A Unified Approach to the Flavour Puzzle

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Particle Physics Seminar @ University of Warwick, 10\textsuperscript{th} November 2022
Outline

1. Review of the flavour puzzle(s)

2. Flavour model building in the LHC era
   a) Horizontal flavour symmetries
   b) Flavour-deconstructed gauge symmetries
   c) Unified (gauge & flavour) symmetries

3. Electroweak flavour unification via $SU(4) \times Sp(6)_L \times Sp(6)_R$ gauge group

Davighi, Tooby-Smith, 2201.07245
The Flavour Puzzle
The Flavour Puzzle(s)

The matter content of the Standard Model is a source of many mysteries!

Puzzle 1: SM fermions in 5 (6) ad hoc representations of SM gauge group:

\[
q_L \sim (3, 2)_{1/6}, \quad u_R \sim (3, 1)_{2/3}, \quad d_R \sim (3, 1)_{-1/3} \\
l_L \sim (1, 2)_{-1/2}, \quad e_R \sim (1, 1)_1, \quad \nu_R \sim (1, 1)_0
\]

Hints at unification e.g. quark-lepton unification? $SU(5)$? $SO(10)$?

Traditionally, unification was done ignoring flavour.

(More later...)
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Yukawa couplings to the Higgs (Y) break this symmetry:

\[ U(3)^5 \rightarrow G_{acc} := U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \]
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These are very good symmetries of Nature, probing scales \(\Lambda_{\text{acc}} \gg \text{TeV}\)

Example: proton half-life

\[ \tau(p \rightarrow \pi^0e^+) \gtrsim 10^{34} \text{ yr}, \]
\[ \tau(p \rightarrow \bar{\nu}K^+) \gtrsim 10^{34} \text{ yr} \]

If due to dim-6 SMEFT, \(\Lambda \gtrsim 10^{13} \text{ TeV}\).

Strongest bound we have on NP!
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The breaking by \( Y \) to \( G_{\text{acc}} \) is not “random”, but very structured:

- Mass hierarchies: \( m_3 \gg m_2 \gg m_1 \)
- Small mixing angles: \( V_{us} \sim \lambda \sim 0.2, \quad V_{cb} \sim \lambda^2, \quad V_{ub} \sim \lambda^3 \)

This structure is highly suggestive of a dynamical BSM theory of flavour!
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BUT: indirect limits from flavour physics sets New Physics scale much higher; \( \mathcal{O}(10^{4-5}) \) TeV for flavour changing processes in 1-2 sector, even higher for Lepton Flavour Violation (LFV)

Barbieri, 2103.15635
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BUT: indirect limits from flavour physics sets New Physics scale much higher; \( \mathcal{O}(10^{4-5}) \) TeV for flavour changing processes in 1-2 sector, even higher for Lepton Flavour Violation (LFV)

... So, should we really have expected fireworks at the LHC? Certainly, they could not have had “generic” flavour structure. Either way, no fireworks yet...

Barbieri, 2103.15635
Flavour model building
(in the LHC era)
Global symmetries: a big clue for model building

Yukawa matrices have approximate global symmetries: $G_{\text{approx}} = U(2)^5$ acting on light families

Mass hierarchies: $m_3 \gg m_2 \gg m_1$
Small mixing angles: $V_{us} \sim \lambda \sim 0.2$, $V_{cb} \sim \lambda^2$, $V_{ub} \sim \lambda^3$

Exact $U(2)$ limit $\approx$ Observed Yukawa

$U(2)$-breaking spurions

Barbieri et al, 1105.2296
Isidori, Straub, 1202.0464
Fuentes-Martin et al, 1909.02519
Yukawa matrices have approximate global symmetries: $G_{\text{approx}} = U(2)^5$ acting on light families.

