Solving Beautiful Puzzles

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Testing the Standard Model

Passed all tests up to 100 GeV
Testing the Standard Model

Energy/Direct

Precision/Indirect

Precision frontier

Tiny deviations from SM predictions constrain effects of New Physics
The Flavour Puzzle

- Flavour symmetry broken by Yukawa couplings to the Higgs field
- Origin of mixing between families described by unitary CKM matrix
- Visualized by unitary triangles
- Dominant source of CP violation (antiparticle-particle asymmetry)

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]
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\end{pmatrix}
\]

Our understanding of Flavour is unsatisfactory
\[ \bar{\rho} + i \bar{\eta} = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \]
The Flavour Puzzle

\[ \bar{\rho} + i \bar{\eta} = - \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \]

Huge amounts of data + theory advances = Precision frontier
Tiny deviations from SM predictions constrain effects of New Physics
SM or beyond?

Challenge:
Disentangle SM long-distances effects from the effects of new interactions
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- Some anomalies already spotted
SM or beyond?

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Disentangle SM long-distances effects from the effects of new interactions

- Some anomalies already spotted
- Revise previous assumptions: reliable theory uncertainties
- Look for the cleanest observables/methods
Challenge:
Disentangle SM long-distance effects from the effects of new interactions

Puzzles in semileptonic decays
- Inclusive versus Exclusive
- $V_{cb}$ and $V_{ub}$
- LFUV in $R_D$ and $R_{D^*}$

Puzzles in nonleptonic decays
- Missing CP violation
- $B \to \pi K$ puzzle
- $B \to D\pi$ puzzle

Puzzles in rare decays
- Anomalies in $b \to s\ell\ell$

$V_{cb}$

$V_{ub}$

$b \to s$

$R_{D(*)}$

$CDV$
Puzzles in semileptonic decays: $V_{ub}$ and $V_{cb}$

Inclusive versus Exclusive decays

$V_{ub}$

$V_{cb}$
exclusive versus Inclusive Theory

- Theory (Weak interaction): Transitions between quarks/partons

\[ W \rightarrow \ell^- + \bar{\nu}_\ell \]

\[ b \rightarrow c \]
Exclusive versus Inclusive Theory

- Theory (Weak interaction): Transitions between quarks/partons
- Observation: Transitions between hadrons

Challenge:
- Dealing with QCD at large distances/small scales
- Parametrize fundamental mismatch in non-perturbative objects
  - Calculable: Lattice or Light-cone sumrules
  - Measurable: from data
Inclusive $B \rightarrow X_c \ell \nu$: Heavy Quark Expansion (HQE)

- $b$ quark mass is large compared to $\Lambda_{\text{QCD}}$
- Setting up the HQE: momentum of $b$ quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\rightarrow$ (local) Operator Product Expansion (OPE)

\[
d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \quad d\Gamma_i = \sum_k C_i^{(k)} \langle B | O_i^{(k)} | B \rangle
\]

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B | \ldots | B \rangle$ non-perturbative matrix elements $\rightarrow$ string of $iD$
- Operators contain chains of covariant derivatives

\[
\langle B | O_i^{(n)} | B \rangle = \langle B | \bar{b}_v(iD_{\mu_1}) \ldots (iD_{\mu_n}) b_v | B \rangle
\]

- HQE parameters extracted from lepton energy and hadronic mass moments
Decay rate

\[ \Gamma = \Gamma_0 + \frac{1}{m_b} \Gamma_1 + \frac{1}{m_b^2} \Gamma_2 + \frac{1}{m_b^3} \Gamma_3 \cdots \]

- \( \Gamma_0 \): decay of the free quark (partonic contributions), \( \Gamma_1 = 0 \)
- \( \Gamma_2 \): \( \mu_\pi^2 \) kinetic term and the \( \mu_G^2 \) chromomagnetic moment
  
  \[ 2M_B \mu_\pi^2 = - \langle B | \bar{b}_v iD_\mu iD^\mu b_v | B \rangle \]
  \[ 2M_B \mu_G^2 = \langle B | \bar{b}_v (-i\sigma^{\mu\nu}) iD_\mu iD_\nu b_v | B \rangle \]

