Theoretical Models of neutrino parameters.

G.G.Ross, Warwick, October 2008

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Number of light neutrinos	3 ?
Masses + Mixing Angles	6 (15)
CP violating phases	3 (6)

Theoretical Models of neutrino parameters.

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- $q \leftrightarrow l\,?$ — See-saw with sequential domination
- Symmetry? — GUT; Family — Abelian, Non-Abelian, Discrete, String?
- Tri-bi-maximal mixing — Non-Abelian discrete symmetry

Theoretical Models of neutrino parameters.

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- $q \leftrightarrow l$ — See-saw with sequential domination
- Symmetry — GUT; Family — Abelian, Non-Abelian, Discrete, String?
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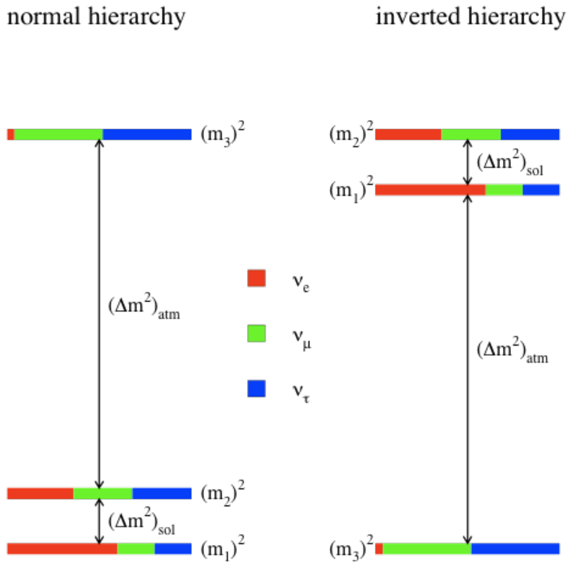
$\Rightarrow$ parameter constraints

DATA :

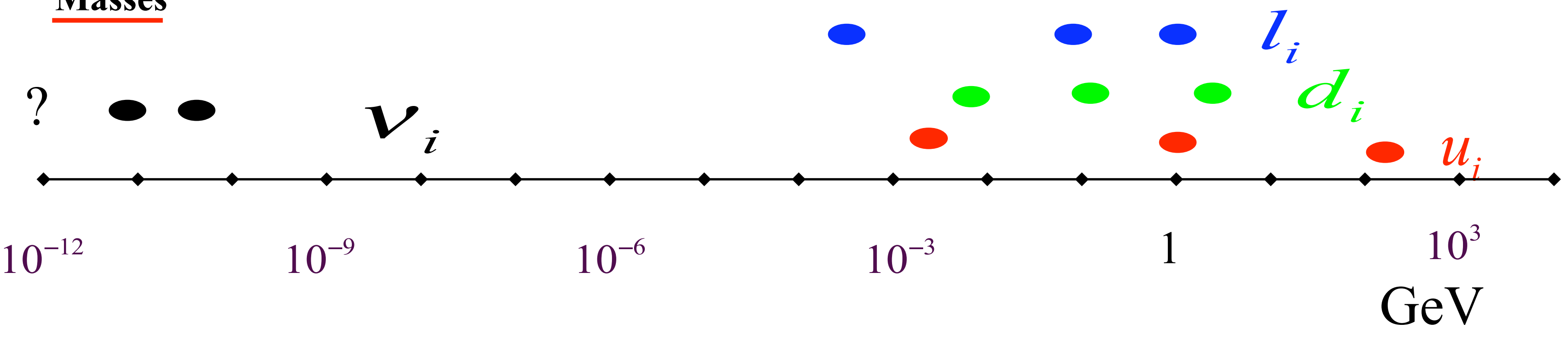
3 neutrino fit



parameter	best fit	2σ	3σ	4σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	7.9	7.3–8.5	7.1–8.9	6.8–9.3
Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	2.6	2.2–3.0	2.0–3.2	1.8–3.5
$\sin^2\theta_{12}$	0.30	0.26–0.36	0.24–0.40	0.22–0.44
$\sin^2\theta_{23}$	0.50	0.38–0.63	0.34–0.68	0.31–0.71
$\sin^2\theta_{13}$	0.000	$\leq$ 0.025	$\leq$ 0.040	$\leq$ 0.058

DATA :

Masses



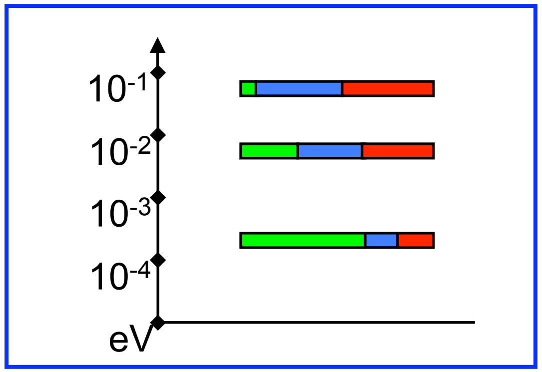
Mixing

Quarks

$$V_{CKM} \approx \begin{pmatrix} 1 & 0.218-0.224 & 0.002-0.005 \\ 0.218-0.224 & 1 & 0.032-0.048 \\ 0.004-0.015 & 0.03-0.048 & 1 \end{pmatrix}$$