An excellent starting point is for the new physics sector (i.e. new gauge symmetries) to only allow renormalizable Yukawa couplings for the third family. Other Yukawas from higher-dimension operators:

$$\mathcal{L} \supset \bar{\psi}_3 H \psi_3 + \left( \frac{w}{\Lambda} \right)^{n_{ij}} \bar{\psi}_i H \psi_j$$

$w =$ vev of some BSM scalar
$\Lambda =$ some heavier NP scale
$n_{ij} =$ an integer
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→ Naturally reconciles puzzle 3 **with puzzle 4** – new physics could still stabilize (or at least not destabilize) the Higgs sector (TeV scale) while being consistent with flavour bounds, and be **hard to discover directly** at ATLAS & CMS due to PDF suppression in pp collisions
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→ Perhaps as likely to see indirect evidence of NP in rare heavy flavoured decays e.g. at LHCb, Belle II
Anchoring the low scale

By and large, any theory of flavour takes the form:

$$\mathcal{L} \supset \bar{\psi}_3 H \psi_3 + \left(\frac{w}{\Lambda}\right)^{n_{ij}} \bar{\psi}_i H \psi_j$$

\(w = \text{vev of some BSM scalar}\)
\(\Lambda = \text{some heavier NP scale}\)
\(n_{ij} = \text{an integer}\)

All the observed hierarchies depend only on ratios of scales. E.g. \(\frac{m_2}{m_3} \sim (\frac{w}{\Lambda})^{n_{22}}\)

Unfortunately, this means the scales of NP responsible for flavour could be very far off...
(the old rationale for postponing the flavour puzzle)

BUT, there are two good reasons for anchoring the low scale \(w\) not too far above TeV

1. Naturalness – a big scale separation would destabilize the Higgs
2. There are already hints of new flavoured physics at TeV scale, in rare B decays
Interlude: a quick review of the flavour anomalies...
Neutral current $b \to sll$

LFUV ratios (clean) @ LHCb

$$R_H = \frac{\text{BR}(B \to H\mu\mu)}{\text{BR}(B \to Hee)}$$

$R_{K^+}[0.05-1.1]$, $2.1\sigma$

$R_{K^0}[1.1-6]$, $3.1\sigma$

$R_{K^*}[0.05-6]$, $0.9\sigma$

$R_{K^0}[1.1-6]$, $2.0\sigma$

$R_{K^+}[1.1-6]$, $2.5\sigma$

$R_{\pi K}[0.1-6]$

Summary from G. Isidori @ Planck 22

BR($B_s \to \mu\mu$) (clean)

In 2021

Pull = 2.3 $\sigma$

Altmannshofer, Stangl 2103.13370
Neutral current $b \rightarrow sll$

LFUV ratios (clean) @ LHCb

$$R_H = \frac{BR(B \rightarrow H\mu\mu)}{BR(B \rightarrow Hee)}$$

In 2022

Pull = $1.6 \sigma$ (agrees with SM now)

Summary from G. Isidori @ Planck 22
Neutral current $b \rightarrow sll$

LFUV ratios (clean) @ LHCb

$$R_H = \frac{BR(B \rightarrow H\mu\mu)}{BR(B \rightarrow H\epsilon\epsilon)}$$

Also tensions in muon-only observables where hadronic uncertainties in TH predictions could be unexpectedly big. E.g. angular analyses for

1. $B \rightarrow K^*\mu\mu$
2. $B_s \rightarrow \phi\mu\mu$

A more or less **coherent** set of deviations in $bsll$ that can be explained by NP contribution to e.g.