- \( \Gamma_3 \): \( \rho_D^3 \) Darwin term and \( \rho_{LS}^3 \) spin-orbit term
  
  \[ 2M_B \rho_D^3 = \frac{1}{2} \langle B | \bar{b}_v [iD_\mu , [ivD , iD^\mu ]] b_v | B \rangle \]
  \[ 2M_B \rho_{LS}^3 = \frac{1}{2} \langle B | \bar{b}_v \{ iD_\mu , [ivD , iD_\nu ] \} (-i\sigma^{\mu\nu}) b_v | B \rangle \]

- \( \Gamma_4 \): 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- \( \Gamma_5 \): 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

\( \Gamma_i \) are power series in \( O(\alpha_s) \)
Moments of the spectrum

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

\[
\langle E^n \rangle_{\text{cut}} = \frac{\int_{E_\ell > E_{\text{cut}}} dE_\ell \ E^n \ \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\text{cut}}} dE_\ell \ \frac{d\Gamma}{dE_\ell}}
\]

Hadronic invariant mass

\[
\langle (M_X^2)^n \rangle_{\text{cut}} = \frac{\int_{E_\ell > E_{\text{cut}}} dM_X^2 \ (M_X^2)^n \ \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\text{cut}}} dM_X^2 \ \frac{d\Gamma}{dM_X^2}}
\]

\[
R^* (E_{\text{cut}}) = \frac{\int_{E_\ell > E_{\text{cut}}} dE_\ell \ \frac{d\Gamma}{dE_\ell}}{\int_0 dE_\ell \ \frac{d\Gamma}{dE_\ell}}
\]

- Moments up to \( n = 3, 4 \) and with several energy cuts available
- Experimentally necessary to use lepton energy cut
State-of-the-art in inclusive $b \rightarrow c$


\[
\Gamma \propto |V_{cb}|^2 m_b^5 \left[ \Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left( \frac{\alpha_s}{\pi} \right)^2 + \Gamma_0^{(3)} \left( \frac{\alpha_s}{\pi} \right)^3 + \frac{\mu_\pi^2}{m_b^2} \left( \Gamma(\pi,0) + \frac{\alpha_s}{\pi} \Gamma(\pi,1) \right) \\
+ \frac{\mu_G^2}{m_b^2} \left( \Gamma(G,0) + \frac{\alpha_s}{\pi} \Gamma(G,1) \right) + \frac{\rho_3^3}{m_b^3} \left( \Gamma(D,0) + \Gamma_0^{(1)} \left( \frac{\alpha_s}{\pi} \right) \right) + \mathcal{O} \left( \frac{1}{m_b^4} \right) + \cdots \right)
\]

- Include terms up to $1/m_b^3$ see also Gambino, Healey, Turczyk [2016]
- Recent progress: $\alpha_s^3$ to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- Recent progress: $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Includes all known $\alpha_s$, $\alpha_s^2$ and $\alpha_s^3$ corrections!

Recent update:

\[
|V_{cb}|^{incl} = (42.16 \pm 0.51) \times 10^{-3}
\]

Towards the ultimate precision in inclusive $V_{cb}$

\[
\Gamma \propto |V_{cb}|^2 m_b^5 \left[ \Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left( \frac{\alpha_s}{\pi} \right)^2 + \Gamma_0^{(3)} \left( \frac{\alpha_s}{\pi} \right)^3 + \frac{\mu^2}{m_b^2} \left( \Gamma(\pi,0) + \frac{\alpha_s}{\pi} \Gamma(\pi,1) \right) \\
+ \frac{\mu_G^2}{m_b^2} \left( \Gamma(G,0) + \frac{\alpha_s}{\pi} \Gamma(G,1) \right) + \frac{\rho_D^3}{m_b^3} \left( \Gamma(D,0) + \Gamma_0^{(1)} \left( \frac{\alpha_s}{\pi} \right) \right) + \mathcal{O} \left( \frac{1}{m_b^4} \right) \right)
\]

Challenge:

- Include higher-order $1/m_b$ and $\alpha_s$ corrections
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109
Alternative $V_{cb}$ determination

