Model independent measurement of the CKM angle $\gamma$
with $B^\pm \to [K^+K^0\pi^+\pi^-]_{D} h^\pm$ at LHCb and BESIII

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Introduction to $CP$ violation
Where is the antimatter in the universe?
Initially equal amounts of matter and antimatter...
... but today we only see matter!
Big Bang and matter-antimatter asymmetry

The difference is very small...

... but the effects we observe today are obviously huge!
How can we explain this?
CP violation

The Nobel Prize in Physics 1980

- CP violation discovery in 1964
- Phys. Rev. Lett. 13, 138
- Observed $K_L^0 \rightarrow \pi^+\pi^-$
- Since, CP violation has also been observed in the $B$, $B_s$ and $D$ systems

Can Standard Model CPV explain the matter-antimatter asymmetry? Or, could it be physics beyond the SM?
The CKM matrix and the Unitary Triangle
In SM, the charged current $W^\pm$ interactions couple (left-handed) up- and down-type quarks, given by

$$\frac{-g}{\sqrt{2}} \left[ \bar{u}_L \, \bar{c}_L \, \bar{t}_L \right] \gamma^\mu W_\mu V_{\text{CKM}} \begin{bmatrix} d_L \\ s_L \\ b_L \end{bmatrix} + \text{h.c.}$$

(a) $t \rightarrow bW^+$

(b) $b \rightarrow cW^-$
The Cabbibo-Kobayashi-Maskawa matrix $V_{\text{CKM}}$, 

$$
\begin{bmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{bmatrix}
$$

must be a unitary matrix: $V_{\text{CKM}}^{\dagger} V_{\text{CKM}} = I \implies$

$$
V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0
$$

Represent this constraint as a triangle in the complex plane: Unitary Triangle
CPV in SM is described by the Unitary Triangle, with angles $\alpha$, $\beta$, $\gamma$.

The angle $\gamma = \arg\left(-\frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}}\right)$ is very important:

1. Negligible theoretical uncertainties: Ideal SM benchmark
2. Accessible at tree level: Indirectly probe New Physics that enter loops
3. Compare with $\alpha$, $\beta$ measurements: Is the Unitary Triangle a triangle?

How to measure $\gamma$?
Measure $\gamma$ through interference effects in $B^\pm \to DK^\pm$

- Superposition of $D^0$ and $\bar{D}^0$
- $b \to u\bar{c}s$ and $b \to c\bar{u}s$ interference $\to$ Sensitivity to $\gamma$

\[
\mathcal{A}(B^-) = \mathcal{A}_B \left( A_{D^0} + r_B e^{i(\delta_B - \gamma)} A_{\bar{D}^0} \right)
\]
\[
\mathcal{A}(B^+) = \mathcal{A}_B \left( A_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)} A_{D^0} \right)
\]

- The magnitude of interference effects governed by $r_B \approx 0.1$

Favoured $B^- \to D^0K^-$

Suppressed $B^- \to \bar{D}^0K^-$
A well known strategy is to consider $D$ decays to a $CP$ eigenstate

For $CP$ eigenstates, $A_{D^0} = A_{D^0}$

$$A_B D^0 K^- \xrightarrow{A_{D^0}} D^0 K^- \xrightarrow{A_B} B^- \xrightarrow{A_B r_B e^{i(\delta_B - \gamma)}} \bar{D}^0 K^- \xrightarrow{A_{D^0}} DK^-$$

$$|A(B^-)|^2 \propto 1 + r_B^2 + 2r_B \cos(\delta_B - \gamma)$$
In $B^\pm \rightarrow [h^+ h^-]_D K^\pm$, we see significant CPV effects.
Doubly Suppressed Cabbibo $D$ decays

Can we enhance the interference effects?

Yes! Use a Doubly Suppressed Cabbibo decay: $A_{D0} = r_D e^{i\delta_D} A_{\bar{D}0}$

\[
\begin{align*}
A_B & \rightarrow D^0 K^- & & r_D e^{i\delta_D} A_{\bar{D}0} \\
B^- & \rightarrow \bar{D}^0 K^- & & A_{\bar{D}0} \\
A_B r_B e^{i(\delta_B - \gamma)} & \rightarrow D^0 K^- & & r_D e^{i\delta_D} A_{\bar{D}0} \\
\end{align*}
\]

\[
|\mathcal{A}(B^-)|^2 \propto r_D^2 + r_B^2 + 2r_B r_D \cos(\delta_B - \gamma + \delta_D)
\]
Doubly Suppressed Cabbibo $D$ decays

$B^{\pm} \rightarrow [K^{\mp}\pi^{\pm}]_{D}K^{\pm}$ has lower statistics, but a spectacular asymmetry!