$$\left( \frac{\bar{s}\gamma_\rho b_L}{\mu L \gamma^\rho \mu L} \right)$$

Conservative estimate in 2021: global significance $\sim 4.3 \sigma$

Expect similar conclusion despite $BR(B_s \rightarrow \mu\mu)$ update

Summary from G. Isidori @ Planck 22

Isidori, Lancierini, Owen, Serra, 2104.05631
Neutral current $b \rightarrow sll$

LFUV ratios (clean) @ LHCb

$$R_H = \frac{BR(B \rightarrow H\mu\mu)}{BR(B \rightarrow Hee)}$$

$R_{K^*0}[0.05 - 1.1]$

$R_{K^0}[0.1 - 6]$

$R_{K^+}[1.1 - 6]$

$R_{\mu\kappa}[1.1 - 6]$

Also tensions in muon-only observables where hadronic uncertainties in TH predictions could be unexpectedly big. E.g. angular analyses for

1. $B \rightarrow K^*\mu\mu$
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A more or less coherent set of deviations in $bsll$ that can be explained by NP contribution to e.g.

$$\left( s_L \gamma^\rho b_L \right) \left( \bar{\mu}_L \gamma^\rho \mu_L \right)$$

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Isidori, Lancierini, Owen, Serra, 2104.05631

New particle explanations. Mass/g $\sim 3$ TeV/0.1 (weak effect)
Charged current $b \rightarrow c\nu$

Measurements continue to show good agreement
Significance remains just above $3\sigma$ after LHCb 2022

Here, LFUV ratios are

$$R_H = \frac{\text{BR}(B \rightarrow H\tau\nu)}{\text{BR}(B \rightarrow H\mu\nu)}$$

2022 update from LHCb:
First measurement of $R_D$ at a collider!

- Measurements continue to show good agreement
- Significance remains just above $3\sigma$ after LHCb 2022
Charged current $b \to clu$

New physics explanations:
- SM process here is tree-level, so the NP effect is BIG! $\text{Mass/g} \sim 3 \text{ TeV/1}$
- The $W'/\text{charged Higgs}$ explanations ruled out by LHC high $p_T$ & $B_c$ lifetime
- Leptoquark is best explanation

- Measurements continue to show good agreement
- Significance remains just above $3\sigma$ after LHCb 2022
These anomalies are, for now, only hints of BSM.

But if they are real, they are the kind of thing we would expect at the LHC, if taking flavour puzzle seriously. Moreover, the anomalies anchor the lowest scale in a BSM theory of flavour \( \sim \)TeV:

Gripaios, 0910.1789

This is one reason why the B anomalies are so interesting to theorists – they could be a window onto the flavour puzzle

With this in mind, let’s return to model building for the flavour puzzle.
Three routes for solving the flavour puzzle

1. Horizontal flavour symmetries
2. Deconstructed gauge symmetries
3. Unified (gauge & flavour) symmetries
1: Horizontal flavour symmetries

A horizontal symmetry *commutes with the SM gauge symmetry*

$$G = G_{\text{SM}} \times G_F$$

Flavour symmetry $G_F$ can be continuous or discrete, abelian or non-abelian. Gauging $G_F$ enforces selection rules on Yukawa couplings e.g. only allowing $Y_{33}$. 

Froggatt, Nielsen, 1979  
+many more  
Grinstein et al, 1009.2049
1: Horizontal flavour symmetries

A horizontal symmetry \textit{commutes with the SM gauge symmetry}

\[ G = G_{\text{SM}} \times G_F \]

Flavour symmetry \( G_F \) can be continuous or discrete, abelian or non-abelian. Gauging \( G_F \) enforces selection rules on Yukawa couplings e.g. only allowing \( Y_{33} \).

\textit{Example: Froggatt—Nielsen mechanism,} where \( G_F = U(1)_F \) with appropriate non-universal charges

Here, the \textit{hierarchies} come from \textit{operator dimensions} in the low energy EFT

The \textit{heavy gauge bosons} from breaking \( G_F \) will all be SM singlets $\rightarrow$ heavy $Z$'s
1: Horizontal flavour symmetries

\[ G = G_{\text{SM}} \times G_F \]