Leptons

$$V_{MNS} = \begin{pmatrix} 0.79-0.88 & 0.48-0.61 & <0.2 \\ 0.27-0.49 & 0.45-0.71 & 0.52-0.82 \\ 0.28-0.5 & 0.51-0.65 & 0.57-0.81 \end{pmatrix}$$



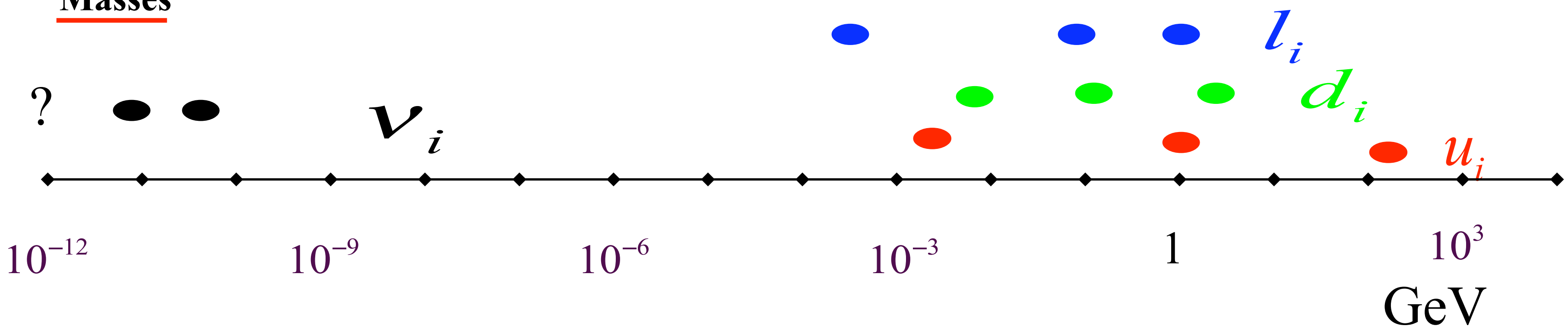
$$\approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \sim 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Tri Maximal Mixing …

Harrison, Perkins, Scott

DATA :

Masses



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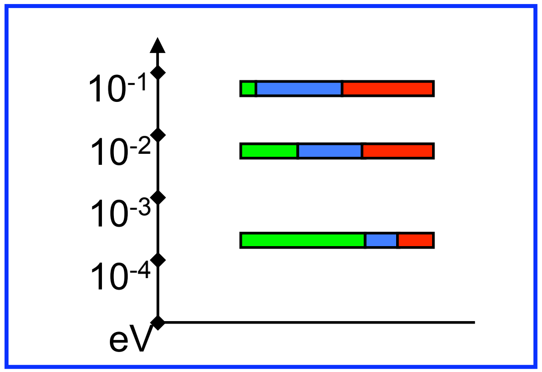

$$\approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \sim 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Tri Maximal
Mixing …
Discrete
Non Abelian
Structure?

DATA :

$$L_{Yukawa} = Y^u_{ij}\, Q^i u^{c,j} H + Y^d_{ij}\, Q^i d^{c,j} \overline{H}$$

$$M^u_{ij} = Y^u_{ij} < H^0 > \qquad M^d_{ij} = Y^d_{ij} < \overline{H}^0 >$$

$$M^u = V_L^\dagger \underline{M^u_{Diag}} V_R$$

$$M^d = U_L^\dagger \underline{M^d_{Diag}} U_R$$

$$\underline{V_{CKM}} = V_L^\dagger U_L$$

The data for quarks is *consistent* with a very symmetric structure :

$$\frac{M^{d,u}}{m^{b,t}} \simeq \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & -\varepsilon^2 \\ -\varepsilon^3 & -\varepsilon^2 & 1 \end{pmatrix} \qquad \begin{aligned} \varepsilon^{d} &= 0.15 \\ \varepsilon^{u} &= 0.05 \end{aligned}$$

$q \leftrightarrow l$ symmetry?

Charged leptons are consistent with a similar form

$$\frac{M^{d,l,u}}{m^{b,\tau,t}} \simeq \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & -a\varepsilon^2 \\ -\varepsilon^3 & -a\varepsilon^2 & 1 \end{pmatrix}$$

$$\varepsilon^{d} = 0.15, \quad a^{d} = 1$$
$$\varepsilon^{l} = 0.15, \quad a^{l} = -3$$
$$\varepsilon^{u} = 0.05, \quad a^{u} = 1$$

Symmetry 1.