Additionally, the partially reconstructed background has an equal but opposite asymmetry
The $B^\pm \to [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode
The $B^\pm \to [K^+K^-\pi^+\pi^-]_D K^\pm$ decay mode

The mode $B^\pm \to [K^+K^-\pi^+\pi^-]_D K^\pm$ has been proposed as a powerful channel for a measurement of $\gamma$

- $D \to K^+K^-\pi^+\pi^-$ has the best of both worlds:
  1. Singly Cabbibo Suppressed decay: Larger branching fraction
  2. Interference effects from over 25 resonance components

- Large interference effects in local regions of the 5D phase space

- First proposed by J. Rademacker and G. Wilkinson
  - FOCUS amplitude model predicts a $14^\circ$ precision with 1000 candidates

- State of the art amplitude analysis by LHCb:
  - JHEP 02 (2019) 126
  - Exploits the huge dataset of charm decays collected by LHCb
The $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$ decay mode

Why do four-body decays have large local interferences?

Many possible decay paths, in different phase space locations, contribute to the total decay amplitude...
The $B^\pm \to [K^+K^-\pi^+\pi^-]_D K^\pm$ decay mode

| Amplitude | $|c_k|$ | arg($c_k$) [rad] | Fit fraction [%] |
|-----------|--------|-----------------|-----------------|
| $D^0 \to [\phi(1020)(\rho-\omega)[L=0]$ | 0.614 ± 0.011 ± 0.031 | 1.05 ± 0.02 ± 0.05 | 19.08 ± 0.60 ± 1.46 |
| $D^0 \to [K^-\pi^+][L=0][K^+\pi^-][L=0]$ | 0.282 ± 0.004 ± 0.008 | -0.60 ± 0.02 ± 0.10 | 18.46 ± 0.35 ± 0.94 |
| $D^0 \to [K_1(1270)^+K^-$ | 0.452 ± 0.011 ± 0.017 | 2.02 ± 0.03 ± 0.05 | 18.05 ± 0.52 ± 0.98 |
| $D^0 \to [K^*(892)^0K^*(892)^0][L=0]$ | 0.259 ± 0.004 ± 0.018 | -0.27 ± 0.02 ± 0.03 | 9.18 ± 0.21 ± 0.28 |
| $D^0 \to [K^*(892)^0K^*(892)^0][L=1]$ | 2.359 ± 0.036 ± 0.624 | 0.44 ± 0.02 ± 0.03 | 6.61 ± 0.15 ± 0.37 |
| $D^0 \to [K^*(892)^0K^*(892)^0][L=2]$ | 0.249 ± 0.005 ± 0.017 | 1.22 ± 0.02 ± 0.03 | 4.90 ± 0.16 ± 0.18 |
| $D^0 \to [K_1(1270)^+K^+$ | 0.220 ± 0.006 ± 0.011 | 2.09 ± 0.03 ± 0.07 | 4.29 ± 0.18 ± 0.41 |
| $D^0 \to [K_1(1270)^+K^+$ | 0.236 ± 0.008 ± 0.018 | 0.04 ± 0.04 ± 0.09 | 2.82 ± 0.19 ± 0.39 |
| $D^0 \to [K^*(1680)^0K^*(892)^0][L=0]$ | 0.823 ± 0.023 ± 0.218 | 2.99 ± 0.03 ± 0.05 | 2.75 ± 0.15 ± 0.19 |
| $D^0 \to [K^*(1680)^0K^*(892)^0][L=1]$ | 1.009 ± 0.022 ± 0.276 | -2.76 ± 0.02 ± 0.03 | 2.70 ± 0.11 ± 0.09 |
| $D^0 \to [K^*(1680)^0K^*(892)^0][L=2]$ | 1.379 ± 0.029 ± 0.373 | 1.06 ± 0.02 ± 0.03 | 2.41 ± 0.09 ± 0.27 |