**Example 2:** Simple $Z'$ models connecting flavour puzzle with the $b \rightarrow sll$ anomalies

- Gauge $G_F = U(1)_X; X = Y_3$. Breaking $U(1)_X \rightarrow$ the $Z'$

Only $Y_{33}$ renormalizable:

\[
\begin{pmatrix}
\vdots & \vdots & \vdots \\
\text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ }
\end{pmatrix}
\approx
\begin{pmatrix}
\vdots & \vdots & \vdots \\
\text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ }
\end{pmatrix}
\]

Allanach, Davighi, 1809.01158; 2103.12056; 2205.12252
1: Horizontal flavour symmetries

\[ G = G_{\text{SM}} \times G_F \]

**Example 2:** Simple \( Z' \) models connecting flavour puzzle with the \( b \to sll \) anomalies

- Gauge \( G_F = U(1)_X; \, X = Y_3 \). Breaking \( U(1)_X \to \) the \( Z' \)

Only \( Y_{33} \) renormalizable:

\[
\begin{pmatrix}
\quad \\
\quad \\
\quad \\
\end{pmatrix} \approx \begin{pmatrix}
\quad \\
\quad \\
\quad \\
\end{pmatrix}
\]

**LHC \( Z' \) production:**

**\( Z' \) decay modes:**

<table>
<thead>
<tr>
<th>Mode</th>
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<th>Mode</th>
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<th>Mode</th>
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<tbody>
<tr>
<td>( \mu^+\mu^- )</td>
<td>0.08</td>
<td>( \tau^+\tau^- )</td>
<td>0.30</td>
<td>( \nu\nu' )</td>
<td>0.08</td>
</tr>
</tbody>
</table>

\[ \text{other } f_i f_j \sim O(10^{-4}) \]

*Mostly heavy flavours!*

**LHC \( Z' \) searches:**

- Allanach, Davighi, [1809.01158; 2103.12056; 2205.12252]
- Allanach, Banks, [2111.06691]
1: Horizontal flavour symmetries

\[ G = G_{\text{SM}} \times G_F \]

**Example 3:** Simple $Z'$ models connecting flavour puzzle with the $b \to sll$ anomalies + absence of LFV!

- Gauge $G_F = U(1)_X; \ X = Y_3 + a(L_2 - L_3).$ Breaking $U(1)_X \to$ the $Z'$

Davighi, [2105.06918](https://arxiv.org/abs/2105.06918)

\[
Y_u = y_t \left( \frac{\Delta_{\alpha}^a \Phi}{\Lambda^2} \frac{V^a}{\Lambda} \right) \quad Y_e = \begin{pmatrix} c_{\frac{3}{2}} \frac{3}{3} & 0 & 0 \\ 0 & c_{\frac{3}{2}} \frac{3}{3} & 0 \\ 0 & 0 & y_\tau \end{pmatrix}
\]

\[
Y_d = y_b \left( \frac{\Delta_{\alpha}^a \Phi}{\Lambda^2} \frac{V^a}{\Lambda} \right) \quad \text{Quark masses +} \quad b \to sll
\]

\[ \Lambda \sim 100 \text{ TeV} \]
1: Horizontal flavour symmetries

\[ G = G_{SM} \times G_F \]

Example 3: Simple \( Z' \) models connecting flavour puzzle with the \( b \to sll \) anomalies + absence of LFV!

- \( G_F = U(1)_X; \ X = Y_3 + a(L_2 - L_3) \). Breaking \( U(1)_X \to \text{the } Z' \)

\[ Y_u = y_l \left( \begin{array}{c} \Delta a \Phi \frac{V^a}{\Lambda^2} \\ 0 \\ 1 \end{array} \right) \quad Y_e = \left( \begin{array}{ccc} c_e \frac{g_3}{\Lambda^3} & 0 & 0 \\ 0 & c_\mu \frac{g_3}{\Lambda^3} & 0 \\ 0 & 0 & y_\tau \end{array} \right) \]

Quark masses + \( b \to sll \)

\( \Lambda \sim 100 \text{ TeV} \)