GUT relations

$$e.g. \quad SU(4) \subset SO(10)$$

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

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GUT relations

$$e.g.\quad SU(4)\subset SO(10)$$

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

$$Det(M^l) = Det(M^d)\,|_{M_X}$$

$$\bar{\psi}^\alpha \psi_\alpha$$

$$\frac{m_b}{m_\tau}(M_X)=1$$

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & -a\varepsilon^2 \\ -\varepsilon^3 & -a\varepsilon^2 & 1 \end{pmatrix}$$

$$\varepsilon^d = 0.15,\quad a^d = 1$$
$$\varepsilon^l = 0.15,\quad a^l = -3$$

Symmetry 1.

GUT relations

$$e.g. \quad SU(4) \subset SO(10)$$

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

$$\langle \Sigma_{45} \rangle$$

$$\shortparallel$$

$$\overline{\psi}^{\alpha} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix} \psi_\alpha$$

$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

Georgi Jarlskog

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & -a\varepsilon^2 \\ -\varepsilon^3 & -a\varepsilon^2 & 1 \end{pmatrix}$$

$$\varepsilon^{d} = 0.15, \quad a^{d} = 1$$

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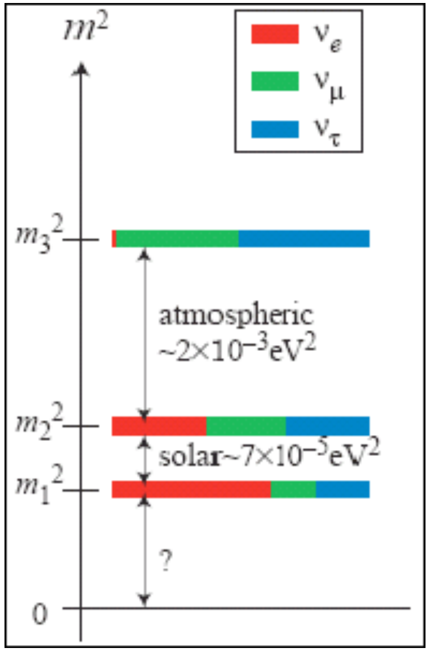
Neutrinos ???

$$L_{eff}^{\nu} = m_3\, \bar{\phi}_{23}^{i} \nu_i \bar{\phi}_{23}^{j} \nu_j + m_2\, \bar{\phi}_{123}^{i} \nu_i \bar{\phi}_{123}^{j} \nu_j$$

$$< \bar{\phi}_{23} >^i = (0,1,-1), \quad < \bar{\phi}_{123} >^i = (1,1,1)$$



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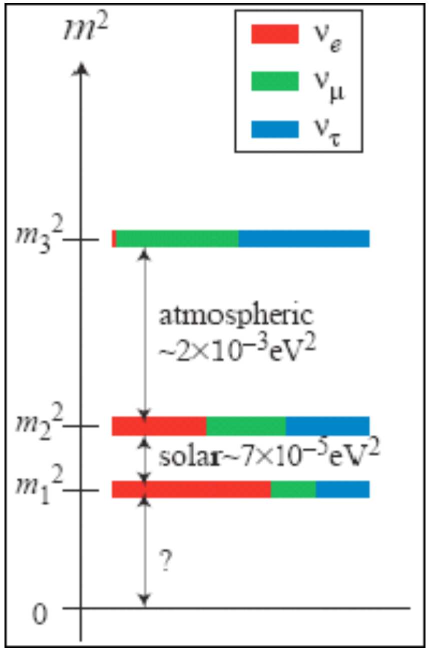

Can one have a unified description of quark, charged lepton and neutrinos?

$$c.f.\ \ L_{Dirac}^{q,l} = m_3\,\bar{\phi}_3^{i}\psi_i\bar{\phi}_3^{j}\psi_j^c + \ldots \qquad <\bar{\phi}_3>^i = (0,0,1) \quad ???$$

Neutrinos ???

$$L_{eff}^{\nu} = m_3\, \bar{\phi}_{23}^{i} \nu_i \bar{\phi}_{23}^{j} \nu_j + m_2\, \bar{\phi}_{123}^{i} \nu_i \bar{\phi}_{123}^{j} \nu_j$$

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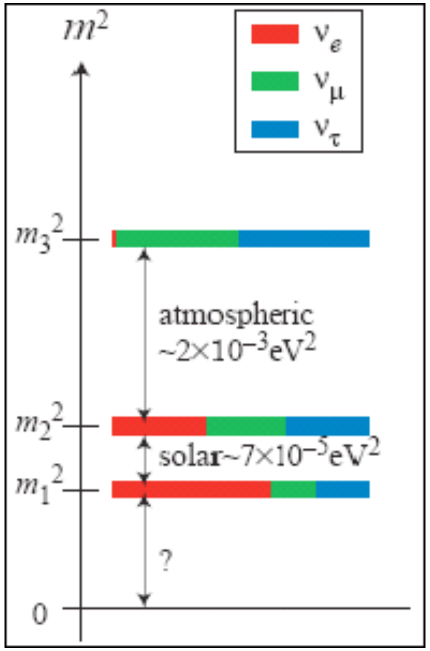