| $D^0 \to [\phi(1020)(\rho-\omega)][L=2]$ | 1.311 ± 0.031 ± 0.018 | 0.54 ± 0.02 ± 0.02 | 2.29 ± 0.08 ± 0.08 |
| $D^0 \to [K^*(892)^0K^*(892)^0][L=2]$ | 0.652 ± 0.018 ± 0.043 | 2.85 ± 0.03 ± 0.04 | 1.85 ± 0.09 ± 0.10 |
| $D^0 \to [\phi(1020)(\pi^+\pi^-)][L=0]$ | 0.049 ± 0.001 ± 0.004 | -1.71 ± 0.04 ± 0.37 | 1.49 ± 0.09 ± 0.33 |
| $D^0 \to [K^*(1680)^0K^*(892)^0][L=1]$ | 0.747 ± 0.021 ± 0.203 | 0.14 ± 0.03 ± 0.04 | 1.48 ± 0.08 ± 0.10 |
| $D^0 \to [\phi(1020)(\rho(1450)^0)][L=1]$ | 0.762 ± 0.035 ± 0.068 | 1.17 ± 0.04 ± 0.04 | 0.98 ± 0.09 ± 0.05 |
| $D^0 \to [\phi(1020)(\rho(1450)^0)][L=2]$ | 1.524 ± 0.058 ± 0.189 | 0.21 ± 0.04 ± 0.19 | 0.70 ± 0.05 ± 0.08 |
| $D^0 \to [\phi(1020)(\rho(1450)^0)][L=1]$ | 0.189 ± 0.011 ± 0.042 | -2.84 ± 0.07 ± 0.38 | 0.46 ± 0.05 ± 0.22 |
| $D^0 \to [\phi(1020)(\rho(1450)^0)][L=2]$ | 0.186 ± 0.014 ± 0.031 | 0.18 ± 0.06 ± 0.43 | 0.45 ± 0.06 ± 0.16 |
| $D^0 \to [\phi(1020)(\rho(1450)^0)][L=3]$ | 0.160 ± 0.011 ± 0.005 | 0.28 ± 0.07 ± 0.03 | 0.43 ± 0.05 ± 0.03 |
| $D^0 \to [K^*(1680)^0K^*(892)^0][L=2]$ | 1.218 ± 0.089 ± 0.354 | -2.44 ± 0.08 ± 0.15 | 0.33 ± 0.05 ± 0.06 |
| $D^0 \to [K^*(1680)^0K^*(892)^0][L=3]$ | 0.195 ± 0.015 ± 0.035 | 2.95 ± 0.08 ± 0.29 | 0.27 ± 0.04 ± 0.05 |
| $D^0 \to [\phi(1020)(f_2(1270))^0][L=1]$ | 1.388 ± 0.095 ± 0.257 | 1.71 ± 0.06 ± 0.37 | 0.18 ± 0.02 ± 0.07 |
| $D^0 \to [K^*(892)^0K^*(1430)^0][L=1]$ | 1.530 ± 0.086 ± 0.131 | 2.01 ± 0.07 ± 0.09 | 0.18 ± 0.02 ± 0.02 |

Sum of fit fractions | 129.32 ± 1.09 ± 2.38 |
χ²/ndf | 9242/8121 = 1.14 |

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... and I really mean a lot of resonances!
The $B^\pm \to [K^+K^-\pi^+\pi^-]_D K^\pm$ decay mode

Our equations suddenly become a lot more complicated.

$\mathcal{A}_{D0}(\Phi)$ now depends on a 5D phase space point $\Phi$.

Defining $\mathcal{A}_{D0} = r_D e^{i\delta_D} \mathcal{A}_{D0}$, $r_D$ and $\delta_D$ are now also functions of $\Phi$!

\[
|\mathcal{A}(B^-)|^2 \propto 1 + r_B^2 r_D^2(\Phi) + 2r_B r_D(\Phi) \cos(\delta_B - \gamma + \delta_D(\Phi))
\]
The $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$ decay mode

$r_D(\Phi)$ and $\delta_D(\Phi)$ can be predicted using the LHCb amplitude model

However, there are many reasons why we should **not** do this:

1. $r_D(\Phi)$ can be measured directly in data at LHCb
2. Amplitude models are just models, which may not reflect reality
3. In fact, the model is fitted to data that knows nothing about $\delta_D(\Phi)$
4. It is **impossible** to assign an objective error to a model!