- Setting up the HQE: momentum of $b$ quark: $p_b = m_b \nu + k$, expand in $k \sim iD$

- Choice of $\nu$ not unique: Reparametrization invariance (RPI)
  - links different orders in $1/m_b \to$ reduction of parameters
  - up to $1/m_b^4$: 8 parameters (previous 13)
    \[ \delta_{RP} \nu_\mu = \delta \nu_\mu \text{ and } \delta_{RP} iD_\mu = -m_b \delta \nu_\mu \]

Caveat: standard lepton energy and hadronic mass moments are not RPI quantities

Alternative determination using only RPI $q^2$ moments including $1/m_b^4$
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Caveat: standard lepton energy and hadronic mass moments are not RPI quantities

Alternative determination using only RPI $q^2$ moments including $1/m_b^4$

Recent progress: First measurement of $q^2$ moments Belle [2109.01685], Belle II [2205.06372]
$q^2$ moments

Centralized moments as function of $q^2_{\text{cut}}$
\[ R^*(q^2_{\text{cut}}) \langle (q^2)^n \rangle_{\text{cut}} \]

\[ \mu_3, \mu_G, \tilde{\rho}_D, r_E, r_G, S_E, S_B, S_{qB}, m_b, m_c \]

\[ \text{Br}(\bar{B} \rightarrow X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[ \Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} \right. \]
\[ + \Gamma_r \frac{r_E^4}{m_b^4} + \Gamma_r \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \frac{s_{qB}^4}{m_b^4} \]

\[ V_{cb} = (41.69 \pm 0.63) \cdot 10^{-3} \]

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]
New $V_{cb}$ determination

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

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- Independent cross check of previous determinations
  - Agreement at $1 - 2\sigma$ level
  - Difference due to input on branching ratio → Need new measurements!
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- First pure data extraction of $1/m_b^4$ terms
- Important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1}\text{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69)\text{GeV}^4$$
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- Inputs for calculations of $B \rightarrow X_u \ell \nu$, $B$ lifetimes and $B \rightarrow X_s \ell \ell$
$B \rightarrow D$ and $B \rightarrow D^*$

- Form factors extracted from lattice, LC sumrules (+data)
- Knowledge on the $q^2$ dependence crucial
- BGL: Boyd, Grinstein, Lebed or CLN/HQE Caprini, Lellouch, Neubert parametrization
  - Start of many discussions Gambino, Jung, Schacht, Bordone, van Dyck, Gubernari, ...
  - BGL: model independent parametrization using analyticity
  - CLN*: uses HQE at $1/m_b +$ assumptions *justified at time of introduction
- Improved HQE treatment including $1/m_c^2$ corrections Bordone, van Dyk, Jung [1908.09398]

$$|V_{cb}|_{\text{excl}} = (40.3 \pm 0.8) \times 10^{-3}$$
Exclusive $V_{cb}$

$B \rightarrow D$ and $B \rightarrow D^*$

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- Recent progress: $B \rightarrow D^*$ form factors at nonzero recoil Fermilab/MILC [2105.14019]
  - tension between the slope of the lattice and experimental data
- Same form factors determine SM predictions for $R_{D(*)}$
- New experimental and lattice data needed!
The challenge of $V_{ub}$

**Exclusive $B \rightarrow \pi \ell \nu$**

- Only one form factor
- Combining Lattice QCD [FNAL/MILC, RBC/UKQCD] and QCD sum rules

**Recent update:**
Leljak, Melic, van Dyk [2102.07233]

$$|V_{ub}|_{\text{excl}} = (3.77 \pm 0.15) \cdot 10^{-3}$$
The challenge of $V_{ub}$

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- Combining Lattice QCD [FNAL/MILC, RBC/UKQCD] and QCD sum rules

**Recent update:**
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$|V_{ub}|_{\text{excl}} = (3.77 \pm 0.15) \cdot 10^{-3}$

**Inclusive $B \rightarrow X_u \ell \nu$**

- Experimental cuts necessary to remove charm background
- Local OPE as in $b \rightarrow c$ cannot work
- Switch to different set-up using light-cone OPE
- Introduce non-perturbative shape functions ($\sim$ parton DAs in DIS)
- Different frameworks: BLNP, GGOU, DGE, ADFR