See-Saw

Quarks, charged leptons, neutrinos can have similar Dirac mass :

$$L_{Dirac}^{q,l,\nu} = \alpha\, \psi_i \bar{\phi}_3^{i} \psi_j^c \bar{\phi}_3^{j} + \beta \left(\psi_i \bar{\phi}_{123}^{i} \psi^c{}_j \bar{\phi}_{23}^{j} + \psi_i \bar{\phi}_{23}^{i} \psi^c{}_j \bar{\phi}_{123}^{j} \right) + \gamma\, \psi_i \bar{\phi}_{123}^{i} \psi^c{}_j \bar{\phi}_{23}^{j} \Sigma_{45} \qquad \alpha > \beta$$

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3+\varepsilon^4 & -\varepsilon^3+\varepsilon^4 \\ \varepsilon^3+\varepsilon^4 & a\varepsilon^2+\varepsilon^3 & -a\varepsilon^2+\varepsilon^3 \\ -\varepsilon^3+\varepsilon^4 & -a\varepsilon^2+\varepsilon^3 & 1 \end{pmatrix}$$

$\varepsilon^d = 0.15,\ \ a^d = 1$

$\varepsilon^l = 0.15,\ \ a^e = -3$

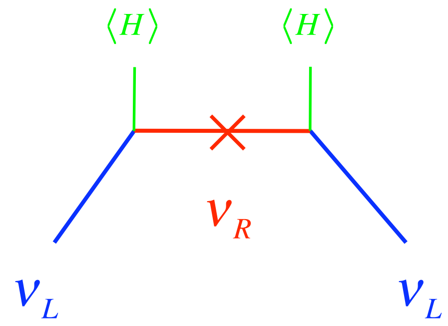
$\varepsilon^u = 0.05,\ \ a^u = 1$

$\varepsilon^\nu = 0.05,\ \ a^\nu = 0$

- "See-saw" with sequential domination $M_\nu = M_D^\nu M_M^{-1} M_D^{\nu T}$

Minkowski
Gell-Mann,
Ramond,
Slansky;
Yanagida,
King

$$M_M \approx \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix} \qquad M_1 < M_2 << M_3$$



$$L_{Dirac}^\nu = \alpha\, \psi_i \bar{\phi}_3^i \psi_j^c \bar{\phi}_3^j + \beta \left(\psi_i \bar{\phi}_{123}^i \psi^c{}_j \bar{\phi}_{23}^j + \psi_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{123}^j \right) \quad \alpha > \beta$$

$$\boxed{L_{eff}^\nu = \frac{\beta^2}{M_1} \psi_i \phi_{123}^i \psi_j \phi_{123}^j + \frac{\beta^2}{M_2} \psi_i \phi_{23}^i \psi_j \phi_{23}^j + \frac{(\alpha+\beta)^2}{M_3} \psi_i \phi_3^i \psi_j \phi_3^j}$$

- "See-saw" with sequential domination $M_\nu = M_D^\nu M_M^{-1} M_D^{\nu T}$

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$$M_M \approx \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix} \qquad M_1 << M_2 << M_3$$

small

$$L_{Dirac}^\nu = \alpha\, \psi_i \bar{\phi}_3^i \psi_j^c \bar{\phi}_3^j + \beta \left(\psi_i \bar{\phi}_{123}^i \psi^c{}_j \bar{\phi}_{23}^j + \psi_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{123}^j \right) \quad \alpha > \beta$$

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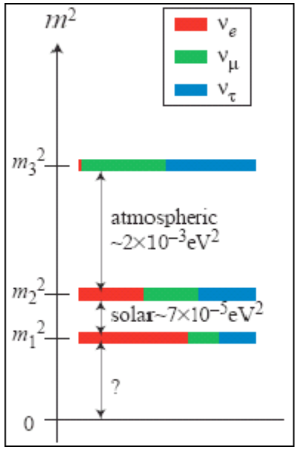
Symmetry 2.

Family symmetry



Non-Abelian family symmetry

Promote ϕ_i to fields transforming under $SU(3)_{\mathrm{family}}$

Vacuum alignment $\phi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\phi_{23} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ $\phi_{123} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\Rightarrow$ Discrete non Abelian symmetry

List of models with discrete flavour symmetry

(incomplete, by symmetry)

S_3: Pakvasa et al.(1978), Derman(1979), Ma(2000), Kubo et al.(2003), Chen et al.(2004), Grimus et al.(2005), Dermisek et al (2005), Mohapatra et al.(2006), Morisi(2006), Caravaglios et al.(2006), Haba et al(2006),…

S_4: Pakvasa et al.(1979), Derman(1979), Lee et al.(1994), Mohapatra et al.(2004),Ma(2006), Hagedorn et al.(2006), Caravaglios et al.(2006), Lampe(2007), Sawanaka(2007), …

A_4: Wyler(1979), Ma et al.(2001), Babu et al.(2003), Altarelli et al.(2005-8), He et al.(2006), Bazzocchi, Morisi, et al.(2007/8), King et al.(2007),…

D_4: Seidl(2003), Grimus et al.(2003/4), Kobayashi et al.(2005), …

D_5: Ma(2004), Hagedorn et al.(2006), …

D_n: Chen et al.(2005), Kajiyama et al.(2006), Frampton et al.(1995/6,2000), Frigerio et al (2005), Babu et al.(2005), Kubo(2005),...