We wish to do a **model independent** measurement
Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode
Binned analysis of the $D \to K^+K^-\pi^+\pi^-$ mode

- Solution: Split phase space into bins, labelled by $i = 1, 2, ...$
- Study the CP asymmetry separately in each bin
- For the decays $D^0 \to K_S^0\pi^+\pi^-$ and $K_S^0K^+K^-$, the binning scheme may be visualised on a Dalitz plot
Binned analysis of the $D \to K^+K^-\pi^+\pi^-$ mode

Can you find the asymmetries?

$B^- \to [K_S^0\pi^+\pi^-]_D K^-$

$B^+ \to [K_S^0\pi^+\pi^-]_D K^+$
Binned analysis of the $D \rightarrow K^+ K^- \pi^+ \pi^-$ mode

Back to rate equation:

$$|A(B^-)|^2 \propto 1 + r_B^2 r_D^2$$

$$+ 2 r_B r_D \left( \cos(\delta_B - \gamma) \cos(\delta_D) - \sin(\delta_B - \gamma) \sin(\delta_D) \right)$$

Integrate rate over a local region $\Phi_i$, which we call bin $i$:

$$N_i^- \propto F_i + r_B^2 \bar{F}_i$$

$$+ 2 r_B \sqrt{F_i \bar{F}_i} \left( \cos(\delta_B - \gamma) c_i - \sin(\delta_B - \gamma) s_i \right)$$

Amplitude averaged strong phase

$$c_i \equiv \frac{\int_i d\Phi |A_{D0}|^2 |A_{D0}^-| \cos(\delta_D(\Phi))}{\sqrt{\int d\Phi |A_{D0}|^2 \int d\Phi |A_{D0}^-|^2}}$$
Binned analysis of the $D \to K^+ K^- \pi^+ \pi^-$ mode

To “decouple” the interference effects in $B^+$ and $B^-$, define the $CP$ violating observables

$$x_{\pm} \equiv r_B \cos(\delta_B \pm \gamma), \quad y_{\pm} \equiv r_B \sin(\delta_B \pm \gamma)$$

Our final equation, which relates the $CP$ observables to experimentally measured yields, is

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2 \sqrt{F_i \bar{F}_i} (x_- c_i - y_- s_i)$$

Amplitude averaged strong phase

$$c_i \equiv \frac{\int \, d\Phi |A_{D0}| |A_{D0}^\ast| \cos(\delta_D(\Phi))}{\sqrt{\int \, d\Phi |A_{D0}|^2 \int \, d\Phi |A_{D0}^\ast|^2}}$$
Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

Bin yield

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_i c_i - y_i s_i)$$

The strategy for measuring $\gamma$ is now clear:

1. Measure bin yields $N_i^\pm$ in $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$ decays
2. Do a likelihood maximisation to determine $F_i, \bar{F}_i, c_i, s_i, x^\pm$ and $y^\pm$
3. From $x^\pm$ and $y^\pm$, extract $r_B, \delta_B$ and $\gamma$
4. Publish new measurement of $\gamma$!
Strong phase input from charm factories
Unfortunately, it is unlikely that this fit will converge...

Sensitivity to $c_i$ and $s_i$ is very limited with current statistics
Unfortunately, it is unlikely that this fit will converge...

Sensitivity to $c_i$ and $s_i$ is very limited with current statistics

Instead, we can join forces with BESIII and measure $c_i$ and $s_i$ directly

This has never been done for $D^0 \to K^+ K^- \pi^+ \pi^-$

More on this later!
Constraining $F_i$ with $B^\pm \rightarrow D\pi^\pm$
Constraining $F_i$ with $B^\pm \to D^{\pi^\pm}$

- The fractional bin yields $F_i$ are yields in the absence of $CP$ violation.
- In principle we can measure these directly at both LHCb and BESIII.