**Recent update:**
Belle [2102.00020]

$|V_{ub}|_{\text{incl}} = (4.10 \pm 0.28) \cdot 10^{-3}$
Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem
- In progress: include known $\alpha_s^2$ corrections
- Moments of shape functions can be linked to HQE parameters in $b \to c$
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \to c$
- Shape function is non-perturbative and cannot be computed
  - In progress: new flexible parametrization
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**In progress:**

Gunawardana, Lange, Mannel, Paz, Olschewsky, KKV [in progress]

$|V_{ub}|_{incl} = \text{Stay Tuned!}$
Recently a lot of attention for the $V_{cb}$ puzzle! [Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari]
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Recent progress: $B_s \rightarrow K \mu \nu$ [LHCb [2012.05143], Khodjamirian, Rusov [2017]]

Unlikely to be due to NP Jung, Straub [2018]

New data necessary: stay tuned!
• NP would also influence the moments of the spectrum
• Requires a simultaneous fit of hadronic parameters and NP *In progress.*
Puzzles in nonleptonic decays
The challenge of nonleptonic $B$ decays

- Nonleptonic decays are important probes of CP violation
  - Direct CP violation due to different strong and weak phases
  - Mixing-induced CP violation in neutral decays probe mixing phase $\phi_{d,s}$
  - Sensitivity to NP in loops (penguins)

- CP violation in the SM is too small and peculiar!
  - CKM CP violating effects only from flavour changing currents
  - Flavour diagonal CP violation tiny in SM (EDMs)
  - Large CP asymmetries with processes with tiny BRs and vice versa
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Challenge: Calculation of Hadronic matrix elements
How to handle nonleptonic B decays?

**QCD Factorization** Beneke, Buchalla, Neubert, Sachrajda

- Disentangle perturbative (calculable) and non-perturbative dynamics using HQE
- Systematic expansion in $\alpha_s$ and $1/m_b$ (studied up to $\alpha_s^2$) Bell, Beneke, Huber, Li

\[
\langle \pi^+ \pi^- | Q_i | B \rangle = T^I_i \otimes F^{B \rightarrow \pi^+} \otimes \Phi_{\pi^-} + T^{II}_i \otimes \Phi_{\pi^-} \otimes \Phi_{\pi^+} \otimes \Phi_B
\]

- Non-perturbative form factors and LCDAs
  - from data, lattice or Light-Cone Sum Rules
- No systematic framework to compute power corrections (yet?)
- Strong phases suffer from large uncertainties
- Theoretical challenge: reliable computations of observables
- Include QED corrections Beneke, Boer, Toelstede, KKV [2020]
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**Flavour symmetries (Isospin or $SU(3)$)**

- Many studies e.g. Fleischer, Jaarsma, KKV, Malami [2017,2018]
- Recent progress: Global $SU(3)$ fit to $B \rightarrow PP$ decays Huber, Tetlalmatzi-Xolocotzi [2111.06418]
$B \rightarrow \pi K$ puzzle
**The $B \to K\pi$ Puzzle**

- e.g. Buras, Fleischer, Recksiegel, Schwab [2004, 2007]; Fleischer, Jaeger, Pirjol, Zupan [2008]

(Longstanding) Puzzling patterns in $B \to \pi K$ data

- Penguin dominated; Electroweak penguins contribute at same level as tree!

\[ \delta(\pi K) \equiv A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-) \]

- Recent LHCb measurement for $A_{CP}(K^- \pi^0)$
  LHCb Collaboration, PRL 126, 091802 [2021]

- Confirms and enhances the observed difference
  - $\delta(\pi K)^{\text{exp}} = (11.5 \pm 1.4)\%$
  - $8\sigma$ from 0
The $B \rightarrow K\pi$ Puzzle

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- Not clean!
  - color-suppressed tree contributes
  - $\delta(\pi K)^{\text{QCD}} = (2.1^{+2.8}_{-4.6})\%$ [Bell, Beneke, Huber, Li]
  - or via $SU(3)$ [Fleischer, Jaarsma, Malami, KKV [2018]]
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- Hint for NP in the EWP sector?
Isospin sumrule

\[ \Delta(\pi K) \equiv A_{CP}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^0 K^-) \]

