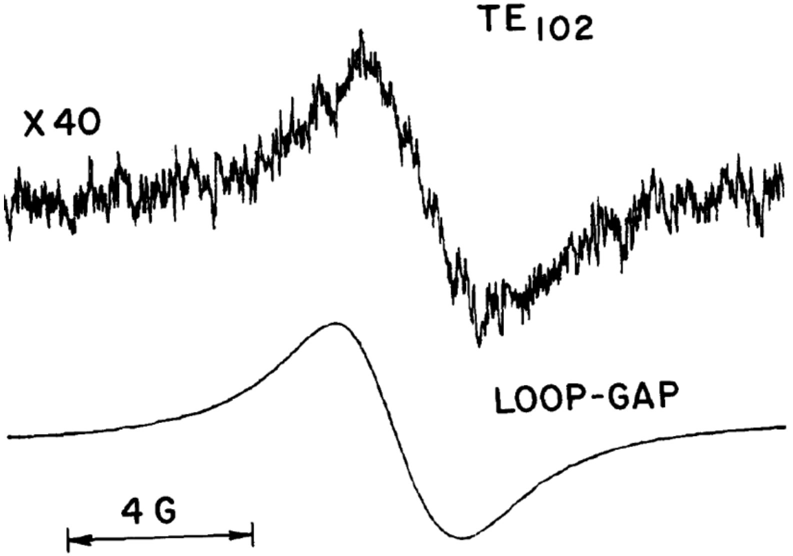
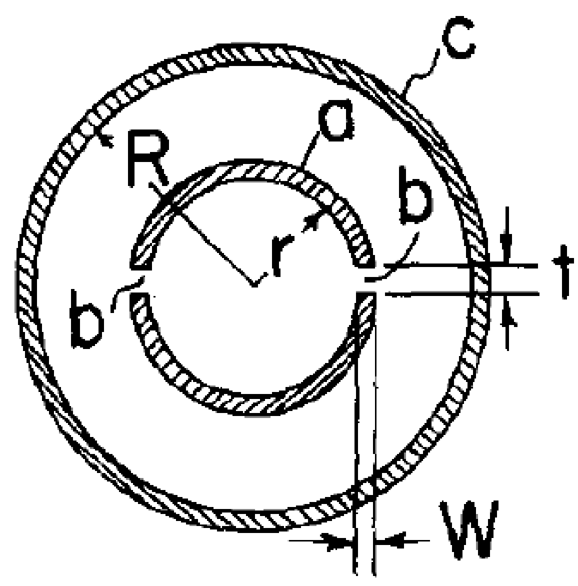


FIG. 2. Comparison of spectra of a point sample of DPPH measured at X band in the Varian Mul-urpose (rectangular TE_{102}) cavity and in a loop-gap resonator of dimensions $r = 0.6$ mm, $Z = 5$ mm. e incident power was held constant at a low nonsaturating level. The loop-gap resonator yields 37 es greater signal, indicating that $\Lambda \approx 6.1$.



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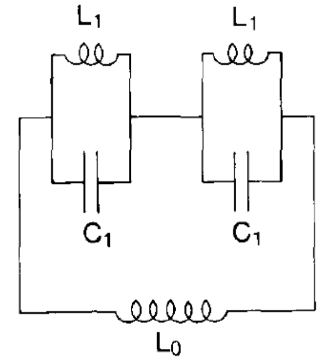
ADVANTAGES

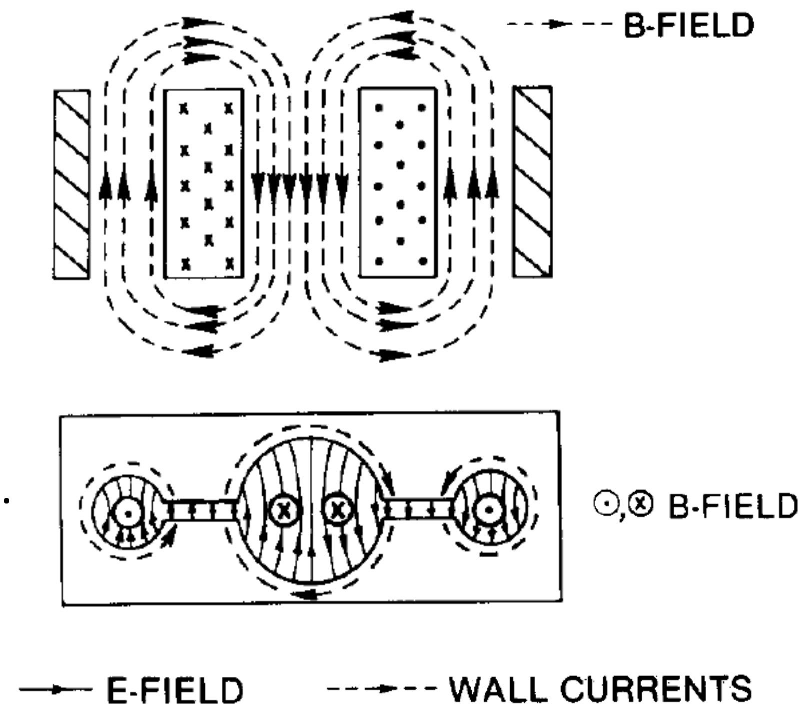
- Large Filling Factor
- Good S/N for small samples
- Reasonable physical size at low frequencies
- Large B_1 per square root Watt
- Fairly uniform B_1 over the sample
- Easy to achieve low Q for large bandwidth and short ring-down time
- LGR's from ~100 MHz to 94 GHz