Four strategies:

1. Calculate from amplitude model.
2. Measure in $B^- \to D^0 \mu^- \bar{\nu}_\mu$ at LHCb.
3. Measure with flavour tagged $D^0$ decays at BESIII.
4. Measure in $B^\pm \to D^{\pi^\pm}$. 
Constraining $F_i$ with $B^\pm \to D\pi^\pm$

- The fractional bin yields $F_i$ are yields in the absence of $CP$ violation.
- In principle we can measure these directly at both LHCb and BESIII.

Four strategies:

1. Calculate from amplitude model. Avoid model dependence.
2. Measure in $B^- \to D^0 \mu^- \bar{\nu}_\mu$ at LHCb.
3. Measure with flavour tagged $D^0$ decays at BESIII.
4. Measure in $B^\pm \to D\pi^\pm$. 
Constraining $F_i$ with $B^\pm \rightarrow D^{\pi^\pm}$

- The fractional bin yields $F_i$ are yields in the absence of $CP$ violation.
- In principle we can measure these directly at both LHCb and BESIII.

Four strategies:

1. **Calculate from amplitude model** Avoid model dependence
2. **Measure in $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$ at LHCb** Different acceptance effects
3. **Measure with flavour tagged $D^0$ decays at BESIII**
4. **Measure in $B^\pm \rightarrow D^{\pi^\pm}$**
Constraining $F_i$ with $B^\pm \to D\pi^\pm$

- The fractional bin yields $F_i$ are yields in the absence of $CP$ violation.
- In principle we can measure these directly at both LHCb and BESIII.

Four strategies:

1. Calculate from amplitude model. Avoid model dependence.
2. Measure in $B^- \to D^0 \mu^- \bar{\nu}_\mu$ at LHCb. Different acceptance effects.
3. Measure with flavour tagged $D^0$ decays at BESIII.
4. Measure in $B^\pm \to D\pi^\pm$. Small CPV effects?

Martin Tat (University of Oxford)
Constraining $F_i$ with $B^{\pm} \to D_{\pi^{\pm}}$

- The fractional bin yields $F_i$ are yields in the absence of $CP$ violation.
- In principle we can measure these directly at both LHCb and BESIII.

Four strategies:

1. Calculate from amplitude model. Avoid model dependence.
2. Measure in $B^- \to D_0^0 \mu^- \bar{\nu}_\mu$ at LHCb. Different acceptance effects.
3. Measure with flavour tagged $D_0$ decays at BESIII.
4. Measure in $B^{\pm} \to D_{\pi^{\pm}}$. Small CPV effects?

No problem, include $B^{\pm} \to D_{\pi^{\pm}}$ as a signal channel.
**Constraining $F_i$ with $B^\pm \to D\pi^\pm$**

- $B^\pm \to D\pi^\pm$ has an identical topology to $B^\pm \to DK^\pm$

- CPV effects are highly suppressed because $r_B^{D\pi} \approx 0.005$

- Branching fraction more than 10 times larger

- As a signal channel, we add another 4 free parameters to our fit:

  \[ x_{\pm}^{D\pi} = r_B^{D\pi} \cos(\delta_B^{D\pi} - \gamma), \quad y_{\pm}^{D\pi} = r_B^{D\pi} \sin(\delta_B^{D\pi} - \gamma) \]
Constraining $F_i$ with $B^\pm \to D_{\pi}^\pm$

To avoid degeneracy, reduce this to 2 additional parameters using this parameterisation:

$$x_\xi = \text{Re}(\xi), \quad y_\xi = \text{Im}(\xi), \quad \xi = \frac{r^D_{\pi} e^{i\delta^D_{\pi}}}{r^D_{K} e^{i\delta^D_{K}}}$$

In summary:

1. Both $B^\pm \to DK^\pm$ and $B^\pm \to D_{\pi}^\pm$ are signal channels, with $x_{DK}^\pm$, $y_{DK}^\pm$, $x_\xi$ and $y_\xi$ as CP observables

2. $B^\pm \to DK^\pm$ has lower statistics, but higher CPV effects

3. $B^\pm \to D_{\pi}^\pm$ has higher statistics and constrain $F_i$ in the fit, but sensitivity to CPV is limited
Binning scheme
We need to split the phase space into bins

But how do we navigate through a 5D space? How do we decide on the bin boundaries?