T’: Frampton et al.(1994,2007), Aranda et al.(1999,2000),Feruglio et al.(2007), Chen et al.(2007), …

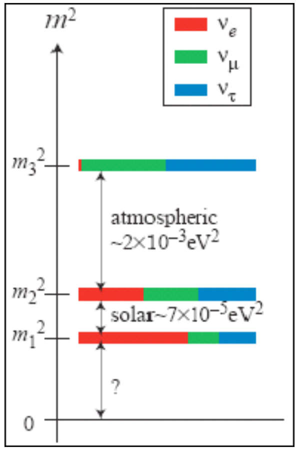
Δ_n: Kaplan et al.(1994), Schmaltz(1994), Chou et al.(1997), de Madeiros Varzielas et al(2006/7).

T_7: Luhn et al.(2007).

Adapted from Lindner, Manchester

Symmetry 2.

Family symmetry



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$\Rightarrow$ Discrete non Abelian symmetry e.g. $\Delta(27) \subset SU(3)_f$, $\quad \phi_i$ triplets

ϕ_i	$Z_3\phi_i$	$Z_3'\phi_i$
ϕ_1	$\rightarrow \phi_2$	$\rightarrow \phi_1$
ϕ_2	$\rightarrow \phi_3$	$\rightarrow \alpha\phi_2$
ϕ_3	$\rightarrow \phi_1$	$\rightarrow \alpha^2\phi_3$

$$\Delta(27) \rightarrow \begin{cases} Z_3, & <\phi> = (1,1,1) \quad \lambda > 0 \\ Z_3', & <\phi> = (0,0,1) \quad \lambda < 0 \end{cases}$$

$$V(\phi) = m^2\,\phi^\dagger\phi + \ldots + \lambda\, m^2 \phi^{i\dagger}\phi_i \phi^{i\dagger}\phi_i$$

A complete model

$$\Delta(27)\otimes SO(10)\otimes G \qquad (G = R\otimes U(1))$$

Varzielas, GGR

- $\psi_i^c, \psi_i \subset (16,3) \Rightarrow$ No mass while SU(3) unbroken

- Spontaneous symmetry breaking

c.f. Georgi-Jarskog

$$\begin{array}{cccc} \overline{\phi}_3^{\,i}, & \overline{\phi}_{23}^{\,i}, & \overline{\phi}_{123}^{\,i}, & H_{45} \\ (1,\overline{3}) & (1,\overline{3}) & (1,\overline{3}) & (45,1) \\ \begin{pmatrix}0\\0\\1\end{pmatrix} & \begin{pmatrix}0\\-1\\1\end{pmatrix}\varepsilon M & \begin{pmatrix}1\\1\\1\end{pmatrix}\varepsilon^2 M & M \end{array}$$

- $$P_Y = \frac{1}{M^2}\overline{\phi}_3^{\,i}\psi_i\overline{\phi}_3^{\,j}\psi_j^c H + \frac{1}{M^3}\overline{\phi}_{23}^{\,i}\psi_i\overline{\phi}_{23}^{\,j}\psi_j^c H H_{45} + \frac{1}{M^2}\overline{\phi}_{23}^{\,i}\psi_i\overline{\phi}_{123}^{\,j}\psi_j^c H + \frac{1}{M^2}\overline{\phi}_{123}^{\,i}\psi_i\overline{\phi}_{23}^{\,j}\psi_j^c H$$