$$f'_{2t} = \frac{1}{2}\left(\frac{L_0 + 2L_1}{L_0 L_1 C_1}\right)^{1/2}$$

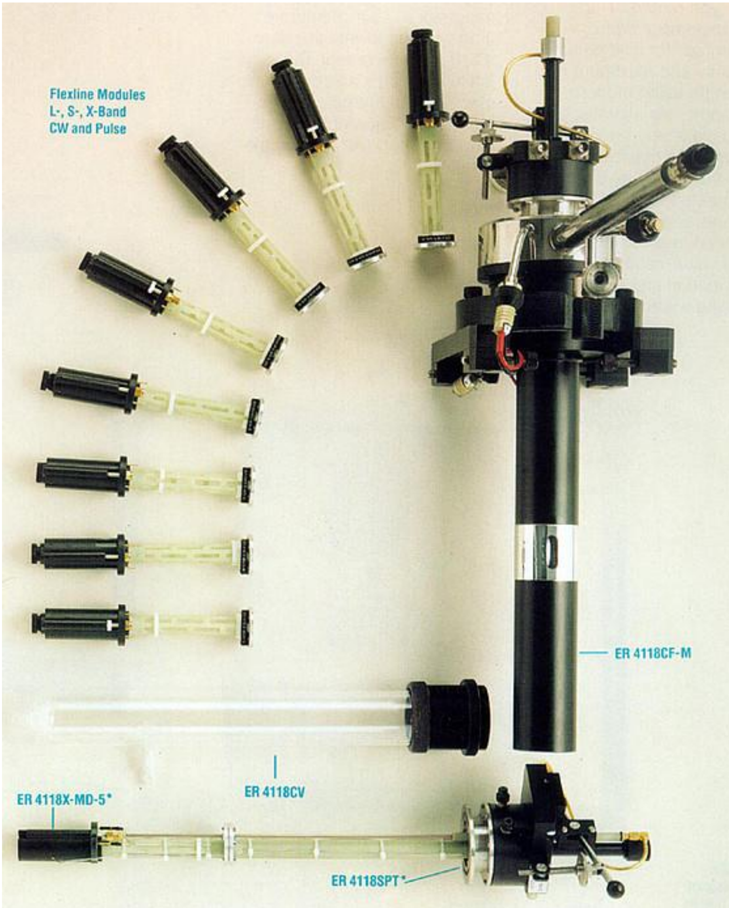


DISADVANTAGES

- Lower Q than cavity
- Small gaps may lead to arcing at high powers
- Require careful sample positioning
- LGR heating if thermal mass small
- Large frequency shift as coupling changed.

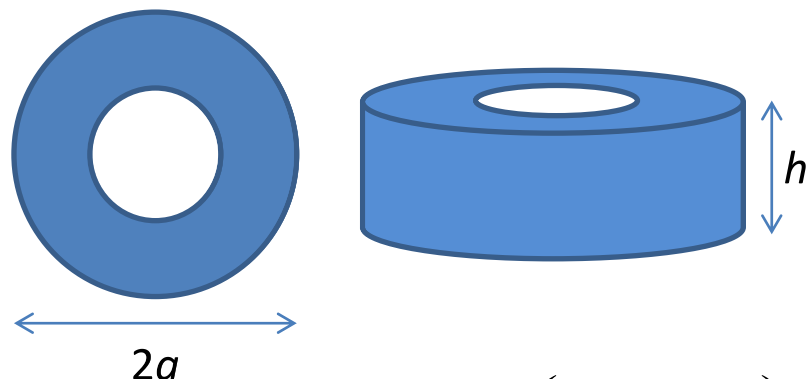


Dielectric Resonator



Dielectric objects can function as resonators.

A disk-shaped dielectric resonator (DR) with a vertical hole up the middle operating in the transverse-electric $TE_{01\delta}$ mode will have a vertically directed microwave magnetic field extending through its centre where the sample is located and will confine the electric field, mostly within the dielectric.



$$f_{GHz} = \frac{34}{a_{mm}\sqrt{\varepsilon_r}}\left(\frac{a}{h} + 3.45\right)$$