Let the amplitude model guide us!
Binning scheme

Back to the amplitude averaged strong phase:

\[ c_i \equiv \frac{\int_i d\Phi |A_{D0}| |A_{\bar{D}0}| \cos(\delta_D(\Phi))}{\sqrt{\int d\Phi |A_{D0}|^2 \int d\Phi |A_{\bar{D}0}|^2}} \]

- If the strong phase varies significantly within a bin, the interference effects will be diluted when integrating.
- We need to group regions of similar strong phase into the same bin.
- This was done for \( K^0_S h^+ h^- \), resulting in colourful “butterfly” plots.
Back to our yield formula:

\[ N_i^- \propto F_i + r_B^2 \bar{F}_i + 2 \sqrt{F_i \bar{F}_i} (x_- c_i - y_- s_i) \]

- In the charm system, \( CP \) is (approximately) conserved, so each \( D^0 \) decay has a corresponding identical \( CP \) conjugated decay.
- Split each bin \( i \) into two “\( CP \) mirror bins”, labelled by \( \pm i \).
- In \( K^0_S h^+ h^- \), this is indicated by the black symmetry line.
- Under \( CP \), \( \delta_D \rightarrow -\delta_D \), so \( c_i \rightarrow c_i \) and \( s_i \rightarrow -s_i \).
Binning scheme

A binning scheme must satisfy the following:
- Minimal dilution of strong phases when integrating over bins
- Enhance interference between $B^\pm \to D^0 K^\pm$ and $B^\pm \to \bar{D}^0 K^\pm$

How to bin a 5-dimensional phase space?

1. For each $B^\pm$ candidate, use the amplitude model to calculate

$$\frac{A(D^0)}{A(D^0)} = r_D e^{i\delta_D}$$

2. Split $\delta_D$ into uniformly spaced bins

3. Use the symmetry line $r_D = 1$ to separate bin $+i$ from $-i$

4. Optimise the binning scheme by adjusting the bin boundaries in $\delta_D$
Bin number

\[ \ln(r_D) \]

\[ Q = 0.90 \]

Bins \( i < 0 \) on top, \( i > 0 \) below

\( \Delta \delta_D \) [rad]
Mass fits and yield extraction
In the end, this analysis is a counting experiment

Counting strategy:

1. Perform a “global fit” of all $B^\pm$ candidates
2. Fix all shape parameters
3. Sort $B^\pm$ candidates by charge and bins
4. Perform a “CP fit” simultaneously, but only let bin yields float
5. From the bin yields, determine $x_{\pm}^{DK}$, $y_{\pm}^{DK}$, $x_\xi$ and $y_\xi$
Signal yield:

\[ B^\pm \rightarrow DK^\pm : \quad 3026 \pm 38 \]

\[ B^\pm \rightarrow D\pi^\pm : \quad 44349 \pm 218 \]
Mass fits and yield extraction

**CP fit setup**

- No measurement of $c_i$ and $s_i$ available yet, use model predictions
- Fix mass shape from global fit
- Split by $B^{\pm}$ charge and $D$ phase space bins (64 categories)

1. CP observables $x^{DK}_{\pm}$, $y^{DK}_{\pm}$, $x^{D\pi}_{\xi}$, $y^{D\pi}_{\xi}$ (6 parameters)
2. Fractional bin yields $F_i$ (15 parameters)
3. Low mass and combinatorial background (128 parameters)
4. Yield normalisation (4 parameters)

In total: 153 free parameters
$CP$ fit results and $\gamma$
Useful cross check to compare measured bin asymmetries against bin asymmetries predicted by the fitted CP observables

The $B^\pm \rightarrow DK^\pm$ mode show non-zero bin asymmetries, and the non-trivial distribution is driven by the change in strong phases across phase space.
The $B^\pm \to DK^\pm$ contours are distinct, indicating CP violation

The $B^\pm \to D\pi^\pm$ mode has very low sensitivity to CP violation
Interpretation of $\gamma$

We can interpret our $CP$ observables in terms of the physics parameters $\gamma, r_{B}^{DK}, \delta_{B}^{DK}, r_{B}^{D\pi}, \delta_{B}^{D\pi}$

\[
\begin{align*}
\gamma &= (116^{+12}_{-14})^\circ, \\
\delta_{B}^{DK} &= (81^{+14}_{-13})^\circ, \\
r_{B}^{DK} &= 0.110^{+0.020}_{-0.020}, \\
\delta_{B}^{D\pi} &= (298^{+62}_{-118})^\circ, \\
r_{B}^{D\pi} &= 0.0041^{+0.0054}_{-0.0041},
\end{align*}
\]