By gauging a combination of lepton numbers, we have excellent protection against LFV, despite LFUV!

\[ \mu \to e\gamma, \text{ due to dim } >12 \text{ operators. Need} \]

\[ \frac{\Lambda}{\sqrt{\epsilon_\Phi}} \frac{a-3}{2} \gtrsim 58000 \text{ TeV} \quad \text{(Satisfied for order-1 WCs)} \]

\[ l_j \to 3l_i, \text{ due to dim } >15 \text{ operators.} \]

\[ \Delta BR(\mu \to 3e) \sim \frac{m_\mu^5}{768\pi^3 \Gamma_\mu} \frac{1}{\Lambda^4} \epsilon^{2a} \lesssim 10^{-29} \quad \text{etc tiny!!} \]

Davighi, 2105.06918
2: Deconstructed gauge symmetries

The SM gauge symmetry, which is flavour-universal, could be \textit{deconstructed} in the UV:

\[ G = G_1 \times G_2 \times G_3 \]

SM Fermions:

\[
\begin{align*}
\psi_1 &\sim (R, 1, 1) \\
\psi_2 &\sim (1, R, 1) \\
\psi_3 &\sim (1, 1, R)
\end{align*}
\]

Multi-scale symmetry breaking pattern generates structure:

The \textbf{hierarchies} can come from different operator dimensions, and/or from \textbf{different scales associated with each family}.

\[ \Lambda_1 > \Lambda_2 > \Lambda_3 \]

The “ladder of scales” does not destabilize Higgs mass! 

Allwicher, Isidori, Thomsen, \texttt{2011.01946}
2: Deconstructed gauge symmetries

$$G = G_1 \times G_2 \times G_3$$

**Example:** 'Pati—Salam cubed' model: Bordone et al. 1712.01368, 1805.09328

$$G_i = [SU(4) \times SU(2)_L \times SU(2)_R]_i$$

The Pati-Salam $SU(4)$ also **unifies quarks and leptons** (puzzle 1!)

Pati, Salam, 1974

Symmetry breaking via the `4-3-2-1 model’:

$$G \rightarrow SU(4)_3 \times SU(3)_{1+2} \times SU(2)_L \times U(1)_Y \rightarrow G_{SM}$$

gives $U_1$ leptoquark coupled mostly to 3rd family

$\rightarrow$ **only** single mediator for both $b \rightarrow sll$ and $b \rightarrow c\tau\nu$ anomalies!

Butazzo et al, 1706.07808
Angelescu et al, 1808.08179

Dvali, Shiffman, hep-ph/0001072
Panico, Pomarol, 1603.06609
Barbieri, 2103.15635
2: Deconstructed gauge symmetries

\[ G = G_1 \times G_2 \times G_3 \]

The origin of deconstruction? Flavour as a 5th dimension, with a 4D brane for each family

Higgs embedded as a composite pNGB!

→ Flavour puzzle & hierarchy problem solved together at TeV scale, while unifying quarks and leptons, consistent with all LHC constraints: Puzzles 1, 3 and 4

Fuentes-Martín, Isidori, Lizana, Selimovic, Stefanek, 2203.01952
3: Unifying gauge and flavour symmetries

None of the previous approaches address Puzzle 2: why 3 families in the first place? (In the 5d model, “why 3 families?” becomes “why 3 branes?”)
None of the previous approaches address Puzzle 2: why 3 families in the first place?

**Gauge-flavour unification**

- **3 generations** of SM fermions start life as *one particle* in the UV
- This UV fermion is acted on by a *large gauge symmetry* $G_{123}$
- So in the UV, different generations are indistinguishable – **opposite to deconstruction** approach!
- At intermediate energies, $G_{123}$ spontaneously broken: *flavour emerges* as a low-energy remnant
Electroweak Flavour Unification

Davighi, Tooby-Smith, 2201.07245
Davighi, 2206.04482
First challenge: embed the SM in an **anomaly-free gauge theory** that unifies the generations

\[ G_{123} = ?? \]
Gauge Flavour Unification

**First challenge:** embed the SM in an anomaly-free gauge theory that unifies the generations

A comprehensive analysis of Lie algebras reveals it is not possible to unify either $SU(5)$ or $SO(10)$ GUT with flavour.