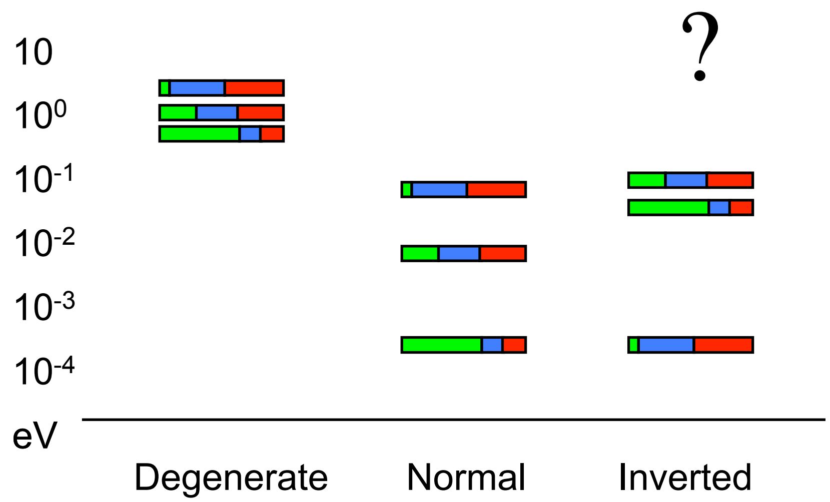
only terms allowed by G

Neutrino Parameters

Neutrino Parameters

- Tri-bi-maximal mixing

  Degenerate and normal spectra favoured



Neutrino Parameters

- **Tri-bi-maximal mixing**

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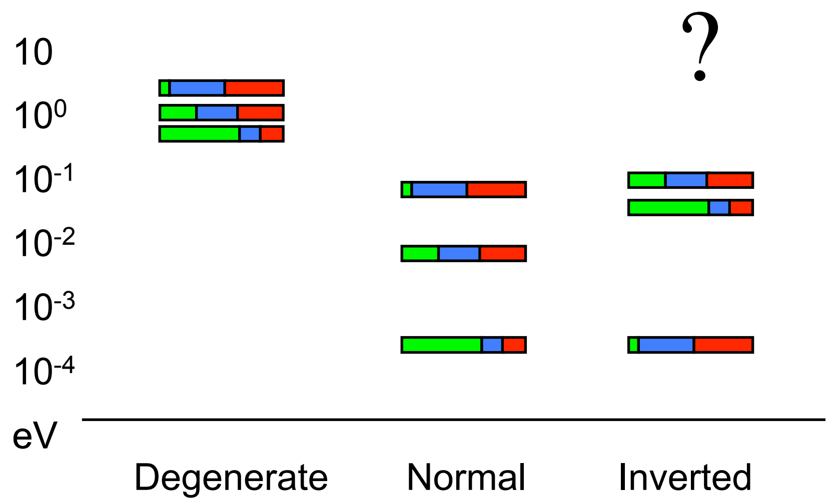


- **Mixing angles**

$$\sin^2\theta_{12} \approx \frac{1}{3} \pm 0.03$$

$$\sin^2\theta_{23} \approx \frac{1}{2} \pm 0.03$$

$$\sin\theta_{13} \approx \sqrt{\frac{m_e}{2m_\mu}} = 0.053 \pm 0.05 \quad (3 \pm 3^o)$$

From charged lepton mixing

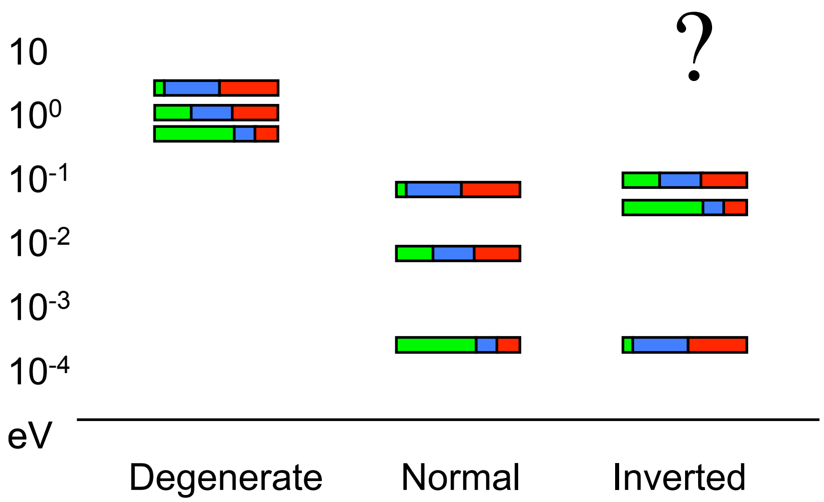
King; GGR, Varzielas

◆

Neutrino Parameters

- Tri-bi-maximal mixing

Degenerate and normal spectra favoured



- Mixing angles

$$\sin^2\theta_{12} \approx \frac{1}{3} \pm 0.03 \longrightarrow \theta_{12} + \frac{1}{\sqrt{2}}\frac{\theta_C}{3}\cos(\delta-\pi) \simeq 35.26 \pm 2^o$$

$3\cos(\delta-\pi)$

Antusch, King

From charged lepton mixing

$$\sin^2\theta_{23} \approx \frac{1}{2} \pm 0.03$$

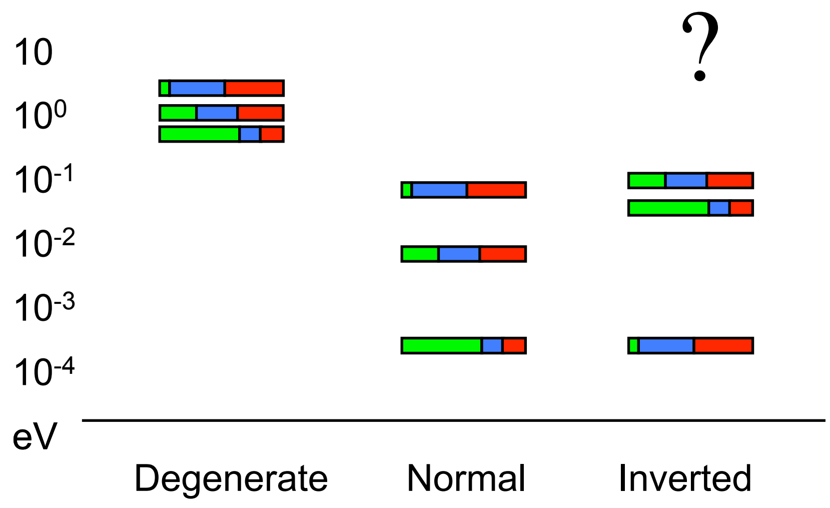
$$\sin\theta_{13} \approx \sqrt{\frac{m_e}{2m_\mu}} = 0.053 \pm 0.05 \quad (3 \pm 3^o)$$