However, the latest $\gamma$ and charm combination result is:

\[
\gamma = (63.8^{+3.5}_{-3.7})^\circ
\]

What went wrong?!
Interpretation of $\gamma$

$\gamma = (116^{+12}_{-14})^\circ$

Do we trust the model predicted $c_i$ and $s_i$, or their uncertainties? No!
Let’s go and measure $c_i$ and $s_i$ at BESIII!
Strong phase analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ at BESIII
Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

- BESIII: Beijing Spectrometer III, a detector at the Beijing Electron-Positron Collider II, located at IHEP
- $e^+e^-$ collider at the $\psi(3770) \rightarrow D^0\bar{D}^0$ threshold
  - 2010-2011: 3 fb$^{-1}$
  - 2022: 5 fb$^{-1}$
  - Expect 20 fb$^{-1}$ in total by end of 2024
Strong phase analysis of $D^0 \rightarrow K^+K^−\pi^+\pi^−$ at BESIII

- Double-tag analysis: Reconstruct signal $(KK\pi\pi)$ and tag mode
- $D^0\bar{D}^0$ pair is quantum correlated

![Diagram]

$\psi(3770)$

$D^0 \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0$

- Equivalently, we can consider $D_+D_−$
  - $D_{±} = \frac{1}{\sqrt{2}}(D^0 ± \bar{D}^0)$ are CP eigenstates

![Diagram]

$\psi(3770)$

$D_+ \rightarrow \psi(3770) \rightarrow D_+\bar{D}_−$

The $DD$ pair is quantum correlated, spooky action at a distance!
Tag mode can be a flavour tag

- $K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-\pi^+\pi^-\pi^+$, $K^-e^+\nu_e$

Flavour tags do not exhibit quantum correlation effects
Tag mode can be a CP even tag:
- $KK$, $\pi\pi$, $\pi\pi\pi^0$, $K_S\pi^0\pi^0$, $K_L\pi^0$, $K_L\omega$

$D \rightarrow K^+K^-$, which is CP even, forces $D \rightarrow K^+K^-\pi^+\pi^-$ to be CP odd.
Tag mode can be a CP odd tag

- $K_S\pi^0$, $K_S\omega$, $K_S\eta$, $K_S\eta'$, $K_L\pi^0\pi^0$

\[ D \rightarrow K_S^0\pi^0, \text{ which is } CP \text{ odd, forces } D \rightarrow K^+K^-\pi^+\pi^- \text{ to be } CP \text{ even } \]
Quantum correlation can modify the effective branching fraction:

\[
\frac{N^{DT}}{N^{ST}} = B(D^0 \rightarrow KK\pi\pi)(1 \pm c_1)
\]

\[c_1\] is the cosine of the strong phase, averaged over the whole phase space.
Our next task is to change the phase space inclusive analysis,

\[
\frac{N^\text{DT}}{N^\text{ST}} = B(D^0 \rightarrow KK\pi\pi) \quad \text{(flavour tag)}
\]

\[
\frac{N^\text{DT}}{N^\text{ST}} = B(D^0 \rightarrow KK\pi\pi)(1 \pm c_1) \quad \text{(CP tag)}
\]

into a binned phase space analysis:

\[
\frac{N_i^\text{DT}}{N^\text{ST}} = B(D^0 \rightarrow KK\pi\pi)F_i \quad \text{(flavour tag)}
\]

\[
\frac{N_i^\text{DT}}{N^\text{ST}} = B(D^0 \rightarrow KK\pi\pi)(F_i + \bar{F}_i \pm 2\sqrt{F_i\bar{F}_i}c_i) \quad \text{(CP tag)}
\]