To unify gauge and flavour symmetries, we should start from **Pati-Salam** gauge group:

\[ SU(4) \times SU(2)_L \times SU(2)_R \]

\[ \Psi_L \sim (4, 2, 1)^\oplus^3, \quad \Psi_R \sim (4, 1, 2)^\oplus^3 \]

---

Allanach, Gripaios, Tooby-Smith, 2104.14555

Pati, Salam, 1974
Gauge Flavour Unification

First challenge: embed the SM in an anomaly-free gauge theory that unifies the generations

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To unify gauge and flavour symmetries, we should start from Pati-Salam gauge group:

$$SU(4) \times SU(2)_L \times SU(2)_R$$

$$\Psi_L \sim (4, 2, 1)^{\otimes 3}, \quad \Psi_R \sim (4, 1, 2)^{\otimes 3}$$

Pati, Salam, 1974

Two options:

1. Unify colour and flavour: $SU(4) \rightarrow SU(12)$: $\Psi_L \sim (12, 2, 1), \Psi_R \sim (12, 1, 2)$

2. Unify electroweak and flavour: $SU(2)_L \times SU(2)_R \rightarrow Sp(6)_L \times Sp(6)_R$: $\Psi_L \sim (4, 6, 1), \Psi_R \sim (4, 1, 6)$
Gauge Flavour Unification

First challenge: embed the SM in an **anomaly-free gauge theory** that unifies the generations

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To unify gauge and flavour symmetries, we should start from **Pati-Salam** gauge group:

\[
SU(4) \times SU(2)_L \times SU(2)_R
\]

\[
\Psi_L \sim (4, 2, 1)^\oplus 3, \quad \Psi_R \sim (4, 1, 2)^\oplus 3
\]

Pati, Salam, *1974*

**Reminder:**
The Lie group $Sp(6)$ is a subgroup of $SU(6)$:

\[
Sp(6) = \{U \in SU(6) | U^T \Omega U = \Omega\}, \text{ where } \Omega = \begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}
\]

Allanach, Gripaios, Tooby-Smith, *2104.14555*

Two options:

1. Unify colour and flavour: $SU(4) \to SU(12)$:
   \[
   \Psi_L \sim (12, 2, 1), \quad \Psi_R \sim (12, 1, 2)
   \]

2. Unify electroweak and flavour: $SU(2)_L \times SU(2)_R \to Sp(6)_L \times Sp(6)_R$:
   \[
   \Psi_L \sim (4, 6, 1), \quad \Psi_R \sim (4, 1, 6)
   \]
Electroweak Flavour Unification (EWFU)

- Gauge group: \( G_{123} = SU(4) \times Sp(6)_L \times Sp(6)_R \)

- SM fermions:

\[
\Psi_L \sim (4, 6, 1)^\sim \begin{pmatrix}
    u_1^r & u_2^r & u_3^r & d_1^r & d_2^r & d_3^r \\
    u_1^g & u_2^g & u_3^g & d_1^g & d_2^g & d_3^g \\
    u_1^b & u_2^b & u_3^b & d_1^b & d_2^b & d_3^b \\
    v_1 & v_2 & v_3 & e_1 & e_2 & e_3
\end{pmatrix}, \quad \Psi_R \sim (4, 1, 6)^\sim \text{ similar}
\]
Electroweak Flavour Unification (EWFU)

By unifying all matter, such a gauge theory explains puzzles 1 and 2 “out of the box”.

But with so much unification, can we also explain puzzle 3?