King; GGR, Varzielas

Neutrino Parameters

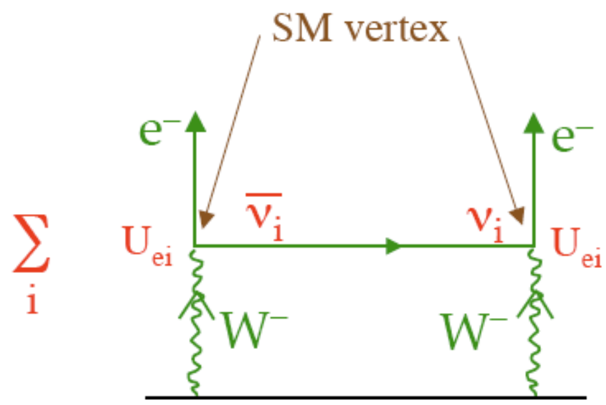
- Tri-bi-maximal mixing

  Degenerate and normal spectra possible



- Neutrinoless double β decay

  (3 light neutrinos)



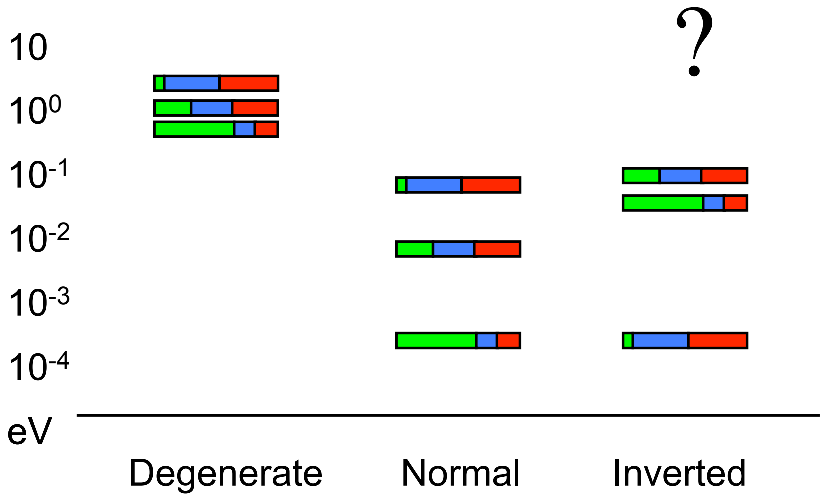
Mass (ν_i)

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left|\sum m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

Neutrino Parameters

- Tri-bi-maximal mixing

  Degenerate and normal spectra possible



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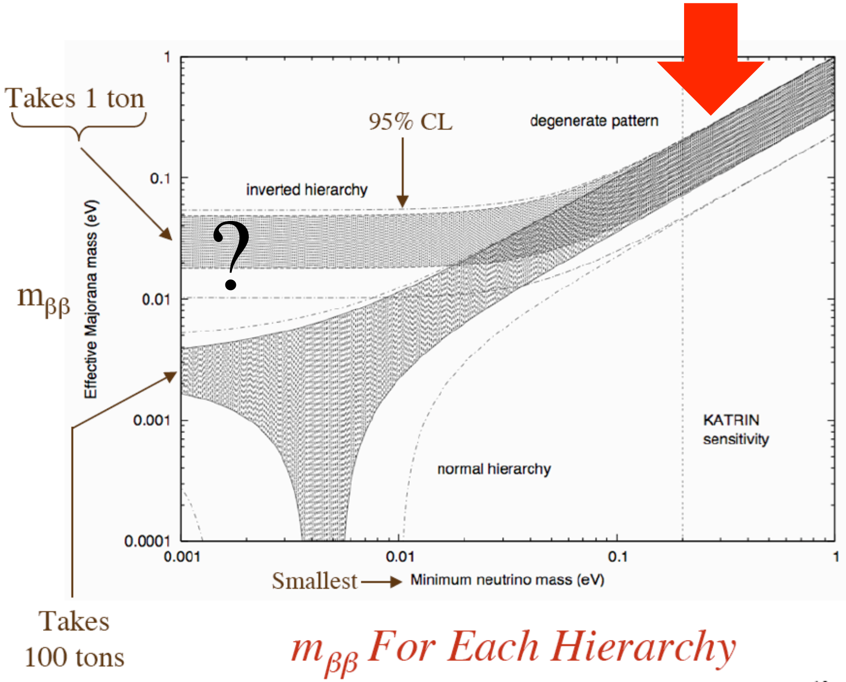
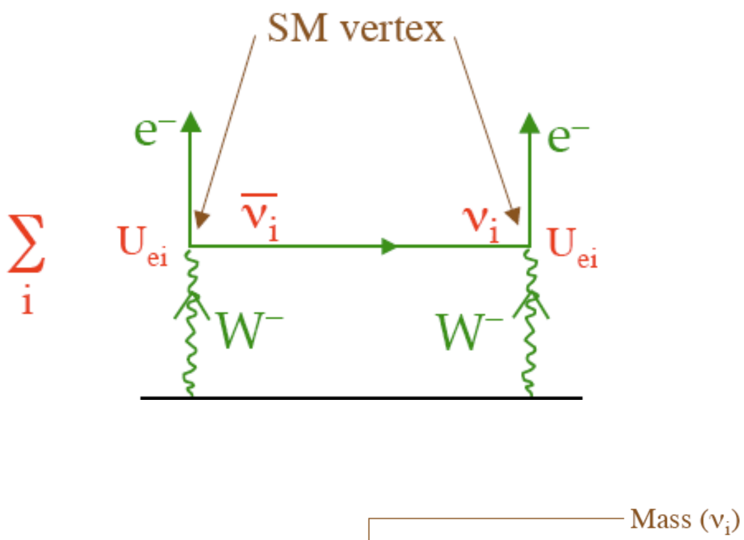