1. \(F_i\): Measure using flavour tags
2. \(c_i\): Determine from asymmetry of \(CP\) even and odd tags
Our next task is to change the phase space inclusive analysis,
\[
\frac{N^{DT}}{N^{ST}} = \mathcal{B}(D^0 \to KK\pi\pi) \quad \text{(flavour tag)}
\]
\[
\frac{N^{DT}}{N^{ST}} = \mathcal{B}(D^0 \to KK\pi\pi)(1 \pm c_1) \quad \text{(CP tag)}
\]
into a binned phase space analysis:
\[
\frac{N^{DT}_i}{N^{ST}} = \mathcal{B}(D^0 \to KK\pi\pi)F_i \quad \text{(flavour tag)}
\]
\[
\frac{N^{DT}_i}{N^{ST}} = \mathcal{B}(D^0 \to KK\pi\pi)(F_i + \bar{F}_i \pm 2\sqrt{F_i\bar{F}_i}c_i) \quad \text{(CP tag)}
\]

1. \(F_i\): Measure using flavour tags
2. \(c_i\): Determine from asymmetry of CP even and odd tags
3. \(s_i\): Analogous to \(c_i\), but requires binning of tag mode
Our next task is to change the phase space inclusive analysis,

\[
\frac{N_{DT}}{N_{ST}} = B(D^0 \to K K \pi \pi) \quad (\text{flavour tag})
\]

\[
\frac{N_{DT}}{N_{ST}} = B(D^0 \to K K \pi \pi \pi \pi) \quad (\text{CP tag})
\]

into a binned phase space analysis:

\[
\frac{N_{i,DT}}{N_{i,ST}} = B(D^0 \to K K \pi \pi) F_i \quad (\text{flavour tag})
\]

\[
\frac{N_{i,DT}}{N_{i,ST}} = B(D^0 \to K K \pi \pi \pi \pi) (F_i + \bar{F}_i \pm 2 \sqrt{F_i \bar{F}_i} c_i) \quad (\text{CP tag})
\]

1. \(F_i\): Measure using flavour tags
2. \(c_i\): Determine from asymmetry of \(CP\) even and odd tags
3. \(s_i\): Analogous to \(c_i\), but requires binning of tag mode
Summary and conclusion

1. I have presented a CPV study of $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D h^\pm$

2. Multi-body decays, such as $D^0 \rightarrow K^+K^-\pi^+\pi^-$, have a great potential for measuring $\gamma$

3. The optimised binning scheme, developed with an amplitude model, successfully identified regions with large, local $CP$ asymmetries

4. However, amplitude model predictions of $\delta_D$ should not be trusted
The fit results, using model predicted strong phases, were found to have a $3\sigma$ tension with the current LHCb combination.

External inputs from charm factories, such as BESIII, are crucial to constrain charm strong phases.

Combined, the LHCb and BESIII analyses will lead to the first model independent measurement of $\gamma$ in this channel.

Work is ongoing in similar four-body modes:
- $D^0 \to \pi^+\pi^-\pi^+\pi^-$
- $D^0 \to K_S^0\pi^+\pi^-\pi^0$

Thanks for your attention!
Backup slides

Backup slides
The LHCb detector
The LHCb detector

LHCb: A beauty experiment with a lot of charm
VELO: Vertex locator to reconstruct $B$ and $D$ vertices
RICH: Identify $B$ and $D$ daughter particles
Event selection

Event selection
Event selection

Decay topology

Look for:

1. 5 charged tracks
2. Displaced $B$ vertex
3. 1 bachelor track with good PID information
4. Displaced $D$ vertex with invariant mass within 25 MeV of the $D^0$ mass

$$B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_{D^0}h^\pm$$
Event selection

Offline selection has 3 stages

Initial cuts:
1. Invariant $D$ and $B$ mass cuts
2. Momentum and RICH requirements

Boosted Decision Tree (BDT)
- Signal sample: Simulation samples
- Background sample: Upper $B$ mass sideband
- 28 variables describing kinematics, impact parameters, vertex quality

Final selection
1. $D$ Flight distance
2. Particle Identification of bachelor
3. $K_S^0$ veto
Event selection

**TMVA overtraining check for classifier: BDTG**

- **Signal (test sample)**
- **Background (test sample)**
- **Signal (training sample)**
- **Background (training sample)**

Kolmogorov-Smirnov test: signal (background) probability = 0.178 (0.02)

BDT is highly efficient at rejecting combinatorial background
Very important, combinatorial background is large in multi-body decays
The invariant $B$ mass, after online selection, show no visible signal...
Event selection

B$^{\pm}$ → DK$^{\pm}$ invariant mass

... but the BDT does a great job cleaning this up!