Puzzle 3

Mass hierarchies: $m_3 \gg m_2 \gg m_1$
Small mixing angles: $V_{us} \sim \lambda \sim 0.2$, $V_{cb} \sim \lambda^2$, $V_{ub} \sim \lambda^3$
EWFU: generating the Yukawa structure

Two steps:

1. Flavour **deconstruction** of $G_{123}$ at very high scale

   \[ SU(4) \times Sp(6)_L \times Sp(6)_R \]

   \[ \langle S_{L,R} \rangle \]

2. Break the flavour non-universal intermediate symmetry to $G_{SM}$

   \[ SU(3) \times SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \times Sp(4)_{R,1+2} \times U(1)_R \]

   \[ \langle \Phi_{L,R} \rangle \]

   \[ SU(3) \times SU(2)_L \times U(1)_Y \]

\[ \epsilon_{L}^{12}, \epsilon_{L}^{23}, \epsilon_{R}^{12}, \epsilon_{R}^{23} \]

The scalar sector is almost minimal:

\[ S_L \sim (1, 14, 1), S_R \sim (4, 1, 6), \Phi_L \sim (1, 14, 1), \Phi_R \sim (1, 1, 14) \]
**EWFU: quark masses and mixings**

**Mass hierarchies:** \( m_3 \gg m_2 \gg m_1 \)

**Small mixing angles:** \( V_{us} \sim \lambda \sim 0.2, \quad V_{cb} \sim \lambda^2, \quad V_{ub} \sim \lambda^3 \)

**Puzzle 3**

EWFU model generates Yukawa structures:

\[
\frac{M_f}{v} \sim \begin{pmatrix}
\epsilon_{L}^{12} & \epsilon_{L}^{12} & \epsilon_{L}^{23} & \epsilon_{R}^{23} \\
\epsilon_{R}^{23} & \epsilon_{L}^{12} & \epsilon_{R}^{12} & \epsilon_{R}^{23} \\
\epsilon_{L}^{23} & \epsilon_{R}^{12} & \epsilon_{L}^{23} & \epsilon_{R}^{23} \\
\epsilon_{L}^{23} & \epsilon_{R}^{23} & \epsilon_{R}^{23} & 1
\end{pmatrix}
\]

Extract observables using perturbation theory:

\[
\begin{align*}
m_1 & \sim \epsilon_{L}^{12} \epsilon_{R}^{12} \epsilon_{L}^{23} \epsilon_{R}^{23}, \\
m_2 & \sim \epsilon_{L}^{23} \epsilon_{R}^{23}, \\
m_3 & \sim 1, \\
V_{ub} & \sim \epsilon_{L}^{12} \epsilon_{L}^{23} \\
V_{cb} & \sim \epsilon_{L}^{23} \\
V_{us} & \sim \epsilon_{L}^{12}
\end{align*}
\]
EWFU: quark masses and mixings

Mass hierarchies: \( m_3 \gg m_2 \gg m_1 \)

Small mixing angles: \( V_{us} \sim \lambda \sim 0.2, \ V_{cb} \sim \lambda^2, \ V_{ub} \sim \lambda^3 \)

**Extract observables using perturbation theory:**

\[
\frac{M^f}{v} \sim \begin{pmatrix}
\epsilon_{12}^L \epsilon_{23}^L & \epsilon_{12}^L \epsilon_{23}^R \\
\epsilon_{23}^L \epsilon_{12}^R & \epsilon_{23}^R \\
\epsilon_{12}^R & 1
\end{pmatrix}
\]

**Mixing angles** → \( \epsilon_{12}^L \sim \lambda, \ \epsilon_{23}^L \sim \lambda^2 \)

**Mass hierarchies** → \( \epsilon_{12}^R \sim \lambda^2, \ \epsilon_{23}^R \sim \lambda \)

Corresponds to a ladder of symmetry breaking scales separated by steps of \( \frac{1}{\lambda} \sim 5 \sim O(1) \)