- \[ - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{QCD} + \delta \Delta(\pi K) \]

- Sensitive to new physics effects: \( \Delta(\pi K)^{QCD} = (0.5 \pm 1.1)\% \) [Bell, Beneke, Huber, Li]

- QED effects: \( \delta \Delta(\pi K) = -0.42\% \) [Beneke, Boer, Toelstede, KKV [2020]]

- Isospin sumrule also robust against QED effects!
Isospin sumrule

\[
\Delta(\pi K) \equiv A_{CP}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^0 K^-) \\
- \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{QCD} + \delta\Delta(\pi K)
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- Isospin sumrule also robust against QED effects!
- Updates of modes with neutral pions necessary \(\rightarrow\) Belle II
Isospin sumrule

e.g. Gronau [2005]; Gronau, Rosner [2006]

\[
\Delta(\pi K) \equiv A_{CP}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^0 K^-)
\]

\[
- \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{CP}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{QCD} + \delta\Delta(\pi K)
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- Isospin sumrule also robust against QED effects!
- Updates of modes with neutral pions necessary \(\rightarrow\) Belle II
- Mixing-induced CP asymmetry in \(B \rightarrow \pi^0 K^0\) provides additional test Fleischer, Jaarsma, Malami, KKV [2016,2018]
$B \rightarrow D\pi$ puzzle
Discrepancies between data and theory for $B_s \rightarrow D_s^{+\ast}\pi^-$ and $B \rightarrow D^{+\ast}K^-$ puzzle

- pure tree decays (no color-suppressed nor penguin contributions)
- NNLO predictions in QCDF Huber, Kraenkl [1606.02888]
- Same form factors as for exclusive $V_{cb}$
- Updated and extended calculations give $\sim 4\sigma$ deviation Bordone, Gubernari, Huber, Jung and van Dyk, [2007.10338]

see also Cai, Deng, Li, Yang [2103.04138], Endo, Iguro, Mishima [2109.10811], Gershon, Lenz, Rusov, Skidmore [2111.04478]
Discrepancies between data and theory for $B_s \rightarrow D_s^{+} \pi^-$ and $B \rightarrow D^{+}(*) K^-$

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- QED corrections cannot explain the tension* Beneke, Boer, Finauri, KKV [2107.03819]
- Possible NP explanations have been studied Iguro, Kitahara [2008.01086], Bordone, Grejlo, Marzocca [2103.10332]
- Also puzzling patterns in $B_s \rightarrow D_s K$ are revealed Fleischer, Malami [2110.04240]

Interesting puzzle that requires both experimental and theoretical attention!
The Challenge of QED Corrections
Electromagnetic Effects

\[ \Gamma[\bar{B} \to M_1 M_2](\Delta E) \equiv \Gamma[\bar{B} \to M_1 M_2 + X_s]|_{E_{X_s} \leq \Delta E}, \]

- IR finite observable (width) must include ultra-soft photon radiation
- \( X_s \) are soft photons with total energy less than ultrasoft scale \( \Delta E \)
- Factorizes in non-radiative amplitude and ultrasoft function

\[ \Gamma[\bar{B} \to M_1 M_2](\Delta E) = |A(\bar{B} \to M_1 M_2)|^2 \sum_{X_s} |\langle X_s |(\bar{S}^{(Q_B)} S_{v_1}^{(Q_{M_1})} S_{v_2}^{(Q_{M_2})})|0\rangle|^2 \theta(\Delta E - E_{X_s}) \]

Simple classification:

- Ultra-soft photons: eikonal approximation, well understood
  \[ \Delta E \ll \Lambda_{QCD} \]
- NEW: Non-universal, structure dependent corrections Beneke, Boer, Toelstede, KKV [2020]
- Both effects important: virtual photons can resolve the structure of the meson!
\[ \Gamma[\bar{B} \to M_1 M_2](\Delta E) \equiv \Gamma[\bar{B} \to M_1 M_2 + X_s]_{|E_{X_s} \leq \Delta E}, \]