$m_{\beta\beta}$ *For Each Hierarchy*



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See saw parameters

See saw parameters

Masses + Mixing Angles 6 (15)
CP violating phases 3 (6)

$$\kappa = Y_\nu^T M^{-1} Y_\nu$$

Insufficient information

$$P = Y_\nu^\dagger Y_\nu$$

RGE for sleptons (SUSY)

In *principle* all parameters determined by neutrino structure and radiative corrections (assuming degenerate sleptons at GUT scale)

Davidson, Ibarra

See saw parameters

Masses + Mixing Angles 6 (15)
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Davidson, Ibarra

Thermal Leptogenesis (Baryogenesis)

$$M_{\nu_R} > 10^9 GeV, \quad T_{\text{Reheat}} > 10^{10} GeV \quad \text{(hierarchical spectrum)}$$

$\beta\beta$ phase

$$\cancel{C}P = \cancel{C}P(\delta, \phi', \phi_{i=1,2,3})$$ see saw phases

ν factory phase

See saw parameters

Masses + Mixing Angles	6	(15)	(8)
CP violating phases	3	(6)	(3)

$$\kappa = Y_\nu^T M^{-1} Y_\nu$$

Insufficient information

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Davidson, Ibarra

Thermal Leptogenesis (Baryogenesis)

Fukugita, Yanagida

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$\beta\beta$ phase

$$\cancel{C}P = \cancel{C}P(\delta, \phi', \phi_{i=1,2,3})$$

see saw phases

ν factory phase

Sequential see saw and symmetry (texture zero) simplifies structure…

Sequential see saw and (1,1) texture zero

(1,1) texture zero

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3+\varepsilon^4 & -\varepsilon^3+\varepsilon^4 \\ \varepsilon^3+\varepsilon^4 & a\varepsilon^2+\varepsilon^3 & -a\varepsilon^2+\varepsilon^3 \\ -\varepsilon^3+\varepsilon^4 & -a\varepsilon^2+\varepsilon^3 & 1 \end{pmatrix}$$

Sequential see saw and (1,1) texture zero

Quark sector zero $\Rightarrow$ $|V_{us}| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$

(1,1) texture zero

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} < \varepsilon^4 & \varepsilon^3 + \varepsilon^4 & -\varepsilon^3 + \varepsilon^4 \\ \varepsilon^3 + \varepsilon^4 & a\varepsilon^2 + \varepsilon^3 & -a\varepsilon^2 + \varepsilon^3 \\ -\varepsilon^3 + \varepsilon^4 & -a\varepsilon^2 + \varepsilon^3 & 1 \end{pmatrix}$$

Sequential see saw and (1,1) texture zero

Lepton sector zero $\Rightarrow \sin^2\theta_{23} \simeq \frac{1}{2}, \quad \sin\theta_{13} \simeq \sqrt{\frac{m_e}{2m_\mu}}$

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$$\boxed{Y_B \propto \sin^2\theta_{13}\sin(2\delta-\phi)}$$

Abada et al

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- Other model implications

  SUSY models: sparticles have related mass structure

  ($m^2 \phi_i^\dagger \phi_i \ldots$ degeneracy split by small fermion mass related terms)

  $FCNC: \quad L_i, B_i \ violation - close\ to\ present\ bounds$

  ($\mu \rightarrow e\gamma$, mercury EDM within a factor of 10 of present limits)

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