... And there is **enough freedom** in the EFT coefficients to fit all the data.
Work in progress
Phenomenology of EWFU

EWFU offers a new solution to

- **Puzzle 1:** why the peculiar set of 5+1 SM fermion reps?
- **Puzzle 2:** why three copies of each?
- **Puzzle 3:** why is the flavour symmetry broken in such a special way? Mass and mixing angle hierarchies

But what about **puzzle 4**? TeV scale new physics?
Phenomenology of EWFU

But what about puzzle 4? TeV scale new physics?

\[ SU(4) \times Sp(6)_L \times Sp(6)_R \]

Small scale separation, \( \mathcal{O}(10^{-1} - 10^{-2}) \), fixed by Yukawa hierarchy

But what about puzzle 4? TeV scale new physics?

Recall there are two good reasons for \( \nu_\Phi \) to be low (\( \sim \)TeV):

1. Naturalness
2. Persistent anomalies in low-energy data
Phenomenology of EWFU: the 45 gauge bosons

\[ SU(4) \times Sp(6)_L \times Sp(6)_R \]
\[ SU(3) \times SU(2)_{L1} \times SU(2)_{L2} \times SU(2)_{L3} \times Sp(4)_{R1+2} \times U(1)_R \]
\[ SU(3) \times SU(2)_L \times U(1)_Y \]

**Gauge bosons (VERY HEAVY):**
- \( \langle S_L \rangle \): \((W', Z')\) triplets x3
  - \( Z' \sim (1, 1)_0 \) x3
- \( \langle S_R \rangle \): \( U_1 \sim (3, 1)_{2/3} \) leptoquark (flavour universal)
  - \( Z^\pm \sim (1, 1)_1 \) x3
  - \( Z' \sim (1, 1)_0 \) x5

**Gauge bosons (LIGHTER):**
- \( \langle \Phi_L \rangle \): \((W', Z')\) triplets x2
- \( \langle \Phi_R \rangle \): \( Z^\pm \sim (1, 1)_1 \) x3
  - \( Z' \sim (1, 1)_0 \) x4
Phenomenology of EWFU: the 45 gauge bosons

Gauge bosons (LIGHTER):
\[ \langle \Phi_L \rangle: \quad (W', Z') \text{ triplets } \times 2 \]
\[ \langle \Phi_R \rangle: \quad Z^\pm \sim (1, 1)_1 \times 3 \]
\[ Z' \sim (1, 1)_0 \times 4 \]

The lightest gauge bosons are **flavoured versions of EW gauge bosons, LH and RH**. For example, LH:
- one triplet coupled to 1\text{st} and 2\text{nd} families with opposite sign, mass \( m_{12} \sim g_L \epsilon_{L}^{12} \Lambda_H \)
- one triplet coupled to 2\text{nd} and 3\text{rd} families with opposite sign, mass \( m_{23} \sim g_L \epsilon_{L}^{23} \Lambda_H \)

New sources of quark flavour violation and LF(U)V!

What’s next for me?

**Characterise the flavour + high \( p_T \)** pheno of these flavoured EW gauge bosons.
Conclusions

• The existence of 3 generations and the rich Yukawa structure are fascinating puzzles that beg for a BSM explanation.

• Points to new physics coupled predominantly to heavy generations. Reasons for a TeV scale “anchor”: (a) Naturalness of EW sector, (b) hints of new physics in the B anomalies.

• 3 approaches to flavour puzzle, all with TeV scale flavoured new physics:
  1. Horizontal flavour symmetry → flavoured Z’s
  2. Deconstructed Pati—Salam model → flavoured $U_1$ LQ
  3. Electroweak flavour unification → flavoured EW gauge bosons
Conclusions

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A key message:
Continued high $p_T$ searches with heavy flavour final states + continued precision measurements of rare decays will probe all these solutions to the flavour puzzle!
Thank you!