- IR finite observable (width) must include \textit{ultra-soft photon} radiation
- \(X_s\) are soft photons with total energy less than \textit{ultrasoft scale} \(\Delta E\)
- Factorizes in \textit{non-radiative} amplitude and \textit{ultrasoft} function

\[ \Gamma[\bar{B} \to M_1 M_2](\Delta E) = |A(\bar{B} \to M_1 M_2)|^2 \sum_{X_s} |\langle X_s | (\bar{S}_v^{(Q_B)} S_{v_1}^{(Q_{M_1})} S_{v_2}^{(Q_{M_2})}) |0 \rangle|^2 \theta(\Delta E - E_{X_s}) \]

\textbf{Simple classification:}

- Ultra-soft photons: eikonal approximation, well understood
  \[ \Delta E \ll \Lambda_{QCD} \]

- Often done: Assume pointlike approximation up to the scale \(m_B\) \[Baracchini, Isidori\]
  \[\rightarrow\] fails to account for all large logarithms (and scales)!
  \[\rightarrow\] photons with energy \(\gtrsim \Lambda_{QCD}\) probe the partonic structure of the mesons
Ultrasoft Contribution

- Ultrasoft effects dress branching ratio

\[ U(M_1 M_2) = \left( \frac{2\Delta E}{m_B} \right)^{-\frac{\alpha_{em}}{\pi}} \left( Q_B^2 + Q_{M_1}^2 \left[ 1 + \ln \frac{m_{M_1}}{m_{Bq}} \right] + Q_{M_2}^2 \left[ 1 + \ln \frac{m_{M_2}}{m_B} \right] \right) \]

- Recover the standard eikonal/QED factor Beneke, Boer, Toelstede, KKV [2020]

- \( \Delta E \) is the window of the \( \pi K \) invariant mass around \( m_B \)

- Theory requires \( \Delta E \ll \Lambda_{QCD} = 60 \text{ MeV} \)
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\]

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  \[\rightarrow U(\pi^+K^-) = 0.914, \ U(\pi^0K^-) = U(K^+\pi^0) = 0.976 \text{ and } U(\pi^-\bar{K}^0) = 0.954\]
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- Experimentally usoft effects included using PHOTOS

- Challenging to compare theory with experiment! In progress...
Solving Beautiful Puzzles
We are in the High-precision Era in Flavour Physics!
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- Reached impressive precision
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- Rethink our previous assumptions to reach eXtreme precision
- Many interesting puzzles still to be solved
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- Stay tuned for new data and updated theory predictions
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Close collaboration between theory and experiment necessary!
Backup
Moments of the spectrum

- Tension between the slope of the lattice and experimental data
- Same form factors determine SM predictions for $R_{D(\ast)}$
- New experimental and lattice data needed!
• QED gives sub-percent corrections to Branching ratios
Beneficial to consider ratios in which QCD is suppressed

\[ R_L = \frac{2 \text{Br}(\pi^0 K^0) + 2 \text{Br}(\pi^0 K^-)}{\text{Br}(\pi^- K^0) + \text{Br}(\pi^+ K^-)} = R_L^{\text{QCD}} + \cos \gamma \text{Re} \delta_E + \delta_U \]

new structure dependent QED corrections enter linearly, QCD only quadratically

\[ \delta_E = (-1.12 + 0.16i) \cdot 10^{-3} \]
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\[ \delta_U \equiv \frac{1 + U(\pi^0 K^-)}{U(\pi^- \bar{K}^0) + U(\pi^+ K^-)} - 1 = 5.8\% \]
Ratios and isospin sumrules


- Beneficial to consider ratios in which QCD is suppressed

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- Combined QED effect larger than QCD uncertainty!
**Exclusive $B \rightarrow D^{(*)} \ell \bar{\nu}$**

- Form factor required (only for $B \rightarrow D$ available at different kinematic points)
  - BGL: model independent based on unitarity and analyticity
  - CLN: Simple parametrization using HQE relations
- Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone, Gubernari [2104.02094]

**Inclusive $B \rightarrow X_c \ell \nu$**

- Determined fully data driven including $1/m_b$ power corrections

Recently a lot of attention for the $V_{cb}$ puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari

Stay tuned!
**NP in the \( \tau \) sector**

- Affects also inclusive \( B \to X_c \tau \nu \) Rusov, Mannel, Shahriaran [2017]
- Lepton and hadronic moments challenging to measure
- Recently moments of the five-body decay \( B \to X_c \tau(\to \mu \nu \nu) \nu \) investigated Mannel, Rahimi, KKV [2105.02163]
- Would also be influenced by NP [in progress]
- Specific NP scenarios from global fit Mandal, Murgui, Penuela, Pich [2004.06726]
Contribution from five-body charm decay to $b \rightarrow c \ell \nu$ via

\[ B(p_B) \rightarrow X_c(p_X_c)(\tau(q_\tau)) \rightarrow \mu(q_\mu)\nu_\mu(q_\nu_\mu)\nu_\tau(q_\nu_\tau))\bar{\nu}_\tau(q_{\bar{\nu}_\tau}) \]

- Phase space suppressed:
  \[ \frac{\Gamma_{\text{tot}}(b \rightarrow c \tau(\rightarrow \ell \bar{\nu}_\ell \nu_\tau))}{\Gamma_{\text{tot}}(b \rightarrow c \ell \bar{\nu})} \sim 4.0\% \]
- Experimentally effects diminished by cutting on the invariant mass of the $B$
- Can be calculated exactly in the HQE

\[ \frac{d^8\Gamma}{dq^2 dq_{\nu_\tau}^2 dp_{X_c}^2 d\Omega d\Omega^* d\Omega^{**}} = - \frac{3G_F^2 |V_{cb}|^2 \sqrt{\lambda(q^2 - m_\tau^2)(m_\tau^2 - q_{\nu_\tau}^2)}B(\tau \rightarrow \mu\nu\nu)}{2^{17} \pi^5 m_\tau^8 m_b^3 q^2} W_{\mu\nu} L_{\mu\nu} \]

- $L_{\mu\nu}$ five-body leptonic tensor (narrow-width limit for $\tau$)
- $W_{\mu\nu}$ standard hadronic tensor including HQE parameters

- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]
Shape functions

- Leading order shape functions

\[ 2m_B f(\omega) = \langle B(\nu) | \bar{b}_\nu \delta(\omega + i(n \cdot D)) b_\nu | B(\nu) \rangle \]

- Charged Lepton Energy Spectrum (at leading order)

\[ \frac{d\Gamma}{dy} \sim \int d\omega \theta(m_b(1 - y) - \omega) f(\omega) \]

- Moments of the shape function are related to HQE \( (b \to c) \) parameters:

\[ f(\omega) = \delta(\omega) + \frac{\mu^2}{6m_b^2} \delta''(\omega) - \frac{\rho_D^3}{m_b^3} \delta'''(\omega) + \cdots \]

- Shape function is non-perturbative and cannot be computed
Shape functions

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

\[ d\Gamma = H \otimes J \otimes S \]

\[ \rightarrow H: \text{Hard scattering kernel at } \mathcal{O}(m_b) \]
\[ \rightarrow J: \text{universal Jet function at } \mathcal{O}(\sqrt{m_b \Lambda_{QCD}}) \]
\[ \rightarrow S: \text{Shape function at } \mathcal{O}(\Lambda_{QCD}) \]

- Framework to include radiative corrections (+ NNLL resummation)
- Introduces 3 subleading shape functions
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- Framework to include radiative corrections (+ NNLL resummation)
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- Other approach: OPE with hard-cutoff \(\mu\) Gambino, Giordano, Ossola, Uraltsev
  - Use pert. theory above cutoff and parametrize the infrared
  - Different definition of the shape functions

- Shape functions have to be parametrized and obtained from data
• Too many to count: exclusive $B \to D^{(*)}$ in combination with

$$R_{D^{(*)}} = \frac{B \to D^{(*)}_{\tau\nu}}{B \to D^{(*)}_{\mu\nu}}$$

• For inclusive $b \to c$ less analyses
  - RH-current, scalar and tensor NP contributions to rate Jung, Straub [2018]
  - RH-current to moments Feger, Mannel, et. al. [2010]
  - NP for moments KKV, Fael, Rahimi [in progress]